

Pricing effects of ambiguous private information*

Scott Condie[†] Jayant Ganguli[‡]

November 3, 2011

Abstract

Ambiguous private information need not be revealed by market prices in a rational expectations equilibrium. This partial revelation property is due to inertia with respect to information on the part of the recipient who has non-smooth ambiguity averse preferences. We consider an otherwise standard asset-pricing framework to study such informational inefficiency and provide conditions under which it arises. The asset-pricing implications are the following: (1) informationally inefficient prices may be less volatile than informationally efficient ones, (2) informational inefficiency in prices may lead to discontinuous changes in asset prices and in price volatility even when the volatility of asset fundamentals does not change, and (3) the price impact of a given trade may be larger when prices are informative than when price are uninformative and price impact can change discontinuously. We show how regimes of information revelation and non-revelation can be related to changes in wealth shares, public information, and individual learning. Informational inefficiency does not require the presence of noise and the nature of partial revelation of ambiguous signals differs from partial revelation of signals due to noise.

*This is work in progress and we welcome comments. Acknowledgments to be added.

[†]Department of Economics, Brigham Young University, Provo, UT 84602, USA. Telephone: +1 801 422 5306, Fax: +1 801 422 0194, Email: ssc@byu.edu

[‡]School of Economics, University of Nottingham, Nottingham NG7 2RD, UK. Telephone: +44 (0) 115 8466393, Fax: +44 (0) 115 9514159, Email: jayant.ganguli@nottingham.ac.uk

1 Introduction

Asset markets are continually beset by new information. This includes information about counterparty, sovereign, market and other risks. The quality of this information can vary widely both across asset classes and through time. In addition to allocating ownership rights to assets, markets also convey information through the observation of relative prices for assets.

Information however, is not homogeneous in its usefulness. For instance, suppose a trader learns that a firm has filed a patent application on a potentially profitable technology. While informative about the firm's future prospects and its stock market value, this news is not as useful as the knowledge of the outcome of the application in assessing the likelihood of increases in the firm's value. Information about a patent application has been submitted might properly be modeled as being ambiguous. That is, the trader does not exactly know the probability of success or failure in the patent process and may instead consider the information as being useful about the range of possible probabilities of success.

Another instance is provided by non-traded assets, like labor income, whose payoff is uncertain and correlated with that of traded financial assets. If private information about labor income arrives and is perceived to be ambiguous, due to uncertainty about its source or its systemic or idiosyncratic implications, then this will translate into ambiguous private information about the traded assets.

This paper investigates whether uninformed investors are able to glean such ambiguous information from the traded financial assets' price. If not, under what conditions would such information be revealed and what effect might the nature of this information have on other properties of market prices? Do prices that reveal information behave differently from prices that do not reveal information?

We present a model that answers the first question negatively and provide insights into the answers to the next three questions. We show that ambiguous information need not be revealed in equilibrium and that when it is revealed this occurs because those who receive the signal have sufficient wealth, the ambiguity about the signal is sufficiently small, or the signal is sufficiently extreme.

This has the following implications for asset prices. Informative prices can be more volatile than uninformative prices. Prices may display discontinuous movements and large volatility variation. The price impact of a trade may be larger when

prices are informative than when prices are not informative.

The model we use to investigate the role of market prices as aggregators and communicators of information is that of rational expectations equilibrium (REE) as formalized by Allen (1981), Radner (1979), and Grossman and Stiglitz (1980) among others. Existing analyses of REE are primarily in a framework with investors whose decision-making is modeled by the Savage (1954) and Anscombe and Aumann (1963) subjective expected utility (SEU) framework which does not allow for sensitivity to ambiguity.

As noted by Ellsberg (1961), the SEU framework precludes any role for ambiguity and this has important behavioral implications.¹ Following Ellsberg (1961), several representations for decision-making under ambiguity have been developed.² The Gilboa and Schmeidler (1989) representation with multiple priors is one of the most well-known and studied in this class of models.

Incorporating concern for ambiguity in models of financial markets has provided a number of insights. Epstein and Schneider (2010) is a recent survey of the growing literature on the effects of ambiguity and ambiguity aversion in financial markets, particularly non-smooth ambiguity aversion. As noted there, much of this work has been conducted in the context of representative agent or homogeneous information models.³

In this paper, market participants are allowed to differ in their ambiguity attitude and there is differential information since the ambiguity averse participants receive private information. This information need not be revealed due to the recipient's ambiguity aversion, as modeled by the Gilboa and Schmeidler (1989). This non-smooth ambiguity aversion leads to inertia with respect to information on the part of the recipient and as a consequence, the information may not be revealed. The mechanism is distinct from the portfolio inertia in prices property identified by Dow and da Costa Werlang (1992b), but related since both utilise non-smoothness of the representation.

¹Keynes (1921) and Knight (1921) emphasized the distinction between risky situations with known probabilities and uncertain situations with unknown probabilities.

²It is useful to note that Ellsberg (1961), p.657, describes ambiguity as an informational phenomenon –“a quality depending on the amount, type, reliability and “unanimity” of information, and giving rise to one’s degree of “confidence” in an estimate of relative likelihoods.”

³Chapman and Polkovnichenko (2009) present evidence of how a representative investor framework can give significantly different estimates for the equity premium and risk free rate if heterogeneity present in the underlying economy is ignored.

The model is based on Condie and Ganguli (2011a), who showed that the nature of partial revelation considered here has the desirable property of being robust in the context of general financial market economies. The partial revelation property studied here does not rely on the presence of noise or taste shocks, which are commonly used methods for generating partial revelation in financial market models with unambiguous information. Dow and Gorton (2008) provide a very nice recent discussion of this mechanism and also of mechanisms used in strategic interaction models to generate informationally inefficient prices. We do not discuss the strategic interaction model mechanisms here since we analyse a general equilibrium financial market model with price-taking investors.

As we show, partial revelation of ambiguous information leads to properties of equilibrium price that are different from those found in models of partial revelation with noise traders. In the present model, partial revelation takes the form of a subset of signal values not being revealed with the rest being revealed. The revealed signals are those with relatively extreme values while the non-revealed signals are those with intermediate values. This form of partial revelation implies the possibility of discontinuous price changes and large discontinuous variations in price volatility.

This is in contrast to noise-based partial revelation, where all signal values are obscured by the noise shock in equilibrium and as such the partial revelation does not have implications for price volatility beyond what noise traders add. That is, noise traders contribute to the properties of equilibrium prices in these models but in a way that is qualitatively similar to how noise traders would alter prices in a setting without private information. Overall, the differences in partial revelation due to ambiguous information and noise-based partial revelation suggest that in principle, these may forms of partial revelation may be useful in different ways, possibly even complementary, in studying financial markets.

There is a small but growing literature examining the informational efficiency of financial market prices in the presence of ambiguity. This includes Tallon (1998), Caskey (2008), Ozsoylev and Werner (2011), Mele and Sangiorgi (2011), Easley, O'Hara, and Yang (2011), Condie and Ganguli (2011a), and Condie and Ganguli (2011b). However, in these papers except the last three, any partial revelation property is driven by noise traders.⁴

⁴de Castro, Pesce, and Yannelis (2010) define a new concept called maximin rational expectations equilibria and prove universal existence, incentive compatibility, Pareto efficiency of these equilibria.

Smooth representations of ambiguity averse preferences such as in Klibanoff, Marinacci, and Mukerji (2005), Maccheroni, Marinacci, and Rustichini (2006), and Hansen and Sargent (2007) will not generate the partial revelation we study here since these will not exhibit inertia with respect to information. The experimental work of Ahn, Choi, Kariv, and Gale (2011) and Bossaerts, Ghirardato, Guarneschelli, and Zame (2010) also provide persuasive evidence in support of non-smooth models of ambiguity aversion in financial market settings.

The paper proceeds as follows. We first develop the financial market model in section 2. Section 3 describes the nature of partial revelation in this framework and the conditions needed for partial revelation to be possible. Section 5 discusses some comparative statics which illustrate the properties of this form of partial revelation further. Section 4.1 discusses the implications for price volatility and swings. Section 6 discusses how the model of section 2 can be used to think about ambiguous information to non-tradeable labor income and section 7 concludes.

2 A model of ambiguous private information

There are two types of investors indexed by $n \in \mathcal{N} = \{A, E\}$ who live for 3 periods and trade assets in the market. Time is indexed by $t = 0, 1, 2$.

There is one asset whose payoff is certain and denoted by V_f , called the risk-free asset or bond.⁵ This asset is in zero net supply. There is another asset whose payoff or terminal value denoted by V is uncertain and it is assumed to have unit net supply. Each investor n is endowed with a fraction $x_0^n > 0$ of the uncertain asset at time 0. Trade occurs in period 1 with the resolution of uncertainty occurring in period 2.

We assume that $\ln V$, denoted v henceforth, is normally distributed with mean μ and variance σ^2 . In period 0 all investors have identical information about the expected value of the uncertain assets. However, the two types of traders differ in their perception of information as we describe next.

Prior beliefs of A- and E-investors. Both types of investors believe that v is normally distributed with variance σ^2 . Both types are uncertain about the mean of v and their beliefs over μ are given by a normal distribution that has mean μ_0 and precision ρ_0 .

⁵It would perhaps be more appropriate to use the term ‘uncertainty-free’ to describe this asset in our setting, but we stay with the usual terminology.

Private and public information. However, in period 0 type A investors receive *a private signal* that conveys information about the mean μ of the log payoff. The signal takes the form $s = \mu + \epsilon$, where ϵ is a stochastic error term.⁶ The signal is interpreted differently by the A investors and the E-investors, if the latter observe it. This differential interpretation is related to the signal error term ϵ .

Both types of investor agree that the signal error ϵ is distributed normally with precision ρ_ϵ . However, they have different assessments of the mean μ_ϵ of the error term. Type A investors perceive *ambiguity in the signal* in the sense that they know only that $\mu_\epsilon \in [-\delta, \delta]$ where $\delta > 0$. We denote the A investors' assessment of the mean by μ_ϵ^A .

This ambiguity in the signal reflects the possibility that the signal provides biased information about the payoff of the asset. A investors may doubt the unbiasedness of a signal because of concerns about the signal source, because the information is intangible in the sense of Daniel and Titman (2006), or because the relationship between the signal and the asset is ambiguous, among other possibilities (see for example, the discussion in Epstein and Schneider (2008) and Illeditsch (2010)).⁷ As we discuss in section 6, ambiguous private information about a non-traded asset like labor income, whose payoff is correlated with that of the stock will also lead to a signal structure like that above.

On the other hand, type E investors believe the signal is unbiased, i.e. $\mu_\epsilon = 0$. We denote the E-investors' assessment of the mean by $\mu_\epsilon^E (= 0)$. We are interested in how ambiguity in the signals perceived by A investors affects the informational efficiency of prices and what implications this has for financial market variables.

In addition to the private signals received by A investors, all investors may receive information from *a public signal* as well. This signal takes the form $\zeta = \mu + \epsilon_\zeta$, where ϵ_ζ is normally distributed with mean 0 and precision ρ_ζ . For simplicity, we assume that there is no ambiguity in the public signal given that ambiguity is present in the private signals. Moreover, since we focus on the effects of partially-revealing prices that do not reveal ambiguous private signals and the public signal is public, this additional generality is not added.

⁶A similar signal structure without ambiguity appears in Peress (2009) relating to the analysis of Peress (2004).

⁷In Epstein and Schneider (2008) and Illeditsch (2010), ambiguity in the signals is captured through an interval of signal variances, rather than through an interval of signal means as done here.

2.1 Decision making

The investors maximize the expected utility of terminal wealth W_2 . Their von Neumann-Morgenstern utility, denoted by u^n , is in the constant relative risk aversion class (CRRA) with common CRRA coefficient γ , i.e.

$$u^n(W_2) = \frac{W_2^{1-\gamma}}{1-\gamma}. \quad (1)$$

Since the A investors receive signals which they perceive as ambiguous, their updated beliefs will not be represented by a single probability distribution and will instead be represented by a set of distributions as we discuss in section 2.1.1.

If investors perceive ambiguity after incorporating all information from private and public signals and from prices, their decision-making is modeled using the Gilboa and Schmeidler (1989) representation. Denoting by M^n the set of distributions representing investor n 's beliefs given his information, the utility from a portfolio θ^n is

$$U^n(\theta^n) = \min_{m \in M^n} \mathbb{E}_m[u^n(W_2)] = \min_{m \in M^n} \mathbb{E}_m \left(\frac{W_2^{1-\gamma}}{1-\gamma} \right) \quad (2)$$

This representation includes the case of E-investors who do not perceive any ambiguity. In this case, M^E is a single probability distribution and the utility $U^E(\cdot)$ corresponds to the Savage (1954) and Anscombe and Aumann (1963) expected utility representation.

This is a non-smooth representation of decision-making under ambiguity and it has been fruitfully used to study a wide array of financial market phenomena (see for example, the discussion in Epstein and Schneider (2010)). In the present case, building on the result of Condie and Ganguli (2011a), we use the non-smoothness to construct partially revealing rational expectations equilibria different from those obtained via the assumption of noise trading, endowment shocks, or taste shocks or the assumption of higher-dimensional private information. Dow and Gorton (2008) provide a recent discussion of the noise and endowment and taste shocks mechanisms, while Ausubel (1990) presents an application of partial revelation due to higher-dimensional private information.⁸

⁸Allen and Jordan (1998) provide an extensive discussion of the results on existence of rational expectations equilibria, fully or partially revealing, covering the higher, equal, and lower dimensional cases for smooth models of preferences.

In the present set up, the utility U^n is everywhere differentiable except when the terminal wealth from portfolio holdings is not uncertain, i.e. when the investor trades away his holdings of the stock and holds only the risk-free asset. These positions will be key to our analysis since the utility is non-differentiable at this value, which in turn is key for the partial revelation equilibria in the present model as we discuss in section 3.⁹

There has also been work on smooth representations of ambiguity averse preferences, inter alia, by Klibanoff, Marinacci, and Mukerji (2005), Maccheroni, Marinacci, and Rustichini (2006), and Hansen and Sargent (2007). These representations will not generate the partial revelation we study here. The reason for this is very similar to that for standard (smooth) Savage (1954) and Anscombe and Aumann (1963) expected utility preferences which do not exhibit sensitivity to ambiguity. The smoothness of these preferences imply that market-clearing prices in markets populated by only traders with such preferences will always respond to changes in private information, which rules out the possibility of the partial revelation we study here as we further clarify in section 3. See also, Radner (1979), Grossman (1981), Allen and Jordan (1998), and Condie and Ganguli (2011a) for closely related analysis and discussion.

2.1.1 Information and updating

In order to understand the nature of partial revelation in this model, we must first specify how information is incorporated into the beliefs of investors. In the present framework, information is processed and incorporated using an updating rule developed in Epstein and Schneider (2007) and Epstein and Schneider (2008). This updating rule includes standard Bayesian updating with unambiguous beliefs as a special case.

In Bayesian updating with unambiguous beliefs, conditional beliefs about the probability of an uncertain event C are found through the use of a prior distribution that reflects the beliefs of the decision maker prior to receiving the information and a likelihood function that expresses the relationship between the signal and the parameter that the decision maker uses in making decisions. In particular, assume that the probability of the event C depends on a parameter B over which the decision

⁹Though we will not explore this here, other portfolio positions where utility is non-differentiable could be used for studying the kind of partial revelation we present here.

maker has a prior denoted $Pr(B)$. Suppose further that given a parameter value B_0 , the likelihood of receiving a signal s is given by $L(s|B_0)$. Bayes' rule then indicates that the updated distribution of B conditional on having observed the signal s is

$$Pr(B|s) = \frac{Pr(B)L(s|B)}{\int L(s|B)dB}. \quad (3)$$

Epstein and Schneider (2007) consider the set of possible Bayes updates that arise from the set of possible likelihoods and a prior. If the prior is $Pr(B)$ and the set of likelihoods is $\{L(s|B)\}_{L \in \mathcal{L}}$ for some index set \mathcal{L} , then the set of updated beliefs is given by

$$\{Pr(B|s)\} = \left\{ \frac{Pr(B)L(s|B)}{\int L(s|B)dB} \middle| L \in \mathcal{L} \right\} \quad (4)$$

For the model with ambiguous beliefs presented here, the prior over the mean of v is a normal distribution with mean μ_0 . The set of likelihoods is the set of normal distributions with mean $\mu_0 + \mu_\epsilon$ with μ_ϵ in the set $[-\delta, \delta]$.

Standard results on Bayesian updating with normal distributions imply that given μ_0 and $\mu_\epsilon \in [-\delta, \delta]$, the mean of v , conditional on having observed the signal s is normally distributed with mean

$$\mu|s = \frac{\rho_0\mu_0 + \rho_\epsilon(s + \mu_\epsilon)}{\rho_0 + \rho_\epsilon} \quad (5)$$

and precision

$$\rho|s = \rho_0 + \rho_\epsilon. \quad (6)$$

Therefore, the set of updated priors representing the ambiguity of an investor is the set of normal distributions with precision $\rho_0 + \rho_\epsilon$ and means

$$\{\mu|s\} = \left\{ \frac{\rho_0\mu_0 + \rho_\epsilon(s + \mu_\epsilon)}{\rho_0 + \rho_\epsilon} \middle| \mu_\epsilon \in [-\delta, \delta] \right\} \quad (7)$$

This can be extended to the case of observing both private and public signals. The updated set of priors is the set of all normal distributions with precision $\rho_0 + \rho_\epsilon + \rho_\zeta$ and means

$$\{\mu|s, \zeta\} = \left\{ \frac{\rho_0\mu_0 + \rho_\zeta\zeta + \rho_\epsilon(s + \mu_\epsilon)}{\rho_0 + \rho_\zeta + \rho_\epsilon} \middle| \mu_\epsilon \in [-\delta, \delta] \right\} \quad (8)$$

In what follows, we will denote the set of distributions after observing the private

signal s and public signal ζ by $M^n(s)$ for investor n . Given the assumptions about prior beliefs and signals, notice that this set can be indexed by the interval $[\underline{\mu}|s, \bar{\mu}|s]$, which is the interval of means of beliefs about μ as defined above (equations (7) and (8)). We will usually suppress the dependence on the public signal for simplicity of exposition since it is observed by all investors.

2.2 Market prices and rational expectations equilibria

Trade in the assets occurs in period 1 and equilibrium requires that markets for all assets clear. Market prices play the role of information aggregators and communicators through a price function.

A price function \mathbf{P} maps signal values s to prices, i.e. $\mathbf{P}(s) = (P(s), R_f(s))$, where $P(\cdot)$ denotes the price of the uncertain asset while $R_f(\cdot)$ denotes the gross return on the risk-free asset. Information is revealed through prices when the prevailing market prices under two signals that convey different information are different, i.e. the function \mathbf{P} is invertible. When this occurs for all signals, market participants can correctly infer the signal by observing the prices in the market and the price function \mathbf{P} is said to be *fully-revealing*.

The market price may not reveal all privately held information if the function from signal information into equilibrium prices is not invertible. In this case, the function is said to be *partially revealing*. When prices are partially revealing, more than one signal may be consistent with the observed price. Upon observing the market prices (P, R_f) , each investor knows that the signal s is in the set $\mathbf{P}^{-1}(P, R_f)$.

The holdings of investor n in the uncertain and risk-free assets are x_t^n and b_t^n , $t = 0, 1$, respectively with $b_0^n = 0$. Hence, initial wealth for investor n at price P is $W_0^n = x_0^n P$, whereas the terminal or period 2 wealth of investor n at time 2 given choices in period 1 is $W_2^n = x_1^n V + b_1^n V_f$. The fraction of wealth put into the risky asset at time 1 is labeled θ_1^n . By definition

$$x_1^n = \frac{\theta_1^n W_0^n}{P} \quad (9)$$

The market clearing conditions for the assets are

$$\begin{aligned} \sum_n \frac{\theta_1^n}{P} W_0^n &= 1 \\ \sum_n (1 - \theta_1^n) R_f W_0^n &= 0. \end{aligned} \quad (10)$$

We now provide a definition of rational expectations equilibrium (REE) for this setting.

Definition 1. A rational expectations equilibrium is a set of portfolio weights $\{\theta_1^n(s)\}_{n \in \mathcal{N}}$ and a price function \mathbf{P} , which specifies prices $P(s)$ and $R_f(s)$ for each signal s such that the following hold almost surely.

1. Each investor A has information s and $\mathbf{P}^{-1}(P(s), R_f(s))$ and chooses a portfolio θ_1^A that satisfies

$$\theta_1^A \in \operatorname{argmax} U^A(\theta | s, \mathbf{P}^{-1}(P(s), R_f(s))) \quad (11)$$

2. Each investor E has information $\mathbf{P}^{-1}(P(s), R_f(s))$ and chooses a portfolio θ_1^E that satisfies

$$\theta_1^E \in \operatorname{argmax} U^E(\theta | \mathbf{P}^{-1}(P(s), R_f(s))) \quad (12)$$

3. The market clearing equations given in (10) are satisfied.

Given this definition an REE is said to be *fully revealing* when the equilibrium price function is fully revealing and it is said to be *partially revealing* otherwise.

2.3 An approximate solution for investor demand

To solve this model, we will first solve for investor demand by using the method developed by Campbell and Viciera (2002) to approximate asset payoffs. As noted there, this solution method becomes exact as the discrete time interval shrinks to zero. We discuss here the approximation as applicable to type A investors, since this covers the case of type E investors also. Let M^n denote the set of distributions that represent the beliefs of investor n conditional on any information that she may have received. Let σ^2 denote the conditional variance of the investor's log portfolio payoff.

We begin by approximating the return on initial wealth W_0^n as a function of the returns to the individual assets. Throughout, lowercase letters represent the natural log of model variables. Given n 's portfolio $(\theta_1^n, 1 - \theta_1^n)$ and using $R = V/P$ to denote the return on the uncertain asset,

$$W_2^n = W_0^n(\theta_1^n R + (1 - \theta_1^n)R_f). \quad (13)$$

Expressing the returns in natural logs implies

$$\begin{aligned}\ln \frac{\theta_1^n R + (1 - \theta_1^n) R_f}{R_f} &= \ln \left(\theta_1^n \frac{R}{R_f} + 1 - \theta_1^n \right) \\ &= \ln(1 + \theta_1^n (e^{r-r_f} - 1))\end{aligned}\tag{14}$$

We then form a second order Taylor series approximation around the point $r - r_f = 0$ to get

$$\theta_1^n (r - r_f) + \frac{1}{2} \theta_1^n (r - r_f)^2.\tag{15}$$

We use the unconditional expectation of the second-order term and obtain

$$\theta_1^n (r - r_f) + \frac{1}{2} \theta_1^n (1 - \theta_1^n) \sigma^2\tag{16}$$

as our approximation of returns. This function is the lognormally distributed function to the actual market return.

If terminal wealth W_2^n is lognormally distributed then the solution to the individual's optimization problem is equivalent to the solution to

$$\max_{\theta} \min_{m \in M^n} \ln \mathbb{E}_m \left[\frac{(W_2^n)^{1-\gamma}}{1-\gamma} \right].\tag{17}$$

The term $\ln \mathbb{E}_m [(W_2^n)^{1-\gamma}]$ by the lognormality of W_2^n can be rewritten as

$$\begin{aligned}\mathbb{E}_m \ln(W_2^n)^{1-\gamma} + \frac{1}{2} \text{Var} \ln(W_2^n)^{1-\gamma} &= \\ (1-\gamma) \mathbb{E}_m w_2^n + \frac{1}{2} (1-\gamma)^2 \text{Var} \ln W_2^n &= \\ (1-\gamma) \mathbb{E}_m w_0^n + \ln(\theta_1^n R + (1 - \theta_1^n) R_f) + \frac{1}{2} (1-\gamma)^2 \sigma^2.\end{aligned}\tag{18}$$

Since w_0^n and r_f are both non-stochastic and $1 - \gamma$ is a scale factor that won't affect the solution to the problem, solving the optimization problem is equivalent to solving

$$\max_{\theta} \min_{m \in M^n} \mathbb{E}_m \ln(\theta R + (1 - \theta) R_f) - r_f + \frac{1-\gamma}{2} \sigma^2.\tag{19}$$

Using the approximation given in (16), we can rewrite the optimization problem

as

$$\max_{\theta} \min_{m \in M^n} \mathbb{E}_m \theta (r - r_f) + \frac{\theta(1-\theta)}{2} \sigma^2 + \frac{(1-\gamma)\theta^2}{2} \sigma^2. \quad (20)$$

The first order conditions for investor n are given by

$$\begin{aligned} 0 &= \min_{m \in M^n} \mathbb{E}_m r - r_f + \frac{1}{2} \sigma^2 - \gamma \theta \sigma^2 & \text{if } \theta > 0 \\ 0 &\in \{ \mathbb{E}_m r - r_f + \frac{1}{2} \sigma^2 : m \in M^n \} & \text{if } \theta = 0 \\ 0 &= \max_{m \in M^n} \mathbb{E}_m r - r_f + \frac{1}{2} \sigma^2 - \gamma \theta \sigma^2 & \text{if } \theta < 0. \end{aligned} \quad (21)$$

Therefore, investor n will not hold a position in the risky asset if and only if

$$\min_{m \in M^n} \mathbb{E}_m v - r_f + \frac{1}{2} \sigma^2 - p \leq 0 \leq \max_{m \in M^n} \mathbb{E}_m v - r_f + \frac{1}{2} \sigma^2 - p. \quad (22)$$

Using $[\underline{\mu}^n, \bar{\mu}^n]$ to denote the interval of means given by the set of distributions M , investor n will not hold the uncertain asset if

$$\underline{\mu}^n - r_f + \frac{1}{2} \sigma^2 \leq p \leq \bar{\mu}^n - r_f + \frac{1}{2} \sigma^2 \quad (23)$$

This implies that the optimal portfolio weight for the uncertain asset is given by

$$\theta_1^n(M^n) = \begin{cases} \frac{1}{\gamma \sigma^2} \left(\underline{\mu}^n - r_f + \frac{1}{2} \sigma^2 - p \right) & \underline{\mu}^n - r_f + \frac{1}{2} \sigma^2 - p > 0 \\ 0 & \underline{\mu}^n - r_f \leq p - \frac{1}{2} \sigma^2 \leq \bar{\mu}^n - r_f \\ \frac{1}{\gamma \sigma^2} \left(\bar{\mu}^n - r_f + \frac{1}{2} \sigma^2 - p \right) & \bar{\mu}^n - r_f + \frac{1}{2} \sigma^2 - p < 0 \end{cases} \quad (24)$$

In the above expression, note that the case of $\underline{\mu}^n - r_f \leq p - \frac{1}{2} \sigma^2 \leq \bar{\mu}^n - r_f$ corresponds to a situation where the investor trades from his non-zero initial stock position to a zero position in the stock. Thus, this demand does involve trading and is not a no-trade position.

Since we can work with relative prices, we normalize $R_f = 1$, i.e. $r_f = 0$ hereafter and work with the stock price P , with $p \equiv \ln P$ in analysing rational expectations equilibrium prices.

3 Partial revelation

We use $[\underline{\mu}|s, \bar{\mu}|s]$ to denote the updated interval of means using the rule given in (8), where

$$\underline{\mu}|s = \frac{\rho_0\mu_0 + \rho_\epsilon(s - \delta)}{\rho_0 + \rho_\epsilon} \text{ and } \bar{\mu}|s = \frac{\rho_0\mu_0 + \rho_\epsilon(s + \delta)}{\rho_0 + \rho_\epsilon}. \quad (25)$$

For ease of exposition in this section, we will not consider the receipt of a public signal.

Using the above and the optimal portfolio expression provided in (24), the demand for the uncertain asset from type A traders is

$$\theta_1^A(s) = \begin{cases} \frac{1}{\gamma\sigma^2} \left(\underline{\mu}|s + \frac{1}{2}\sigma^2 - p \right) & \underline{\mu}|s + \frac{1}{2}\sigma^2 - p > 0 \\ 0 & \underline{\mu}|s \leq p - \frac{1}{2}\sigma^2 \leq \bar{\mu}|s \\ \frac{1}{\gamma\sigma^2} \left(\bar{\mu}|s + \frac{1}{2}\sigma^2 - p \right) & \bar{\mu}|s + \frac{1}{2}\sigma^2 - p < 0 \end{cases} \quad (26)$$

The above expression exhibits two interesting and complementary facts about the demand θ^A of A investors. The first is that for any given signal value s , there exists a range of prices for which it is optimal for A investors to trade away their stock holdings to a zero position ($\theta^A = 0$). This fact was first noted by Dow and da Costa Werlang (1992b) as portfolio inertia with respect to prices.

The second fact is that for a fixed price p , the A investors will still find it optimal to trade to a zero position even if they observe a different signal $s' \neq s$ instead of s . As we show below, this inertia with respect to information can lead to the existence of a partially revealing rational expectations equilibrium price.¹⁰ Note also that this inertia does not appear in smooth models of ambiguity averse preferences and so these will not lead to the partial revelation property we study here.

Recall that non-revelation of signals s and s' requires that the price is the same for both, i.e. $P(s) = P(s') = P$. Given the optimal portfolio expression provided in (24), at price P (with $p = \ln P$), investor A will trade to a zero-position in the uncertain asset under both signal realisations s and s' if $[\underline{\mu}|s, \bar{\mu}|s] \cap [\underline{\mu}|s', \bar{\mu}|s']$ is non-empty and

$$\max\{\underline{\mu}|s, \underline{\mu}|s'\} \leq p - \frac{1}{2}\sigma^2 \leq \min\{\bar{\mu}|s, \bar{\mu}|s'\}. \quad (27)$$

¹⁰Condie and Ganguli (2011a) use this fact in the context of general financial market exchange economies to establish robust existence of partially revealing rational expectations equilibria.

This observation is key in the existence of partially-revealing equilibria with ambiguous information and is where the property of inertia with respect to information under non-smooth preferences will be utilised.

3.1 A benchmark: unambiguous information and revelation

To begin, it is useful to consider the case of unambiguous information first and note that in this case, private information will (almost surely) be revealed by the market price. This is the message of Grossman (1976), Grossman (1981) and Radner (1979) among others. We present the analysis here to clarify the role of ambiguous information in partial revelation and it will be further useful in discussing the conditions under which ambiguous information is not revealed.

Suppose that all information is unambiguous and so each investor's beliefs can be represented by a single probability distribution.¹¹ That is, suppose A investors also believe the signal is unbiased, $\mu_\epsilon^A = 0$.¹²

In this case, there is no ambiguity and their updated belief about the mean of v is given by a normal distribution with precision $\rho_0 + \rho_\epsilon$ and mean

$$\underline{\mu}|s = \bar{\mu}|s = \mu|s = \frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon}. \quad (28)$$

At stock price P , with $p \equiv \ln P$, the demand of A investors is then

$$\theta_1^A(s) = \frac{1}{\gamma\sigma^2} \left(\frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} + \frac{1}{2}\sigma^2 - p \right). \quad (29)$$

Since $\theta_1^A(s)$ varies (linearly) with s , this will imply that the rational expectations equilibrium market-clearing price will reveal s since it will vary with s monotonically. To see this, suppose first that E-investors do not use any information from price to update their beliefs, i.e. their demand is given by $\theta^E = \mu_0 + 0.5\sigma^2 - p$.

Using the market clearing condition (10) for the stock, the price $p^1(s)$, is

$$\left(x_0^A \frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} + x_0^E \mu_0 \right) + \frac{1-2\gamma}{2}\sigma^2 = p^1(s), \quad (30)$$

¹¹Equivalently, consider a market that begins with all investors having homogeneous information and participating in the market.

¹²As will be clear, the same reasoning applies for any fixed value of $\mu_\epsilon^A \neq 0$.

which is linear in s .

Thus observing this price reveals the signal s to the uninformed investors E. As envisaged in the rational expectations equilibrium framework E-investors then use this information to update their beliefs, which in turn means that their demand is now given by $\theta^E(s) = \frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} + 0.5\sigma^2 - p$.

Using the market clearing condition (10) for the stock again, the price $p^0(s)$ is

$$\frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} + \frac{1 - 2\gamma}{2}\sigma^2 = p^0(s), \quad (31)$$

which is linear in s and hence reveals s to E-investors and in fact is a fully-revealing rational expectations equilibrium price function.

Finally, notice also in the above analysis that a change in information s will change demand almost surely. Thus, there is no inertia in demand with respect to information and (27) will (almost surely) not be satisfied for distinct signals s and s' at any price p , thus ruling out non-revelation of signals. This observation is also helpful in the discussion below where information is ambiguous.

3.2 Ambiguous information and partial revelation

Now consider the receipt of an ambiguous private signal by A traders. Type E-investors do not receive the signal, but may observe it if it is revealed by the market price. We use $\mu_{PR}^E|\cdot$ to generically denote the updated beliefs of E-investors about the mean of v . We will be explicit about the construction of these beliefs shortly.

For now, using similar reasoning as in section 3.1 note that if the price distinguishes two distinct signal values s and s' , i.e. $P(s) \neq P(s')$, then these signals are revealed by the price to E-investors and hence $\mu_{PR}^E|s \neq \mu_{PR}^E|s'$.¹³ On the other hand, if price does not reveal the signals, i.e. $P(s) = P(s')$, then $\mu_{PR}^E|s = \mu_{PR}^E|s'$. As noted above, this non-revelation requires inertia with respect to information, in particular for (27) to hold.

¹³More precisely, $\mu_{PR}^E(s) \neq \mu_{PR}^E|s'$ almost surely given the operation of standard Bayesian updating with unambiguous beliefs.

3.2.1 Non-stock holding by A investors

Since (27) is needed for partial revelation to be possible, we will show that it can be satisfied when A investors exhibit inertia with respect to information. In turn, given the preference representation and structure of information in this model, A investors will exhibit inertia when they trade away stock holding to a zero position.

We now investigate the possibility of A investors trading away their stock holdings to a zero position being consistent with market clearing. First, recall that after observing signal s , A investors belief about the mean of v is given by the interval $[\underline{\mu}|s, \bar{\mu}|s]$, where $\underline{\mu}|s = \frac{\rho_0\mu_0 + \rho_\epsilon(s-\delta)}{\rho_0 + \rho_\epsilon}$ and $\bar{\mu}|s = \frac{\rho_0\mu_0 + \rho_\epsilon(s+\delta)}{\rho_0 + \rho_\epsilon}$.

Now, suppose the signal value s is revealed by the price. In this case, the updated belief $\mu_{PR}^E|s$ of E-investors about the mean of v is $\frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon}$, since they believe the signal is unbiased, i.e. $\mu_\epsilon^E = 0$. Hence, $\mu_{PR}^E|s \in [\underline{\mu}|s, \bar{\mu}|s]$.

First, note that shorting the asset ($\theta^A(s) < 0$) is not consistent with market clearing. For any price P , A investors will short only if $\bar{\mu}|s < p - \frac{1}{2}\sigma^2$. However, in this case, E-investors will also short since $\mu_{PR}^E|s < \bar{\mu}|s$ and so the stock market will not clear. The market will also not clear with only E-investors shorting, i.e. $\mu_{PR}^E|s < p - \frac{1}{2}\sigma^2 \leq \bar{\mu}|s$ since the A investors will want to trade to a zero position at this price.

Hence, markets will only clear with both types of investors trading to non-negative positions in the stock. Suppose both types of investors hold long positions, $\theta^A(s) > 0$ and $\theta^E(s) > 0$ at price P . Then for the stock market to clear, using (10), we get $p - 0.5\sigma^2 = x_0^E \mu_{PR}^E|s + x_0^A \underline{\mu}|s - \gamma\sigma^2$. Moreover, $\theta^A(s) > 0$ requires $p - 0.5\sigma^2 < \underline{\mu}|s$ which, using the above, in turn requires $\mu_{PR}^E|s - \frac{\gamma\sigma^2}{x_0^E} < \underline{\mu}|s$. Using the expressions for $\mu_{PR}^E|s$ and $\underline{\mu}|s$, this simplifies to $\frac{\gamma\sigma^2}{x_0^E} > \frac{\delta\rho_\epsilon}{\rho_0 + \rho_\epsilon}$. This analysis is summarized in the following result.

Proposition 1. *If $\frac{\gamma\sigma^2}{x_0^E} > \frac{\delta\rho_\epsilon}{\rho_0 + \rho_\epsilon}$ then markets clear with $\theta^A(s) > 0$ almost surely.*

The intuition is that A-investors will hold a positive position in equilibrium if market clearing prices include an uncertainty premium given their beliefs. This premium is given by $\frac{\delta\rho_\epsilon}{\rho_0 + \rho_\epsilon}$. On the other hand, if A-investors do not hold a positive position in equilibrium then E-investors have to hold all of the stock.

Since the total number of stockholders is now smaller, those that are holding the stock are holding more risk and must be compensated for it. That is, market clearing prices must include a premium to compensate the E-investors for holding all of the

stock. This premium is given by $\frac{\gamma\sigma^2}{x_0^E}$. Since $0 < x_0^E < 1$, this premium is larger than the usual risk premium $\gamma\sigma^2$ which would be required if only E-investors populated the market.

Thus, if the uncertainty premium required by A-investors for a positive position is less than that the premium required by E-investors for holding all of the stock, then in equilibrium, markets can clear with prices reflecting the lower uncertainty premium required by A-investors and with $\theta^A(s) > 0$ almost surely.¹⁴

If $\theta^A(s) > 0$, then analogous reasoning as in section 3.1 shows that there is no inertia with respect to information and hence the partial revelation condition (27) can not be satisfied. Moreover, using the same reasoning shows that if $\theta^A(s) > 0$ when markets clear for some signal value s , then $\theta^A(s') > 0$ for all $s' \neq s$ given the relation between the updated beliefs of E-investors and A-investors noted earlier, i.e. $\mu_{PR}^E|s \in [\underline{\mu}^A|s, \bar{\mu}^A|s]$ for all s .

As the inertia property requires $\theta^A = 0$ to be consistent with market clearing, the above analysis yields the following result about informational efficiency of prices.

Corollary 1. *If $\frac{\gamma\sigma^2}{x_0^E} > \frac{\delta\rho_\epsilon}{\rho_0 + \rho_\epsilon}$ then any rational expectations equilibrium price function is monotonic in signals and hence fully revealing.*

In light of this result, we next consider market-clearing prices where $\theta^A(s) = 0$, i.e. A-investors trade away their stock holdings to a zero position, to study prices which are not fully revealing. That is, we consider markets where the uncertainty premium required by A-investors to hold the stock is too high relative to the premium required by E-investors to hold the all of the stock.

Suppose markets clear at price P with the E-investors holding the entire stock supply and the A-investors trading away their stock holding to hold a zero position in the stock. Given signal s , the market-clearing price $p(s)$ satisfies

$$p(s) - 0.5\sigma^2 = \mu_{PR}^E|s - \frac{\gamma\sigma^2}{x_0^E}. \quad (32)$$

As noted above, the term $\frac{\gamma\sigma^2}{x_0^E}$ reflects the premium required by E-investors over and above the usual risk premium $\gamma\sigma^2$ by a factor of $\frac{1}{x_0^E}$. We next consider when price given by (32) reveals and does not reveal private signals received by A-investors.

¹⁴This reasoning is related to that in Easley and O'Hara (2009), but not the same since there is no updating of beliefs in Easley and O'Hara (2009).

Finally, note that since the A-investors are endowed with a positive amount of the uncertain asset, i.e. $x_0^A > 0$, the above is not a no-trade outcome. Moreover, there is no indeterminacy in equilibrium prices.

3.2.2 Revelation of large signal values

We now turn to whether the market price may reveal the signal when the A-investors trade to a zero position in the stock. As we will show below, partial revelation takes the form of market price P not revealing intermediate signal values, in particular an interval of signal values, while revealing large signal values, i.e. signal values outside of the interval.

Market prices may reveal relatively large high or low signal values in a rational expectations equilibrium. When relatively high signals arrive, the A-investors will demand a higher price for selling their stock holdings given their updated belief that mean stock payoff is high. In turn E-investors are willing to pay a higher price for the stock holdings if they believe a high enough signal has been received since this would mean a higher (estimate of) mean stock payoff.

For high enough signal values, this means the market clearing price will be responsive to signal values, i.e. will change as the signal value changes. Hence, the market price will reveal high enough signals. Similar reasoning applies to the case of low enough signal values. A-investors would be willing to accept a lower price for selling stock holding since their updated belief about the mean stock payoff is low and E-investors would be willing to only pay lower prices if they believe a low enough signal has been received, meaning a lower (estimate of) mean stock payoff.

On the other hand, as we will show next, for intermediate signal values, price may not reveal signal values as A- and E-investors will trade at the same price for a range of signals. This in turn means that prices need not respond to information and hence will not reveal changes in signals.

3.2.3 Non-revelation of intermediate signal values

As noted above, in the present set up, market clearing requires E-investors to hold the entire stock supply and A-investors to trade away their holdings of the stock.

Hence, the equilibrium price must satisfy

$$p = \mu_{PR}^E + \frac{x_0^E - 2\gamma}{2x_0^E} \sigma^2 \quad (33)$$

Combining this with the condition for stock price to not reveal, i.e. distinguish two distinct signal values s and s' yields

$$\max\{\underline{\mu}|s, \underline{\mu}|s'\} \leq \mu_{PR}^E - \frac{\gamma\sigma^2}{x_0^E} \leq \min\{\bar{\mu}|s, \bar{\mu}|s'\}. \quad (34)$$

These inequalities and the expressions for updated beliefs $\underline{\mu}|s$ and $\bar{\mu}|s$ can be used to characterise the beliefs μ_{PR}^E of the uninformed E investors for unrevealed signals as we describe next.

3.3 Prices and uninformed beliefs for non-revealed signals

Rearranging the above equation shows that for the uninformed investors to hold the entire stock supply in equilibrium with the stock price not changing in response to changes in signal values, the signals must not be too extreme and must lie in some intermediate range or interval with finite bounds.

Using the explicit expressions for $\underline{\mu}|s$ and $\bar{\mu}|s$ and rearranging terms, this range for s is

$$\mu_{PR}^E + \delta + \frac{\rho_0}{\rho_\epsilon}(\mu_{PR}^E - \mu_0) - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon} \frac{\gamma\sigma^2}{x_0^E} \geq s \geq \mu_{PR}^E - \delta + \frac{\rho_0}{\rho_\epsilon}(\mu_{PR}^E - \mu_0) - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon} \frac{\gamma\sigma^2}{x_0^E} \quad (35)$$

Let a denote the lower bound on this range, as given by the second inequality and b denote the upper bound on this range as given by the first inequality. Thus, for the beliefs of type E investors to be consistent with the non-participation of the informed group of A investors at the same price for a range of signal values, the E investors will know that the signal observed by A investors falls within an interval $[a, b]$ of potential signals. This means that E -investors beliefs will be constant over the interval $[a, b]$ and we denote $\mu_{PR}^E|[a, b] = \mu_{PR}^E|s$ for all $s \in [a, b]$.

3.3.1 The price function

Equation (35) in effect implies that the price function p_{PR} under partial revelation is non linear in the signals. In fact, as we will show below, the price function also exhibits discontinuities at $s = a$ and $s = b$.

$$p_{PR}(s) = \begin{cases} \mu_{PR}^E|[a, b] + \frac{x_0^E - 2\gamma}{2x_0^E} \sigma^2 & \text{if } s \in [a, b] \\ \mu_{PR}^E|s + \frac{x_0^E - 2\gamma}{2x_0^E} \sigma^2 & \text{if } s < a \text{ or } s > b \end{cases} \quad (36)$$

The price function is non-linear in signals and discontinuous due to the ambiguity in private signals for A-investors. Ambiguity in private information means some signals are not revealed. This in turn means that uninformed E-investors update beliefs are based only on the information that the signal is one of several possible signals, described by the range $[a, b]$. As we show next, in section 3.3.2, this update is based on “averaging” over the possible signals. This updated belief $\mu_{PR}^E|[a, b]$ is in general strictly between the updated belief based on knowing the signal value is a and the updated belief based on knowing the signal value is b . This in turn implies that the price function is non-linear in signals, with discontinuities at signal values a and b .

The price function in the commonly used Grossman and Stiglitz (1980) framework is linear, which is driven by the assumption of normal distributions, CARA utility with non wealth constraints, and unambiguous beliefs. Other models of noise-based partial revelation with different distributional or utility assumptions such as Mailath and Sandroni (2003) and Barlevy and Veronesi (2003) or wealth constraints such as Yuan (2005) provide non-linear price functions under noise-based partial revelation.

Further, while REE models with noise traders and ambiguity-averse traders such as Ozsoylev and Werner (2011) and Mele and Sangiorgi (2011) feature piecewise-linear price functions, the price function is not discontinuous.¹⁵ In Ozsoylev and Werner (2011), ambiguity-averse traders do not receive any private signals, while in Mele and Sangiorgi (2011) private information eliminates ex-ante ambiguity, unlike the scenario we analyse where information can be ambiguous. In Easley, O’Hara, and Yang (2011), where partial revelation is driven by ambiguity-aversion of traders,

¹⁵See also figure 3 in Easley and O’Hara (2009).

acquiring information also eliminates ambiguity and the REE price function is continuous in private information signals.¹⁶

3.3.2 Uninformed beliefs

The boundaries of the interval $[a, b]$ are determined endogenously according to equation (35). Were the uninformed investors able to observe the signal s , their updated beliefs about the mean of v would be

$$\mu_{PR}^E | s = \frac{\rho_0 \mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} \quad (37)$$

Since the uninformed E-investors are not able to observe the signal, their belief about the mean of v is obtained by using the updated beliefs conditional on the knowledge that the signal is in $[a, b]$. If $f(s|a \leq s \leq b)$ is the marginal probability density function over signals conditional on the signal being between a and b then this expected value is

$$\begin{aligned} \mathbb{E}[\mu | a \leq s \leq b] &= \int_a^b \frac{\rho_0 \mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} f(s|a \leq s \leq b) ds \\ &= \frac{1}{\rho_0 + \rho_\epsilon} \left(\rho_0 \mu_0 + \rho_\epsilon \int_a^b s f(s|a \leq s \leq b) ds \right) \\ &= \frac{1}{\rho_0 + \rho_\epsilon} (\rho_0 \mu_0 + \rho_\epsilon \mathbb{E}[s | a \leq s \leq b]) \end{aligned} \quad (38)$$

By definition $s = \mu + \epsilon$ where μ and ϵ are independent, normally distributed random variables. Under the assumption that the uninformed are expected utility maximizers who think that the signal is unbiased, s is normally distributed with mean μ_0 and precision $\rho_0 + \rho_\epsilon$. The expected value of s conditional on s being in the interval $[a, b]$ is therefore

$$\mathbb{E}[s | s \in [a, b]] = \mu_0 + \Delta(a, b) \quad (39)$$

where

$$\Delta(a, b) = \frac{\phi\left(\sqrt{\frac{\rho_0 \rho_\epsilon}{\rho_0 + \rho_\epsilon}}(a - \mu_0)\right) - \phi\left(\sqrt{\frac{\rho_0 \rho_\epsilon}{\rho_0 + \rho_\epsilon}}(b - \mu_0)\right)}{\Phi\left(\sqrt{\frac{\rho_0 \rho_\epsilon}{\rho_0 + \rho_\epsilon}}(b - \mu_0)\right) - \Phi\left(\sqrt{\frac{\rho_0 \rho_\epsilon}{\rho_0 + \rho_\epsilon}}(a - \mu_0)\right)} \sqrt{\frac{\rho_0 \rho_\epsilon}{\rho_0 + \rho_\epsilon}}, \quad (40)$$

¹⁶In particular, in Easley, O'Hara, and Yang (2011) one set of 'simple' traders is ambiguous about the trading strategy of another set of 'opaque' traders, which in turn leads to an REE price function which is not fully informative for the simple traders.

with ϕ and Φ denoting the standard normal density and distribution functions respectively. The above is derived from the properties of the truncated normal distribution (see e.g. Johnson and Kotz (1970)).

Simplification on expression (38) gives

$$\mu_{PR}^E|[\alpha, b] = \mathbb{E}[\mu|a \leq s \leq b] = \mu_0 + \frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon} \Delta(a, b). \quad (41)$$

The term

$$\frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon} \Delta(a, b) \quad (42)$$

represents the change in beliefs of an expected utility maximizing investor when that investor knows only that the ambiguity averse investors received a signal that has caused them to not participate in the market. The cutoffs a and b are obtained endogenously in the model.

Plugging these beliefs into the non-participation condition (35) gives

$$\frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon} \left(\mu_0 - \frac{\gamma\sigma^2}{x_0^E} \right) - \frac{\rho_0}{\rho_\epsilon} \mu_0 + \delta + \Delta(a, b) \geq s \geq \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon} \left(\mu_0 - \frac{\gamma\sigma^2}{x_0^E} \right) - \frac{\rho_0}{\rho_\epsilon} \mu_0 - \delta + \Delta(a, b). \quad (43)$$

Since by definition the left hand side of this inequality is b and the far right hand side is a , the signal bounds that are consistent with the behavior of the informed agents are found by solving the system of equations

$$\begin{aligned} \mu_0 - \delta + \Delta(a, b) - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon} \frac{\gamma\sigma^2}{x_0^E} &= a \\ \mu_0 + \delta + \Delta(a, b) - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon} \frac{\gamma\sigma^2}{x_0^E} &= b \end{aligned} \quad (44)$$

A few things become apparent from this system of equations. The first is that the length of the interval of received signals that the uninformed believe the informed to have received is related to the amount of ambiguity in the signal. To see this, subtract the second equation from the first and rearrange to obtain

$$b - a = 2\delta. \quad (45)$$

From this, this system can be reduced to a system of one equation in one unknown

since $2\delta = b - a$, which is exogenous. This system becomes

$$F(a) = \mu_0 - \delta + \Delta(a, a + 2\delta) - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon} \frac{\gamma \sigma^2}{x_0^E} - a = 0. \quad (46)$$

This implicit equation in a can be solved numerically in general. Below we present the solution for a special case of risk neutral investors, which also allows us to differentiate the effects of ambiguous information, as reflected in the aversion to ambiguity of the investors, from that which is unambiguous.

In the risk neutral case, i.e. $\gamma = 0$, the fixed point problem for the solution of beliefs when the signal is not revealed becomes

$$F(a) = \mu_0 - \delta + \Delta(a, a + 2\delta) - a = 0. \quad (47)$$

From inspection we see that $\Delta(a, a + 2\delta)$ is zero when a and $a + 2\delta$ are symmetric around μ_0 . Since the interval distance must be 2δ , we note that values $a = \mu_0 - \delta$, $b = \mu_0 + \delta$ are a solution to this system. We summarize this discussion as follows.

Proposition 2. *The existence of an interval $[a, b]$ of signal values that are not revealed by market price and hence the existence of partially revealing rational expectations equilibrium price can be obtained by solving (46). When the investors are risk-neutral, i.e. $\gamma = 0$, the equation (46) has the solution $a = \mu_0 - \delta$ and hence $b = \mu_0 + \delta$.*

In the risk-neutral case, the set of signals that do not get revealed in equilibrium is $[\mu_0 - \delta, \mu_0 + \delta]$. As the ambiguity of the private signal grows, the interval of signals that are not revealed in equilibrium expands out from the prior mean μ_0 . Note however, that in the case where the non-revelation region is symmetric around μ_0 the update to beliefs that comes from investor E not participating provides no meaningful information by which E updates beliefs. That is, the update to belief $\Delta(a, a + 2\delta)$ is always zero. This will not be the case generally when $\gamma \neq 0$.

By the continuity of the functions involved in the fixed point problem being considered since a solution to (46) exists for $\gamma = 0$, a solution will also exist for values of $\gamma \neq 0$ that are small enough. For $\gamma \neq 0$, the update $\Delta(a, a + 2\delta)$ will not usually be zero since a and $b = a + 2\delta$ will not usually be symmetric around μ_0 .

It is apparent from the above analysis that large signal values, captured by $s < a$ or $s > b$ will be revealed by the market prices, while intermediate signals values,

$s \in [a, b]$ will not be revealed in the partially revealing REE. Using the expressions for beliefs μ_{PR}^E obtained above, we thus have the following.

Proposition 3. *The price function under partial revelation is given by*

$$p_{PR}(s) = \begin{cases} \frac{\rho_0\mu_0 + \rho_\epsilon(\mu_0 + \Delta(a, b))}{\rho_0 + \rho_\epsilon} + \frac{x_0^E - 2\gamma}{2x_0^E} \sigma^2 & \text{if } s \in [a, b] \\ \frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} + \frac{x_0^E - 2\gamma}{2x_0^E} \sigma^2 & \text{if } s < a \text{ or } s > b \end{cases} \quad (48)$$

where the values a and b are obtained by solving (46).

As noted earlier, the price function is non-linear in signals and exhibits discontinuities at signal values a and b . When the signal is not revealed, E-traders updated beliefs are based only on the knowledge that the signal could be any one of those in $[a, b]$. The updated belief based on this information $\mu_{PR}^E|[a, b]$ lies strictly between the updated belief based on the signal value a , $\mu_{PR}^E|a$, and the updated belief based on the signal value b , $\mu_{PR}^E|b$. This in turn implies the discontinuity and non-linearity of the price function as shown in figure 1, which depicts the price function for different values of γ .

It is possible to draw some interesting conclusions about the market even without explicit solutions to the above equation, and we proceed to these in section 4 next. Before doing, note that given the price function in (48), it is possible to contrast the partial revelation due to ambiguous information with the widely studied noise-based partial revelation. In the present model, partial revelation involves an intermediate range of signal values not being revealed and relatively extreme values being revealed. As we show in section 4.1, this form of partial revelation implies the possibility of discontinuous price changes and large discontinuous variations in price volatility.

Under noise-based partial revelation all signal values are obscured by the noise shock in equilibrium and as such the partial revelation does not have implications for price volatility beyond what noise adds. That is, noise contributes to the properties of equilibrium prices in these models but in a way that is qualitatively similar to how noise traders would alter prices in a setting without private information.

Another feature which distinguishes partial revelation discussed here from that in some common noise-based models is that information on volume does not change the informational properties of the prices here. In CARA-normal models of noise-

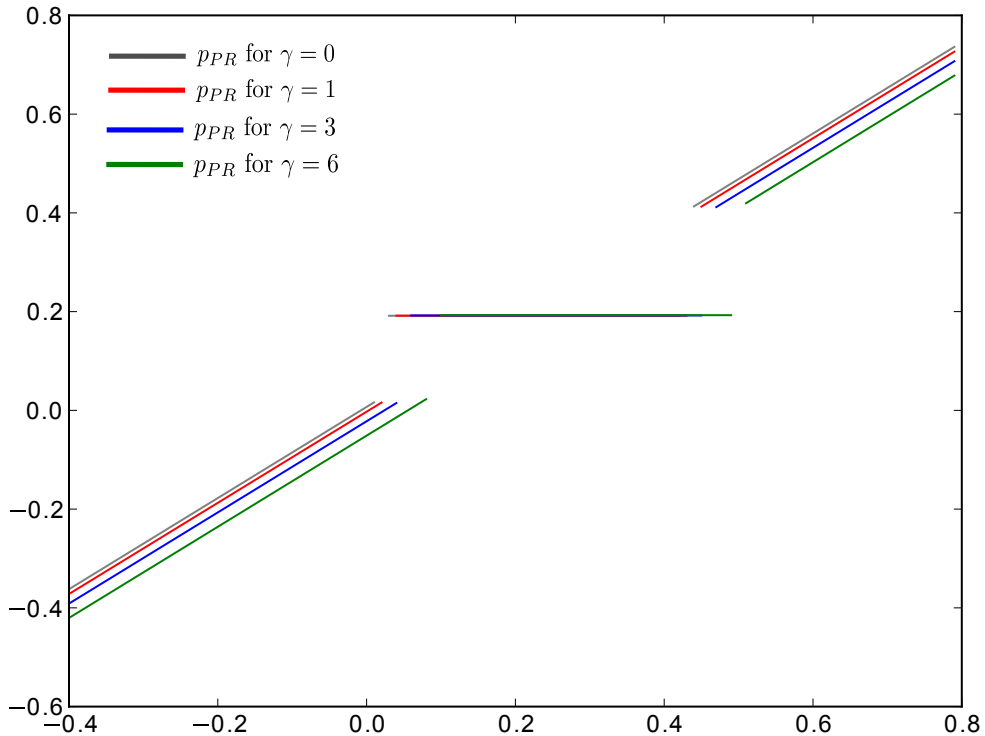


Figure 1: Partial revelation price function p_{PR}

based partial revelation adding trading volume information makes partially revealing prices fully revealing. See Blume, Easley, and O’Hara (1994) and Schneider (2009) for a discussion of this issue. Overall, the differences in partial revelation due to ambiguous information and noise-based partial revelation suggest that in principle, these differing forms of partial revelation may be differentially useful, possibly even complementary, in studying financial markets.

Noise-based partial revelation is also used to provide a resolution to the Grossman and Stiglitz (1980) paradox of costly information acquisition. The present model does not address this issue. It is not clear that all information used in financial markets involves a direct cost. One example would be information from a non-traded asset like labor income, whose payoff is correlated with, and hence informative about, the stock payoff. We discuss this in the present model’s context in section 6. Also, work by Bernardo and Judd (2000), Muendler (2007), and Krebs (2007) suggests that

the co-existence of informationally efficient prices and costly information is not paradoxical outside of the widely-used CARA-normal models, where wealth effects are absent and normality assumptions yields linearity of the equilibrium price function.

4 Pricing implications

4.1 Price volatility and swings

The revelation and non-revelation of privately held signals has implications for the volatility of equilibrium stock prices. To see this, we consider now the variance of p_{PR} conditional on the signal being revealed or not revealed. When the signal is not revealed, i.e. $s \in [a, b]$, then the price is constant at

$$p_{PR}(s) = \frac{\rho_0 \mu_0 + \rho_\epsilon (\mu_0 + \Delta(a, b))}{\rho_0 + \rho_\epsilon} + \frac{x_0^E - 2\gamma}{2x_0^E} \sigma^2. \quad (49)$$

Hence, price volatility conditional on signals not being revealed is zero, i.e.

$$\text{Var}[p_1 | s \in [a, b]] = 0. \quad (50)$$

On the other hand, if the signal is revealed, i.e. $s < a$ or $s > b$ then the equilibrium price is

$$p_{PR}(s) = \frac{\rho_0 \mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} + \frac{x_0^E - 2\gamma}{2x_0^E} \sigma^2 \quad (51)$$

Thus, conditional on the signal being outside the non-revelation interval the variance of price is

$$\text{Var}(p_{PR} | s < a \text{ or } s > b) = \left(\frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon} \right)^2 \text{Var}(s | s < a \text{ or } s > b) \quad (52)$$

We can use the properties of the truncated normal distribution, to obtain an expression for $\text{Var}(s | s < a \text{ or } s > b)$. First, note that $\text{Var}(s | s < a)$ is

$$\text{Var}(s | s < a) = \frac{1}{\rho_\epsilon} \left[1 - \frac{\phi(\sqrt{\rho_\epsilon}(\mu_0 - a))}{1 - \Phi(\sqrt{\rho_\epsilon}(\mu_0 - a))} \left(\frac{\phi(\sqrt{\rho_\epsilon}(\mu_0 - a))}{1 - \Phi(\sqrt{\rho_\epsilon}(\mu_0 - a))} - \sqrt{\rho_\epsilon}(\mu_0 - a) \right) \right] \quad (53)$$

and $\text{Var}(s|s > b)$ are

$$\text{Var}(s|s > b) = \frac{1}{\rho_\epsilon} \left[1 - \frac{\phi(\sqrt{\rho_\epsilon}(b - \mu_0))}{1 - \Phi(\sqrt{\rho_\epsilon}(b - \mu_0))} \left(\frac{\phi(\sqrt{\rho_\epsilon}(b - \mu_0))}{1 - \Phi(\sqrt{\rho_\epsilon}(b - \mu_0))} - \sqrt{\rho_\epsilon}(b - \mu_0) \right) \right] \quad (54)$$

To calculate the variance of s conditional on s not being an element of $[a, b]$, denote the probability density function of s by $f(s)$ and note that the conditional density of s is

$$f(s|s < a \text{ or } s > b) = \frac{f(s)}{F(a) + 1 - F(b)} \quad (55)$$

Using $\hat{F}(a) = \frac{F(a)}{F(a) + 1 - F(b)}$ and $\hat{F}(b) = \frac{1 - F(b)}{F(a) + 1 - F(b)}$, the conditional variance is then defined as

$$\begin{aligned} \text{Var}(s|s < a \text{ or } s > b) &= \int_{-\infty}^a (s - \mu_0)^2 \frac{f(s)}{F(a) + 1 - F(b)} ds + \int_b^{\infty} (s - \mu_0)^2 \frac{f(s)}{F(a) + 1 - F(b)} ds \\ &= \hat{F}(a) \int_{-\infty}^a (s - \mu_0)^2 \frac{f(s)}{F(a)} ds + \hat{F}(b) \int_b^{\infty} (s - \mu_0)^2 \frac{f(s)}{F(b)} ds \\ &= \hat{F}(a) \text{Var}(s|s < a) + \hat{F}(b) \text{Var}(s|s > b) \end{aligned} \quad (56)$$

Together, this implies that

$$\text{Var}(p_{PR}|s < a \text{ or } s > b) = \left(\frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon} \right)^2 (\hat{F}(a) \text{Var}(s|s < a) + \hat{F}(b) \text{Var}(s|s > b)) \quad (57)$$

We can summarize the above analysis as follows.

Proposition 4. *The volatility of stock price conditional on revelation of signals is $\text{Var}(p_{PR}|s < a \text{ or } s > b) > 0$ where $\text{Var}(p_{PR}|s < a \text{ or } s > b)$ is given by (57). The volatility of stock price conditional on non-revelation of signals is $\text{Var}(p_{PR}|s \in [a, b]) = 0$.*

The above result leads to the following observations about large movements in price and in price volatility.

Corollary 2. *Periods of non-revelation due to ambiguous signals will have strictly less volatility, ceteris paribus, than market periods when either ambiguous signals are revealed in the market or signals are not perceived to be ambiguous.*

This observation describes two aspects of information transmission through markets as studied here. The first is that partial revelation of ambiguous signals differs

in both the cause and the empirical implications from traditional, noise-based partial revelation. In noise-based partial revelation, every signal value is obscured, i.e. there is only one informational regime, and moreover excess volatility arises directly from the random noise. As such price volatility does not differ due to different informational regimes unlike in the current model.

The second point is the role price movements play in information revelation of any kind. Periods of higher price volatility coincide with the revelation of information. To see this consider that asset price volatility is explained, at least in part, by the arrival of information that changes traders' beliefs about the payoffs of the assets. Under partial revelation this information may not be revealed through market prices depending on the informational regime. These price movements under the revelation regime are the result of the market incorporating private information and are the method by which market prices convey this information to other traders.

As such, these results suggest caution in the face of policy options that might unduly limit market volatility, whether this is the goal of the policy or not. Periods of high price volatility are not necessarily bad if prices are successfully incorporating new information and transmitting that information to market participants. Likewise, for these same reasons, periods of low market volatility are not necessarily desirable.

Moreover, as the next observation shows, periods of low market volatility may portend large price swings followed by periods of high volatility.

Corollary 3. *If partial revelation occurs because of ambiguous information as described in this model, changes in the information content of market prices will coincide with discontinuous jumps in asset prices.*

As discussed above (equation (38)), during periods of non revelation, E-investors will formulate beliefs by averaging over the set of signals that are not observed in equilibrium. Changes in the wealth distribution of traders, public signals or additional private signals can all serve to move the market from non revelation into revelation. This movement into revelation implies that the previously unrevealed information is now incorporated into the beliefs of the uninformed traders and hence into market prices.

Since traders had previously been using only the average of the unrevealed signals, when a signal becomes available to them there is a discontinuous change in

their beliefs about market fundamentals which implies a discontinuous change in market prices. This happens even if the revealed signal is very close in value to the unrevealed signal. This jump in asset prices will be positive if the previously unrevealed signal was better news than the average of the unrevealed signals. Likewise, this price swing will be negative if the unrevealed signal was worse than the average unrevealed signal.

A similar phenomenon occurs when the economy moves from revelation of signals to non revelation of signals. Although all previously revealed signals are known by all market participants, when the market moves into periods of non-revelation, unrevealed signals must be averaged over by E-investors and this can lead to a discontinuous movement in market prices, even if the change in signal value is small.¹⁷

The above two observations mean that the transition out of periods of low market volatility can be hectic. In this model, periods of low market volatility occur when information is not being revealed. When the market changes in such a way that information is revealed, this happens concurrently with a large price swing, followed by a period of relatively high market volatility.

In the model of Illeditsch (2010) with a CARA representative investor and normally distributed payoffs, ambiguous information also leads to discontinuity in the price as a function of signals. Illeditsch (2010) models ambiguous information through a range of signal precisions, rather than through a range of means as done in the present paper. The discontinuity arises at the signal value which confirms the prior mean since the representative investor uses the highest signal precision when the signal is below the prior mean to update his beliefs and the lowest signal precision when it is above the mean. This leads to a discontinuity in the price and hence also a jump in price volatility at the point of discontinuity.

Dow and da Costa Werlang (1992a) provide an example of excess volatility due to ambiguity aversion. There investors update ambiguous prior beliefs via the Dempster-Shafer rule. In this case, the standard variance decomposition formula used in Bayesian updating with unambiguous beliefs need not hold, which then leads to excess price variance as it violates the variance bounds implied by Bayesian updating.

¹⁷This applies even when the fundamental v remains the same, but the signal value changes due to a change in the error term. This discontinuity due to a switch in informational regime can be traced back to the inertia with respect to information property exhibited by A-investors. Epstein and Schneider (2010) discuss how the property of portfolio inertia with respect to prices can lead to a discontinuous change in prices due a change in a representative ambiguity averse investor's portfolio.

Mele and Sangiorgi (2011) find that non-smooth ambiguity aversion can lead to price swings in a model with noise-based partial revelation.

4.2 Price impact

The revelation and non-revelation of signals also has implications for the price impact of a given trade. Price impact of trade can be used as a measure of market liquidity. The volume of trade under partial revelation is given by $x_0^E > 0$ for all signal values since the A-investors trade to a zero position in the stock.

The price impact of this trade differs depending on whether the price p_{PR} reveals the signal or not. For unrevealed signals $s \in [a, b]$, the price change is zero since the same price $p_{PR}(s)$ prevails. On the other hand, for revealed signals $s \notin [a, b]$, the price impact is non-zero since the price $p_{PR}(s)$ changes with the signal value.

Moreover, given the discontinuous price function, there can be a large change in price impact if the information content of price changes. Let $\lambda(s, s') \equiv |p_{PR}(s) - p_{PR}(s')|$ denote the price impact of trade by A-investors due a change in private signal from s to s' . Then,

$$\lambda(s, s') = \begin{cases} 0 & \text{if } s \in [a, b] \text{ and } s' \in [a, b] \\ \frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon} |s - s'| & \text{if } s \notin [a, b] \text{ and } s' \notin [a, b] \\ \frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon} |\mu_0 + \Delta(a, b) - s'| & \text{if } s \in [a, b] \text{ and } s' \notin [a, b]. \end{cases} \quad (58)$$

This discussion can be summarized as follows.

Proposition 5. *The price impact of trade due to changes in information is (i) 0 if neither signal is revealed, (ii) positive if either of the signals is revealed, and (iii) discontinuously large if one signal is not revealed while the other is revealed.*

5 Changes in informational regime

Using the above analysis of partial revelation conditions, we can now analyse the effect of relative wealth shares, individual learning by A-investors, and public signals on the informational efficiency of price. Recall that the conditions for partial

revelation are given by

$$\underline{\mu}|s \leq \mu_{PR}^E|[a, b] - \frac{\gamma\sigma^2}{x_0^E} \leq \bar{\mu}|s \quad (59)$$

which can be rearranged to get

$$\mu_{PR}^E|[a, b] - \bar{\mu}|s \leq \frac{\gamma\sigma^2}{x_0^E} \leq \mu_{PR}^E|[a, b] - \underline{\mu}|s. \quad (60)$$

As can be seen from the above inequalities, the wealth share of E-investors, more generally the uninformed investors, the differences in beliefs between the A- and E-investors, and the ambiguity perceived by A-investors play a role in determining whether or not the signal is revealed.

5.1 Wealth share

A change in the wealth share of E-investors as captured by a change in x_0^E can lead to a switch in informational regimes. Consider the second inequality in (60). If E-investors are relatively wealthy compared to A-investors, as captured by this inequality, then the economy may have a partial revelation information regime.

The intuition for this is that if A-investors are poor enough, then their effect on asset prices is relatively small. This means that in equilibrium it is possible for them to sell off their initial asset endowment and hold only the risk-free asset. E-investors hold onto all the uncertainty in the economy and as noted previously are compensated in the form of a risk premium and a reduced-participant premium.

5.2 Learning by A-investors

So far, we have only discussed the scenario where A-investors observe a signal once. However, the analysis extends in a straightforward manner to the case where the A-investors observe more than one signal and learn.

The updated beliefs in the case of multiple private signals with the same precision, say K signals, is given by set of normal distributions with precision $\rho_0 + K\rho_e$ and means

$$\{\mu|s_1, \dots, s_K\} = \left\{ \frac{\rho_0\mu_0 + \rho_e \sum_{k=1}^K (s_k + \mu_{ek})}{\rho_0 + K\rho_e} : \mu_{ek} \in [-\delta_k, \delta_k] \text{ for all } k = 1, \dots, K \right\}. \quad (61)$$

We denote the interval of means of v by $[\underline{\mu}^A|(s_1, \dots, s_K), \bar{\mu}^A|(s_1, \dots, s_K)]$ in this case. The partial revelation condition then becomes

$$\mu_{PR}^E|[a, b] - \bar{\mu}^A|(s_1, \dots, s_K) \leq \frac{\gamma\sigma^2}{x_0^E} \leq \mu_{PR}^E|[a, b] - \underline{\mu}^A|(s_1, \dots, s_K) \quad (62)$$

In this case, as information accumulates the interval $[\underline{\mu}|(s_1, \dots, s_K), \bar{\mu}|(s_1, \dots, s_K)]$ shrinks. Thus, if the economy is in a partial revelation regime and the A-investors are learning, there may be no change in the regime until enough information accumulates and the interval is reduced to a stage where one of the inequalities is violated. This will imply a switch to a full revelation regime. Thus, a reduction in the ambiguity perceived by A-investors due to individual learning may lead to a switch in the informational efficiency of prices and an accompanying discontinuous change in related variables as noted in section 4.

This shift in information regime accompanied by a seeming shift in individual choices appears similar to a phenomenon in the presence of adjustment costs and sequential decisions noted by Caplin and Leahy (1994). However, in that intertemporal setting, exogenous adjustment costs prevent individuals from changing choices despite changes in information, while here there are no such exogenous costs or non-convexities and it is ambiguity perceived by A-investors which leads to inertia in response to changes in information.

5.3 Public signals

As noted earlier, the discussion so far has excluded a public signal which conveys information about the mean (log) stock payoff. Such a signal affects the beliefs of both A- and E-investors. In particular, it implies that the differences in beliefs as captured by $(\mu_{PR}^E|[a, b] - \underline{\mu}|s)$ and $(\mu_{PR}^E|[a, b] - \bar{\mu}|s)$ are reduced.

When a public signal is received by market participants, the beliefs of both A-investors and E-investors are changed. The A-investors' beliefs about the mean of v after the receipt of a public signal ζ with precision ρ_ζ are given by

$$\{\mu^A|(s, \zeta)\} = \left\{ \frac{\rho_0\mu_0 + \rho_\zeta\zeta + \rho_\epsilon(s + \mu_\epsilon)}{\rho_0 + \rho_\zeta + \rho_\epsilon} : \mu_\epsilon \in [-\delta, \delta] \right\} \quad (63)$$

On the other hand the beliefs of expected utility investors E under non revelation

become

$$\mu_P^E R |([a, b], \zeta) = \frac{\rho_0 \mu_0 + \rho_\zeta \zeta + \rho_\epsilon \Delta(a, b)}{\rho_0 + \rho_\zeta + \rho_\epsilon}. \quad (64)$$

This possibility for transition from non revelation to revelation due to a public signal means that otherwise anomalous price behavior can occur. The receipt of a public signal that is bad news will usually lead to a decline in the price of the stock. However, if that bad news implies that price reveals A's private information, then this revelation may influence price to increase or decrease relative to where they would be were the economy to remain in partial revelation.

6 Non-tradeable labor income

In this section we provide conditions under which the signal structure described previously can be reinterpreted as one in which investors receive signals about a non-tradeable asset such as labor income whose payoff is correlated with the stock parameter about which investors learn. The investors can use the stock to hedge against their labor income fluctuations and in turn use information about labor income to update their information about the stock payoff.¹⁸

Suppose that each participant in the model has access to non-tradeable labor income that provides a return on initial wealth R_l . The return from labor income that the investor receives is correlated with the expected payoff to the risky asset. In particular, $r_l = \ln R_l$ and μ are jointly normally distributed with means (μ_l, μ_0) and covariance η . The precision of r_l is ρ_l .

For this section we assume that investors no longer receive a signal that is directly relevant to the asset. Instead A-investors receive a signal $s = r_l + \epsilon$ that provides information about the value labor income in the future. For clarity, first assume that there is no ambiguity present in this signal. Assume that ϵ is normally distributed with mean 0 and precision ρ_ϵ and that ϵ and r_l are independent. Let $\rho_s = \rho_l \rho_\epsilon / (\rho_l + \rho_\epsilon)$ be the precision of the signal s . Since r_l and μ are correlated, this signal also provides information about the mean payoff to the asset. We assume the joint normality of s , r_l and μ_0 which implies that the covariance of s and μ is η . As

¹⁸Such formulations of hedging motives are commonly used in the rational expectations equilibrium literature, see for example Biais, Bossaerts, and Spatt (2010), Schneider (2009), Goldstein and Guembel (2008), Watanabe (2008), and the references therein. In these papers, the hedging motive is closely tied to the noise which prevents prices from revealing information.

such, the updated distribution of μ given the observation of the private signal s is normal with mean

$$\mu|s = \mu_0 + \frac{\eta}{\sigma_l^2 + \sigma_\epsilon^2}(s - \mu_l) \quad (65)$$

As the previous updated mean can be written as

$$\mu|s = \mu_0 + \frac{\rho_s}{\rho_0 + \rho_s}(s - \mu_0) \quad (66)$$

we see that the form of this equation and that of equation (5) are similar. The difference is that the change in the updated mean of the asset payoff when a signal has been received now depends on the covariance of μ with the signal, which is the covariance of μ with labor incomes. In the previous structure the update depended on the covariance of s with μ which was just the variance of μ (or one over the precision). Both of these updated means can be written as $\mu_0 + \beta(s - \mathbb{E}s)$ although β and $\mathbb{E}s$ differ depending on whether the signal is about the mean payoff directly or about some other variable that is correlated with the mean payoff.

If the mean of the signal (or equivalently, of ϵ) is ambiguous, then the set of updates becomes

$$\{\mu|s\} = \left\{ \mu_0 + \frac{\eta}{\sigma_l^2 + \sigma_\epsilon^2}(s + \mu_\epsilon - \mu_l) : \mu_\epsilon \in [-\delta, \delta] \right\}. \quad (67)$$

6.1 Investor demand with non-tradeable labor income

We model the wealth shock as being a random return on initial wealth. Hence, terminal wealth is given by

$$W_2 = W_0(\theta R + (1 - \theta)R_f + R_l) \quad (68)$$

As before, we approximate the return on initial wealth by a lognormal random variable based on a second order Taylor series approximation of the portfolio return.

We find first the second-order Taylor series approximation of $\ln(\theta R + (1 - \theta)R_f + R_l)/R_f$. This can be rewritten as

$$f(r - r_f, R_l/R_f) = \ln \left[1 + \frac{R_l}{R_f} + \theta(e^{r - r_f} - 1) \right]. \quad (69)$$

After substituting in the appropriate derivatives evaluated at the point $(r-r_f, R_l/R_f) = (0, 0)$, the approximation becomes

$$f(r-r_f, R_l/R_f) \approx \theta(r-r_f) + e^{r_l-r_f} + \frac{\theta-\theta^2}{2}(r-r_f)^2 + \frac{1}{2}e^{2(r_l-r_f)} - \frac{\theta}{2}(r_l-r_f)e^{r_l-r_f} \quad (70)$$

For this paper we will assume that the signal provides information about the asset payoff only through its covariance with the asset's mean payoff. As such, conditional on having observed the signal and updated her prior about the mean of the asset payoff, the covariance of the asset with the return on labor income is zero.

As do Campbell and Viciera (2002), we replace the second order terms with their expectations to get the final form of the approximation

$$f(r-r_f, R_l/R_f) \approx \theta(r-r_f) + \frac{R_l}{R_f} + \frac{\theta-\theta^2}{2}\sigma^2 + \frac{1}{2}\sigma_l^2 \quad (71)$$

Given the assumptions above,

$$\text{Var}(r_w) = \text{Var}(\theta(r-r_f) + r_l - r_f) = \theta^2\sigma^2 + \sigma_l^2. \quad (72)$$

Using M^n to denote the set of distributions and $[\underline{\mu}^n, \bar{\mu}^n]$ to denote the corresponding interval of means, we can use the above in the investor's objective function to get

$$\max_{\theta} \min_{m \in M^n} \mathbb{E}_m \left[\theta(r-r_f) + \frac{R_l}{R_f} + \frac{\theta-\theta^2}{2}\sigma^2 + \frac{1}{2}\sigma_l^2 - r_f + \frac{1-\gamma}{2} [\theta^2\sigma^2 + \sigma_l^2] \right] \quad (73)$$

The terms r_f , $\frac{1}{2}\sigma_l^2$, $\frac{R_l}{R_f}$, and $\frac{1-\gamma}{2}\sigma_l^2$ are monotonic increases which will not affect the optimal choice of θ and hence can be ignored. This leaves the objective function

$$\max_{\theta} \min_{m \in M^n} \mathbb{E}_m \left[\theta(r-r_f) + \frac{\theta-\theta^2}{2}\sigma^2 + \frac{1-\gamma}{2}\theta^2\sigma^2 \right] \quad (74)$$

The first order condition of this objective with respect to θ is

$$\begin{aligned} 0 &= \min_{m \in M^n} \mathbb{E}_m \left[(r-r_f) + \frac{1}{2}\sigma^2 - \theta\sigma^2 + (1-\gamma)\theta\sigma^2 \right] & \text{if } \theta > 0 \\ 0 &\in \left\{ \mathbb{E}_m \left[(r-r_f) + \frac{1}{2}\sigma^2 - \theta\sigma^2 + (1-\gamma)\theta\sigma^2 \right] : m \in M^n \right\} & \text{if } \theta = 0 \\ 0 &= \max_{m \in M^n} \mathbb{E}_m \left[(r-r_f) + \frac{1}{2}\sigma^2 - \theta\sigma^2 + (1-\gamma)\theta\sigma^2 \right] & \text{if } \theta < 0 \end{aligned} \quad (75)$$

which implies a demand of

$$\theta_1^n(M^n) = \begin{cases} \frac{\underline{\mu} - r_f + \frac{1}{2}\sigma^2 - p}{\gamma\sigma^2} & \text{if } \underline{\mu} - r_f + \frac{1}{2}\sigma^2 - p > 0 \\ 0 & \text{if } \underline{\mu} - r_f \leq p - \frac{1}{2}\sigma^2 \leq \bar{\mu} - r_f \\ \frac{\bar{\mu} - r_f + \frac{1}{2}\sigma^2 - p}{\gamma\sigma^2} & \text{if } \bar{\mu} - r_f + \frac{1}{2}\sigma^2 - p < 0 \end{cases} \quad (76)$$

This demand is similar to that in the case when signals are directly related to the asset. The difference here, is that now the range of values for $E(r)$ given the ambiguity in the labor income signal will be slightly different. Despite this difference, given the above, this model with non-tradeable labor income can be mapped directly into the model presented previously by changing a single parameter—the relative importance of the signal (modeled there by $\rho_e/(\rho_0 + \rho_e)$) in the updating of beliefs about the mean of the (log) asset payoff. The rest of the analysis conducted previously will then follow immediately.

7 Concluding remarks

In this paper, we show how partially revealing rational expectations prices may arise when with ambiguous private information is received by ambiguity averse investors in a financial market. This informational inefficiency can arise due to inertia with respect to information, which in turn arises under non-smooth ambiguity averse preferences such as the Gilboa and Schmeidler (1989) representation.

The partial revelation property is different from that generated through noise and can lead to large variation in price and price volatility without a large change in fundamentals or any change in volatility of fundamentals. Moreover, informationally inefficient prices can coincide with lower price impact of trade than informationally efficient prices and there can be large changes in price impact without a large change in fundamentals.. Changes in wealth shares, arrivals of public signals, individual learning by informed investors may directly affect the informational efficiency of prices.

References

- AHN, D., S. CHOI, S. KARIV, AND D. GALE (2011): “Estimating Ambiguity Aversion in a Portfolio Choice Experiment,” mimeo.
- ALLEN, B. (1981): “Generic existence of completely revealing equilibria for economies with uncertainty when prices convey information,” *Econometrica*, 49, 1173–1199.
- ALLEN, B., AND J. S. JORDAN (1998): “The existence of rational expectations equilibrium: a retrospective,” in *Organizations with incomplete information: essays in economic analysis*, ed. by M. Majumdar. Cambridge University Press, Also available as Federal Reserve Bank of Minneapolis Research Department Staff Report 252.
- ANSCOMBE, F., AND R. AUMANN (1963): “A definition of subjective probability,” *Annals of Mathematical Statistics*, 34, 199–205.
- AUSUBEL, L. M. (1990): “Insider Trading in a Rational Expectations Economy,” *The American Economic Review*, 80(5), pp. 1022–1041.
- BARLEVY, G., AND P. VERONESI (2003): “Rational panics and stock market crashes,” *Journal of Economic Theory*, 110(2), 234 – 263.
- BERNARDO, A. E., AND K. L. JUDD (2000): “Asset market equilibrium with general tastes, returns, and informational asymmetries,” *Journal of Financial Markets*, 3(1), 17 – 43.
- BIAIS, B., P. BOSSAERTS, AND C. SPATT (2010): “Equilibrium Asset Pricing and Portfolio Choice Under Asymmetric Information,” *Review of Financial Studies*, 23(4), 1503–1543.
- BLUME, L., D. EASLEY, AND M. O’HARA (1994): “Market statistics and technical analysis: The role of volume,” *Journal of Finance*, 49(1), 153–181.
- BOSSAERTS, P., P. GHIRARDATO, S. GUARNESCHELLI, AND W. ZAME (2010): “Ambiguity in Asset Markets: Theory and Experiment,” *Review of Financial Studies*, 23, 1325–1359.
- CAMPBELL, J., AND L. VICIERA (2002): *Strategic asset allocation: portfolio choice for long-term investors*. Oxford University Press.

- CAPLIN, A., AND J. LEAHY (1994): “Business as usual, market crashes, and wisdom after the fact,” *American Economic Review*, 84, 548–565.
- CASKEY, J. A. (2008): “Information in equity markets with ambiguity-averse investors,” Forthcoming in *The Review of Financial Studies*.
- CHAPMAN, D. A., AND V. POLKOVNICHENKO (2009): “First-Order Risk Aversion, Heterogeneity, and Asset Market Outcomes,” *The Journal of Finance*, 64(4), pp. 1863–1887.
- CONDIE, S., AND J. V. GANGULI (2011a): “Ambiguity and rational expectations equilibria,” *Review of Economic Studies*, 78(3), 821–845.
- CONDIE, S., AND J. V. GANGULI (2011b): “Informational efficiency with ambiguous information,” forthcoming in *Economic Theory*.
- DANIEL, K., AND S. TITMAN (2006): “Market Reactions to Tangible and Intangible Information,” *The Journal of Finance*, 61(4), pp. 1605–1643.
- DE CASTRO, L., M. PESCE, AND N. YANNELIS (2010): “A new perspective to rational expectations: maximin rational expectations equilibrium,” mimeo, University of Illinois, Urbana-Champaign.
- DOW, J., AND S. R. DA COSTA WERLANG (1992a): “Excess volatility of stock prices and Knightian uncertainty,” *European Economic Review*, 36(2-3), 631 – 638.
- DOW, J., AND S. R. DA COSTA WERLANG (1992b): “Uncertainty aversion, risk aversion, and the optimal choice of portfolio,” *Econometrica*, 60(1), 197–204.
- DOW, J., AND G. GORTON (2008): “Noise trader,” in *The New Palgrave Dictionary of Economics*, ed. by L. Blume, and S. Durlauf. Palgrave Macmillan.
- EASLEY, D., AND M. O’HARA (2009): “Ambiguity and Nonparticipation: The Role of Regulation,” *Rev. Financ. Stud.*, 22(5), 1817–1843.
- EASLEY, D., M. O’HARA, AND L. YANG (2011): “Opaque trading, disclosure, and asset prices,” Rotman School of Management Working Paper No. 1945347.
- ELLSBERG, D. (1961): “Risk, ambiguity, and the Savage axioms,” *Quarterly Journal of Economics*, 75, 643–669.

- EPSTEIN, L., AND M. SCHNEIDER (2008): “Ambiguity, information quality and asset pricing,” *Journal of Finance*, 63, 197–228.
- EPSTEIN, L. G., AND M. SCHNEIDER (2007): “Learning under ambiguity,” *Review of Economic Studies*, 74, 1275–1303.
- (2010): “Ambiguity and Asset Markets,” *Annual Review of Financial Economics*, 2(1), 315–346.
- GILBOA, I., AND D. SCHMEIDLER (1989): “Maxmin expected utility with non-unique prior,” *Journal of Mathematical Economics*, 18, 141–153.
- GOLDSTEIN, I., AND A. GUEMBEL (2008): “Manipulation and the Allocational Role of Prices,” *The Review of Economic Studies*, 75(1), pp. 133–164.
- GROSSMAN, S. (1976): “On the efficiency of competitive stock markets where trades have diverse information,” *Journal of Finance*, 31, 573–585.
- GROSSMAN, S. J. (1981): “An Introduction to the Theory of Rational Expectations Under Asymmetric Information,” *Review of Economic Studies*, 48, 541–559.
- GROSSMAN, S. J., AND J. E. STIGLITZ (1980): “On the impossibility of informationally efficient markets,” *American Economic Review*, 70(3), 393–408.
- HANSEN, L., AND T. SARGENT (2007): *Robustness*. Princeton University Press.
- ILLEDITSCH, P. K. (2010): “Ambiguous Information, Portfolio Inertia, and Excess Volatility,” forthcoming in the *Journal of Finance*.
- JOHNSON, N., AND S. KOTZ (1970): *Continuous univariate distributions - 1*. Houghton Mifflin.
- KEYNES, J. M. (1921): *A treatise on probability*. Macmillan.
- KLIBANOFF, P., M. MARINACCI, AND S. MUKERJI (2005): “A smooth model of decision-making under ambiguity,” *Econometrica*, 73, 1849–1892.
- KNIGHT, F. (1921): *Risk, Uncertainty, and Profit*. Houghton, Mifflin.
- KREBS, T. (2007): “Rational expectations equilibrium and the strategic choice of costly information,” *Journal of Mathematical Economics*, 43(5), 532 – 548.

- MACCHERONI, F., M. MARINACCI, AND A. RUSTICHINI (2006): “Ambiguity aversion, robustness, and the variational representation of preferences,” *Econometrica*, 74, 1447–1498.
- MAILATH, G., AND A. SANDRONI (2003): “Market selection and asymmetric information,” *Review of Economic Studies*, 70, 343–368.
- MELE, A., AND F. SANGIORGI (2011): “Ambiguity, information acquisition, and price swings in asset markets,” mimeo.
- MUENDLER, M.-A. (2007): “The Possibility of Informationally Efficient Markets,” *Journal of Economic Theory*, 133, 467–483.
- OZSOYLEV, H., AND J. WERNER (2011): “Liquidity and asset prices in rational expectations equilibrium with ambiguous information,” forthcoming in *Economic Theory*.
- PERESS, J. (2004): “Wealth, Information Acquisition, and Portfolio Choice,” *Review of Financial Studies*, 17(3), 879–914.
- PERESS, J. (2009): “Wealth, Information Acquisition and Portfolio Choice: A Correction,” mimeo, INSEAD.
- RADNER, R. (1979): “Rational expectations equilibrium: Generic existence and the information revealed by prices,” *Econometrica*, 47(3), 655–678.
- SAVAGE, L. J. (1954): *The foundations of statistics*. John Wiley and Sons.
- SCHNEIDER, J. (2009): “A Rational Expectations Equilibrium with Informative Trading Volume,” *Journal of Finance*, 44, 2783–2805.
- TALLON, J.-M. (1998): “Asymmetric information, nonadditive expected utility, and the information revealed by prices: an example,” *International Economic Review*, 39, 329–342.
- WATANABE, M. (2008): “Price Volatility and Investor Behavior in an Overlapping Generations Model with Information Asymmetry,” *The Journal of Finance*, 63(1), pp. 229–272.

YUAN, K. (2005): "Asymmetric price movements and borrowing constraints: a rational expectations equilibrium model of crises, contagion, and confusion," *Journal of Finance*, 60, 379–411.