CONDORCET MEETS ELLSBERG

ANDREW ELLIS

Abstract. The Condorcet Jury Theorem states that given a subjective expected utility electorate with common values, the equilibrium probability that the correct candidate wins goes to one as the size of the electorate goes to infinity. This paper studies strategic voting when voters have pure common values but may be ambiguity averse–exhibit Ellsberg-type behavior–as modeled by maxmin expected utility preferences. It provides sufficient conditions so that the equilibrium probability of the correct candidate winning the election is bounded above by one half in at least one state. As a consequence, no equilibrium aggregates information, in contrast to the Condorcet Jury Theorem. However, information may aggregate even when no voter has subjective expected utility preferences.

1. Introduction

1.1. **Objectives and outline.** When deciding how to vote, an individual may have private information about which of the two candidates will be better. A large literature studies the effect of this information on how a rational voter casts her ballot. Within this literature, one of the most important results is the "Condorcet Jury Theorem" (henceforth, CJT). It states that if each voter maximizes her subjective expected utility (henceforth, SEU) and has common values, then there exists an equilibrium to the voting game in which all private information is revealed for a large enough electorate. If a majority of voters would prefer candidate A over candidate B if they knew the true state of the world, then in an election with n voters the probability that A wins goes to one as n goes to infinity. This result provides an important efficiency justification for democracy as a political system because it provides conditions under which democracy is superior to even a benevolent dictatorship,

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since the probability of selecting the better policy is higher when an election rather than a privately informed dictator picks the policy.

This paper shows that if voters' private information is ambiguous and voters are ambiguity averse, there may not be an equilibrium that aggregates information regardless of the size of the electorate. In fact, Theorem 3 shows that for many voting games, *no equilibrium of the game reveals information.* This contrasts with previous results showing that information always aggregates when voters have SEU preferences. A rational voter conditions her action on the probability her vote changes the outcome of the election. Because of this conditioning, an SEU voter plays a different strategy in equilibrium than the one that her private information alone would suggest is best. However, each vote noisily reveals her private information and, with enough voters, information aggregates. When voters are ambiguity averse, each picks a voting strategy as if to insure herself against altering the outcome in favor of the worse candidate. In equilibrium, no vote reveals information, precluding aggregation.

A large literature, initiated by Ellsberg [1961], criticizes SEU on both normative and descriptive grounds.[1] SEU does not accurately describe preferences when the information provided about the likelihood of events differs in precision. An ambiguous event is an event the decision maker has only vague information about. An ambiguity averse decision maker prefers to bet on events that occur with a known frequency to comparable ambiguous events. For example, a bet on an event E, which is known to occur with $p = .5$, may be preferred both to a bet on the event F and a bet on its complement F^c when no information is provided about F. Ambiguity aversion explains evidence from asset markets that contradicts SEU (for instance, see Epstein and Schneider [2010]).

Many important policy decisions depend on ambiguous events. A policy to cap carbon emissions deals with poorly understood costs, base case emissions, and tails of the probability distribution of temperature changes. The recession of 2008-2009 resulted at least in part from an unprecedented event (systematic default in AAA rated bonds) in the credit market. The

[1]Ellsberg describes an urn containing three balls, exactly one of which is red and the other two may be either blue or green in any proportion. Subjects typically prefer to bet on "red" rather than bet on "blue" but also prefer to bet on "blue or green" rather than "red or green." This behavior is incompatible with SEU.

Federal Reserve decided whether or not to bail out banks and hedge funds based on their beliefs about the poorly understood connection between this event, these companies, and the financial system as a whole. Many foreign policy decisions must be made despite possessing only poor quality information, such as that leading to the 2003 invasion of Iraq.

To accommodate ambiguity averse voters, this paper assumes that voter preference conforms to maxmin expected utility (henceforth, MEU; introduced and axiomatized in Gilboa and Schmeidler [1989]). This model differs from SEU in that it allows a set of probability measures to describe the decision maker's behavior in the face of uncertainty. Voters evaluate an act by taking the expected value using each of the probability measures in the set and then taking the minimum of these expectations. Formally, for some closed and convex set of probability measures Π, the utility of an act f can be written as

$$\min_{p\in\Pi} \mathbb{E}_p[u \circ f].$$

SEU is the special case when Π is singleton. When Π is not singleton, the behavior in the Ellsberg paradox can be rationalized.

Section 2 gives an example that illustrates how ambiguity averse voters behave differently from their SEU counterparts. The example shows that even when sincere voting would maximize ex-ante welfare over *all* strategy profiles in a common values election, voting sincerely may not be an equilibrium. Section 3 introduces ambiguous Poisson games, the class of games that will be used to study voting with large numbers of voters, and proves existence of an equilibrium. Section 4 describes a common values voting game when voter preferences are MEU and presents the paper's main results. Theorem 3 shows that ambiguity aversion can preclude the existence of any equilibrium that aggregates information. Theorem 4 provides sufficient conditions for existence of an equilibrium that aggregates information. Section 5 modifies the setup by allowing voters to abstain strategically. Theorem 6 shows that the main result holds in this setting as well. Proofs are collected in the appendices.

1.2. **Literature review.** In economics and political science, most work on the CJT describes the conditions under which information aggregation is an equilibrium for a voting game. Seminal work by Austen-Smith and Banks [1996] showed that sincere voting is not a Nash equilibrium in general. This led to Feddersen and Pesendorfer [1996, 1997, 1999], McLennan [1998] and Myerson [1998] (among others), who show that there exists a Nash equilibrium for the voting game that aggregates information. All work in this literature assumes SEU.

Other papers that assume instrumental voters and show a failure of the CJT include Feddersen and Pesendorfer [1997], Bhattacharya [2008] and Mandler [2010]. Feddersen and Pesendorfer prove that if the distribution of preferences is not common knowledge, then the CJT will fail for a generic utility function. Bhattacharya drops the assumption of common values and characterizes the distributions of preferences for which information aggregation fails. Mandler shows that if the conditional distribution of signals is unknown, then the CJT may fail as well. In Mandler's setting, even if all the signals were observed by each voter, uncertainty would remain as to which state is correct even as the number of voters goes to infinity. This paper maintains pure common values and a known conditional distribution of signals. As a consequence, the distribution of preferences is known and if all signals were observed by each voter, uncertainty would vanish throughout the electorate.

Condie and Ganguli [2011] also document a failure of information aggregation with ambiguity averse agents. They show that a rational expectations equilibrium for an exchange economy may be partially revealing when agents are ambiguity averse; in contrast, fully revealing equilibria are generic with SEU agents. A key difference from this paper is that agents do not act strategically: they are price takers.

There is a small literature that addresses political economy questions with ambiguity averse voters or candidates. Two papers that consider ambiguity averse voters are Ghirardato and Katz [2006] and Ashworth [2005]. Ghirardato and Katz show that MEU voters can find abstention to be optimal when there is ambiguity about candidates' policies. Ashworth uses a related model of incomplete preferences (Bewley [2002]) to show that campaigns may try to release information to induce voters to abstain rather than to inform voters. Neither paper studies strategic interaction between voters or the correlation between the election's outcome and the true state of the world. Berliant and Konishi [2005] show that ambiguity can cause candidates to avoid revealing their position on certain issues. Bade [2010] considers candidates competing over platforms and shows that a game with MEU candidates is more likely to have an equilibrium in pure strategies than is the corresponding game with SEU candidates.

2. Sincere Voting and Ambiguity

This section offers a brief example showing that ambiguity aversion alters the set of equilibria to voting games. In the example, sincere voting aggregates information and maximizes social welfare but is not an equilibrium. Though the remainder of the paper considers games with population uncertainty and this section does not, the intuition for why voters play the equilibrium strategy in the example is identical to that in games with population uncertainty. A formal definition of the game and equilibrium will be deferred to Appendix A.

2.1. **Example.** Consider an election with 101 voters who vote for one of two candidates, A and B. The candidate with the most votes wins. Suppose there are two states of the world, a and b, and all voters agree that A's policy is better in state a but B's policy is better in state b. Before voting, all voters observe a signal from the set $\{1, 2\}$. They believe that signal 1 occurs with probability 0.6 in state A, that signal 2 occurs with probability 0.6 in state B, and that signals are independently distributed conditional on the state of the world. Ex-ante, voters have MEU preferences with set of priors Π. Because the state space is one dimensional, it is convenient to represent Π as the interval $[\underline{p}, \bar{p}]$ corresponding to the probability each measure in Π assigns to a. Assume that voters get utility equal to 1 if the correct candidate is elected but 0 otherwise and that the interval $[\underline{p}, \bar{p}]$ is symmetric about $\frac{1}{2}$. That is, ex ante preference is represented by

$$\min_{p \in [\underline{p}, \overline{p}]} [p(Pr(A \text{ wins}|a)) + (1-p)Pr(B \text{ wins}|b)]. \quad (2.1)$$

After observing signal t, each voter considers the set of posteriors Π_t consisting of the Bayesian updates of the probability measures in Π. This ensures that each voter is dynamically consistent: if she believes that a voting strategy is optimal conditional on observing either signal, she finds the same strategy optimal ex ante.

Because of the noted symmetry, a voter (strictly) prefers to bet on A over B if she observes signal 1 and vice versa if she observes signal 2. If all voters who observe 1 vote for A and all those who observe 2 vote for B, information aggregates and the objective in (2.1) is maximized. If voters were SEU ($\underline{p} = \bar{p} = \frac{1}{2}$), Theorem 1 of McLennan [1998] would show that this sincere voting strategy is an equilibrium. In that equilibrium, information aggregates and each voter receives the same expected utility in each state, about 0.979. However, if $\underline{p} < 0.4$ and $0.6 < \overline{p}$, sincere voting is not an equilibrium because all players best respond by voting for both A and B with equal probability.

For instance, assume that $\underline{p} = .39$ and $\overline{p} = .61$. After updating, players who observe 1 use $\Pi_1 = [0.49, 0.7]$ and players who observe 2 use $\Pi_2 = [0.3, 0.51]$. Consider the problem of an arbitrary voter when all the others vote sincerely. If this voter observes signal 1, she picks her vote to maximize

$$\min_{p \in [0.49, 0.7]} [pPr(A \text{ wins}|a) + (1-p)Pr(B \text{ wins}|b)]. \tag{2.2}$$

She affects the outcome only when she is pivotal, or when exactly 50 of the others vote for A. Since all others vote sincerely,

$$Pr(A \text{ has 50 votes}|a) = Pr(B \text{ has 50 votes}|b) = \binom{100}{50} .6^{50}.4^{50} = \rho,$$

which is approximately 0.01, and

$$Pr(\text{ 51+ votes for } A|a) = Pr(\text{ 51+ votes for } B|b) = \sum_{j=51}^{100} \binom{100}{j} .6^{j}.4^{100-j} = \theta,$$

which is approximately 0.973. If she votes for A with probability α, then

$$Pr(A \text{ wins}|a) = \theta + \rho\alpha$$

and

$$Pr(B \text{ wins}|b) = \theta + \rho(1-\alpha).$$

Therefore, this voter's utility from voting for A with probability α is

$$\min_{p\in[0.49,0.7]} p[\alpha\rho+\theta]+(1-p)[(1-\alpha)\rho+\theta]. \tag{2.3}$$

If she voted sincerely, she would always vote for A ($\alpha=1$) and her utility would be

$$\min_{p\in[0.49,0.7]} p[\rho+\theta]+(1-p)\theta=\theta+.49\rho,$$

about 0.9779. If she played her other pure strategy, voting for B ($\alpha=0$), she would get utility

$$\min_{p\in[0.49,0.7]} p\theta+(1-p)[\theta+\rho]=\theta+.3\rho,$$

about 0.976 which is less than if she voted for A.

When the voter picks her strategy, the state of the world is determined but unknown. By randomizing, she replaces subjective uncertainty with objective risk. Although she prefers to vote sincerely rather than vote against her signal, voting for A and B with equal probability insures her against ambiguity. By doing so, she receives utility equal to

$$\min_{p\in[0.49,0.7]} p[\theta+.5\rho]+(1-p)[\theta+.5\rho]=\theta+.5\rho,$$

about 0.978, so she prefers this mixture to sincere voting. A symmetric argument shows that the voter also prefers to mix in this way after observing signal 2. Hence, her best response is to randomize between voting for A and B regardless of the signal she observes.

As in the SEU case, each voter picks her strategy based on her "beliefs" about the state of the world if her vote decides the election. If all voters were SEU, each vote would reveal something about the voter's private information, and as the number of voters approached infinity, the outcome of the election would reflect all private information. In this example, the voter minimizes the probability she makes a mistake (conditional on her being pivotal) by randomizing between voting for A and B. She thinks that if she is pivotal, she will make a mistake with probability as high as 0.51 by voting for A or 0.7 by voting for B . By mixing, she makes a mistake with a probability of exactly 0.5. Because the voter is ambiguity averse, she strictly prefers the latter strategy. However, by mixing she does not reveal her signal, so if the whole electorate plays this strategy then information cannot aggregate. Indeed, all voters randomizing as above is an equilibrium to this game.

The rest of the paper considers large games with an electorate that is distributed according to the Poisson distribution. In these games, voters find insurance strategies like the one above optimal. In the ambiguous voting game that corresponds to this example, all equilibria are uninformative (see Theorem 3 and Proposition 1).

2.2. **Implications.** This example provides insight into the games of common interest studied by McLennan [1998]. A game of common interest is one where all players agree on the ranking of alternatives (given the state of the world) and have the same prior beliefs. He proves that if a strategy profile maximizes social welfare, this strategy profile is an equilibrium. The next theorem says that this result does not apply to games with MEU players.

Theorem 1. *There are games of common interest with MEU players where a strategy profile maximizes the ex-ante utility function but is not an equilibrium.*

The proof is the counter-example above (formal definitions and details are in appendix A). An adaptation of Theorem 1 of McLennan [1998] fails because the proof implicitly uses Savage [1954]'s sure thing principle (P2) and MEU players violate this axiom in general.[2] This suggests that the result is not robust to other deviations from SEU, such as rank-dependent expected utility or the smooth ambiguity model. Further, all work that relies on McLennan's result (for instance, Duggan and Martinelli [2001], Peleg and Zamir [2011] or Ahn and Oliveros [2011]) also depends crucially on the assumption of SEU.

3. Ambiguous Poisson Games

This section introduces ambiguous Poisson games. Since standard voting models assume SEU, analyzing the impact of ambiguity aversion requires a generalization of a standard voting game. A natural candidate for such a generalization is *extended Poisson games.* Myerson [1998] shows that these games simplify the analysis of large population games with some underlying uncertainty. He proves that if an extended Poisson voting game has a

[2]McLennan's result would also fail if voter preferences did not satisfy dynamic consistency. That is, they had preferences ex-post that differed substantially from the ex-ante preference. In the example above, voters are dynamically consistent, which rules out this reason.

common prior, common values and informative signals, then there exists an equilibrium that aggregates information (see Theorem 5 for a formal statement). Motivated by this result, this section modifies extended Poisson games to allow for MEU players, providing a formal definition of the game and equilibrium. The notation and definition of equilibrium are adapted from Myerson [1998]. Theorem 2 proves existence of an equilibrium.

For any finite set E, denote by ΔE the set of probability measures on E .

Definition. An *ambiguous Poisson game* Γ is a collection $(\Omega, C, T, U, (\Pi_t)_{t\in T}, r, n)$ where:

- Ω is a finite set of states.
- C is a finite set of actions. Define $Z(C) = \{x \in \mathbb{R}^C : x(c) \in \mathbb{N}_{++} \forall c \in C\}$, the set of all possible realized action profiles (the number of players taking each action).
- T is a finite set of types.
- $U : T \times C \times \Omega \times Z(C) \to \mathbb{R}$ is a bounded function that represents preference over consequences. $U(t, c, \omega, x)$ is the utility for a voter of type t who takes action c when the realized state is ω and the realized action profile is x.
- $\Pi_t \subset \Delta(\Omega)$ is a closed (in the weak* topology), non-empty and convex set, representing the set of posteriors for each type. If Π_t is singleton for all t, then all players are SEU, though they may have different priors.
- $r : \Omega \to \Delta T$ maps each state to a probability measure so that types are drawn independently according to $r(\omega)$ in state ω.
- The number of players is a random variable distributed Poisson with mean $n \in \mathbb{R}_{++}$.

The timing of the game is as follows. First, Nature chooses the number of players according to the Poisson distribution with mean n and chooses the state of the world according to some unknown, unmodeled procedure. Each player learns her type, forms a set of posteriors, and then picks a strategy $s \in \Delta C$ before learning how many other players there are or what actions the other players have taken.[3] Because this happens after the state of the world is

[3]Note that posterior beliefs rather than prior beliefs are taken as a primitive. On the one hand, one could close the model by specifying a set of priors and an updating rule (in the example from section 2, the updating rule is prior-by-prior Bayesian updating). On the other hand, there are no ex-ante actions and

realized but before the player learns it, each player acts as if Nature picked the distribution over states with the goal of minimizing her utility in anticipation of her choice of strategy. A mixed strategy may equalize her expected utility across states, limiting her exposure to Nature's choice. For this reason, she may find a mixed strategy to be the only best response. For a more in depth discussion of this issue, see Lo [1996] or Klibanoff [1996].

As in Myerson [1998], assuming a Poisson population yields convenient properties. Because types are conditionally independent and the population is distributed Poisson, the number of players that take each action c in state ω is also distributed Poisson. Moreover, the number of players taking action $c' \neq c$ in state ω is independent of the number of players taking action c in state ω and each player's conditional expectation does not depend on her private information. If $\lambda(\omega)(c)$ is the expected number of players in state ω that take action c, the probability of any given action profile x in state ω is $p(x|\lambda(\omega))$ where

$$p(x|\lambda) = \prod_{c \in C} \frac{e^{-\lambda(c)} \lambda(c)^{x(c)}}{x(c)!}. \tag{3.1}$$

These properties imply that the best response correspondence for any voter of a given type is the same. A *strategy profile* σ is a map from types to strategies, i.e. $\sigma : T \to \Delta(C)$. A player of type t plays strategy $\sigma_t \in \Delta C$, picked to maximize

$$V_t(\sigma_t, \sigma) = \min_{q \in \Pi_t} \int_\Omega \int_{Z(C)} \sum_{c \in C} \sigma_t(c) U(t, c, \omega, x) dp(x|\lambda(\omega)) dq(\omega) \tag{3.2}$$

where

$$\lambda(\omega)(a) = n \sum_{t \in T} \sigma(t)(a) r(t|\omega). \tag{3.3}$$

Definition. A strategy profile σ^* is an equilibrium for Γ if for each $t \in T$

$$\sigma^*(t) \in \arg\max_{\hat{\sigma} \in \Delta C} V_t(\hat{\sigma}, \sigma^*). \tag{3.4}$$

there isn't a consensus within the literature on which updating rule is best. Since any updating rule and set of priors defines a special case of the model above, there's no need to impose a specific model of updating.

If σ^* is an equilibrium, then every player picks her strategy to maximize the minimum expected utility over all measures in her set of posteriors, given she knows that the other players follow the strategy profile σ^*. When Π_t is singleton for all $t \in T$ this definition is equivalent to the definition in Myerson [1998]. Because each player maximizes her minimum expected utility given her beliefs and all player's beliefs agree, the behavior of each player is as in Lo [1996]'s "Beliefs Equilibrium with Agreement." While he does not consider games with population uncertainty, this definition of equilibrium otherwise coincides with his.

Theorem 2. *For any ambiguous Poisson game* Γ, *there exists a strategy profile* σ^* *that is an equilibrium for* Γ.

The proof is standard.

4. The Condorcet Jury Theorem

This section describes common values voting games with MEU players and discusses the limiting equilibria. Theorem 3 establishes the existence of voting games with no equilibrium that aggregates information. Theorem 4 shows that information aggregates for some voting games where no voter is SEU.

4.1. **Ambiguous voting games.** Candidates A and B each commit to a distinct policy. Voters cast a vote for either candidate, and the candidate with the most votes wins; in a tie, each candidate is selected with equal probability. These voters have common values and are instrumentally rational: they care only about the policy outcome and they have the same preference over policies given the state. Depending on the state of the world, the policy is either good or bad. There are two states, a and b, representing which policy is the good one.

Formally, an *ambiguous voting game* is an ambiguous Poisson game where the action set is $C = \{A, B\}$, the set of states is $\Omega = \{a, b\}$ and the utility function of all types takes value 1 if the candidate elected matches the state and 0 otherwise. The action A is interpreted as a vote for candidate A, and B is interpreted as a vote for B, and the set of types T is interpreted as a set of signals.

Given others play strategy profile σ, the payoff to a voter of type t using strategy $\hat{\sigma} \in \Delta\{A, B\}$ is

$$\begin{aligned} V_t(\hat{\sigma}, \sigma) \quad = \quad & \min_{\pi \in \Pi_t} \{\pi(a)[\hat{\sigma}(A) Pr(A \text{ wins}|a, v_A, \sigma) + \hat{\sigma}(B) Pr(A \text{ wins}|a, v_B, \sigma)] + \\ & + \pi(b)[\hat{\sigma}(A) Pr(B \text{ wins}|b, v_A, \sigma) + \hat{\sigma}(B) Pr(B \text{ wins}|b, v_B, \sigma)]\} \end{aligned}$$

where $Pr(c \text{ wins}|\omega, v_d, \sigma)$ is the probability candidate c wins in state ω if she votes for candidate d and others play strategy profile σ.

As in Section 2, represent Π_t by the interval of probabilities that the measures within it assign to a. That is, $\Pi_t \equiv [p_t, q_t]$ where $p_t = \min_{\rho \in \Pi_t} \rho(a)$ and $q_t = \max_{\rho \in \Pi_t} \rho(a)$.

4.2. **Information aggregation.** Because there is always some possibility of a mistake in a finite electorate, one cannot require full certainty that voters elect the proper candidate. In Poisson games, there is always an e^{-n} probability that the realized number of voters is zero so each candidate wins with at least probability $\frac{e^{-n}}{2}$ regardless of the state and strategy profile played. The following definition says that the probability of electing the wrong candidate vanishes along some sequence of equilibria to the game.

Definition. A sequence of ambiguous voting games $(\Gamma_n)_{n=1}^{\infty}$ satisfies *Full Information Equivalence (FIE)* if there exists a sequence of strategy profiles $(\sigma_n)_{n=1}^{\infty}$ so that σ_n is an equilibrium for Γ_n and for any $\epsilon > 0$ there exists an N so $n > N$ implies that the correct candidate is elected in each state with probability higher than $1 - \epsilon$ when σ_n is played.[4]

4.3. **Limiting Equilibria.** This paper's main result shows that FIE fails for a class of games. All results restrict attention to sequences of ambiguous voting games indexed by mean number of players, where all other primitives remain the same.

Definition. An ambiguous voting game *has voters who lack confidence* if $p_t < \frac{1}{2} < q_t$ for all $t \in T$.

[4]The term "Full Information Equivalence" originally comes from Feddersen and Pesendorfer [1997]. The above definition adapts it to the current setting.

Players' betting preferences reveal the likelihoods they assign to states. For instance, a voter might reveal that a is more likely than b by choosing a bet that pays off when the state is a over a bet that pays off when the state is b. If voters lack confidence, all voters strictly prefer to bet on the outcome of a fair coin toss over betting on either a or b. This is impossible with SEU: if a is at least as likely as b when a and b are the only two states, then a bet on a is at least as good as a fifty-fifty lottery. Even though the voter thinks that a is more likely than b, she thinks that b *may* be more likely than a. This translates into the voting setting as follows. For any fixed voter, if that voter were made a dictator, she would strictly prefer to pick the policy implemented by flipping a fair coin rather than implementing either policy for sure, irrespective of the signal she receives.

Suppose that voters form posterior beliefs by updating a common set of priors Π using prior-by-prior Bayesian updating. If this is the case, then precision of signals and the set of priors both contribute to posterior beliefs. Voters lack confidence when the signals do not provide enough information to offset the prior ambiguity. With very precise signals (there is some t so that $\frac{r(t|b)}{r(t|a)}$ is very high or very low), Π must be very close to $[0,1]$ for voters to lack confidence; conversely, if signals are not very precise ($\frac{r(t|b)}{r(t|a)}$ close to one for all t), then Π can be a much smaller interval. For instance, with the signal structure described in section 2, voters lack confidence whenever $[.4,.6] \subsetneq \Pi$, but if $r(1|a) = r(2|b) = .51$, then voters lack confidence whenever $[.49,.51] \subsetneq \Pi$. If $\Pi = [.45,.55]$, voter lack confidence with the second signal structure but not the first.

When voters lack confidence, no equilibrium aggregates information.

Theorem 3. *If $(\Gamma_n)_{n=1}^{\infty}$ is a sequence of ambiguous voting games with voters who lack confidence, then FIE fails for $(\Gamma_n)_{n=1}^{\infty}$. In particular, for n large enough all equilibria have the property that if the expected vote share for A in state a is greater than $\frac{1}{2}$, the expected vote share for B in state b is less than $\frac{1}{2}$ (and vice versa).*

Remark. This n only needs to be large enough so that no equilibrium in pure strategies exists. For instance, if $T = \{1,2\}$ and $r(1|a) = r(2|b)$, then any positive n is large enough.

The proof is by contradiction. Suppose that there is an equilibrium where the expected winners are correct. The first step characterizes each voter's best response correspondence. If the worst case scenario (the state in which the wrong candidate is most likely to be elected) is independent of the voter's strategy, the voter acts as if she's SEU. If all voters act as if SEU, Myerson [1998] Theorem 2 wold imply that there is an equilibrium that aggregates information.

To rule this out, the next step shows that in equilibrium, the worst case scenario depends on the candidate for whom she votes. To see this, suppose not and also that the equilibrium calls for each voter to play pure strategy. A corollary of Myerson [1998] Theorem 2 is that the expected vote share in each state is the same. As a consequence, any voter thinks that if she abstains, she'd be indifferent between which state occurred. When a player votes for A for sure, her expected utility in state a increases and her expected utility in state b decreases (and vice versa when voting for B). It follows that the worst case scenario depends on her vote. The proof extends this argument to the case where some voters play a mixed strategy.

The proof then analyzes the strategies played in the equilibrium. Because each voter is instrumental, she focuses only on the situations where she is pivotal: a vote for A (respectively, B) would change the outcome of the election from B (A) winning to a tie or from a tie to A (B) winning. Because the worst case scenario changes with her vote and voters lack confidence, no voter wants to play a pure strategy. As in Section 2, randomization insures the voters against making a mistake and altering the election in favor of the wrong candidate. In fact, all but at most one type of voter want to insure themselves by randomizing.

Finally, the proof analyzes the condition for information aggregation. If all voters insure themselves, information cannot aggregate because the insurance strategy is independent of private information. Therefore, it must be that some voter weakly prefers playing another mixed strategy instead of insuring herself. However, the only strategies that are at least as good as the insurance strategy assign higher probability to voting for the same candidate that receives more votes from the insurance strategy. All voters expect to vote for the same candidate regardless of signal. This candidate is the expected winner in both states, which contradicts that the equilibria aggregate information.

Theorem 3 shows that no equilibrium exists where the expected winners are both correct. The following result characterizes one equilibrium to the game.

Proposition 1. *If an ambiguous voting game* Γ *has voters who lack confidence, then the strategy profile* σ *defined by* $\sigma(t)(A) = \frac{1}{2}$ *for all* $t \in T$ *is an equilibrium for* Γ.

Neither this proposition nor Theorem 3 show that this is the only equilibrium. However, Theorem 3 shows that no equilibrium does better than this equilibrium in both states of the world. In this equilibrium, both candidates are elected with equal probability regardless of the state. Therefore, knowing the winner of the election would not change the beliefs of a Bayesian agent.

Theorem 3 provides sufficient conditions for FIE to fail. Since SEU is a special case of MEU, some ambiguous voting games must satisfy FIE. However, SEU is not necessary for FIE. In fact, the Theorem 4 will prove the existence of a sequence of equilibria that aggregates information whenever the game has disjoint* posteriors, defined as follows.

Definition. An ambiguous Poisson game has *disjoint* posteriors* if $[p_t, q_t] \cap [p_{t'}, q_{t'}]$ is either empty or contained in the boundary of both sets for all t and t' in T.

If all voters are SEU, then each Π_t is singleton and the ambiguous Poisson game has disjoint* posteriors. The following remark, a corollary of Lemma 1 (in the appendix), differentiates SEU, disjoint* posteriors and voters who lack confidence.

Remark. Consider an ambiguous voting game Γ. If Γ has singleton posteriors, then all voters act as SEU maximizers and none strictly prefer to randomize for any strategy profile. If Γ has disjoint* posteriors, then at most one type of voter strictly prefers to randomize. If Γ has voters who lack confidence, then there is a strategy profile so that all voters strictly prefer randomizing to playing a pure strategy.

For the following result, assume $0 < p_t \leq q_t < 1$ for all t, and for each ω and t, $r(t|\omega) > 0$. This assumption was not used for Theorem 3, but can be imposed there as well without altering results. If this condition is imposed, any SEU voting game derived from an ambiguous voting game by taking type t's beliefs to be a selection for Π_t will satisfy FIE, even if voters lack confidence in the original game.

Theorem 4. *Let $(\Gamma_n)_{n=1}^{\infty}$ be a sequence of ambiguous voting games that have disjoint* posteriors. If $\exists t \in T$ s.t. $r(t|a) \neq r(t|b)$, then $(\Gamma_n)_{n=1}^{\infty}$ satisfies FIE.*

The proof generalizes the construction in Theorem 2 of Myerson [1998]. As in that paper, the equilibrium consists of a "step strategy": at most one type of voter randomizes, and all others play a pure strategy, determined by how likely they view a relative to the randomizing voter. Because of disjoint* posteriors, at most one type of voter has a strict preference for randomization. The proof shows that in equilibrium, no voter strictly prefers to randomize and the limiting strategy will be the same as in the SEU game with the same signal structure.

Finally, note that Myerson [1998]'s Theorem 2 is a special case of Theorem 4. For completeness, it is restated below using the terminology of this paper.

Theorem 5. *(Myerson [1998]) Let $(\Gamma_n)_{n=1}^{\infty}$ be a sequence of ambiguous voting games such that for each t, Π_t is the singleton probability measure resulting from Bayesian updating of some common prior Q and $\exists t \in T$ s.t. $r(t|a) \neq r(t|b)$. Then $(\Gamma_n)_{n=1}^{\infty}$ satisfies FIE.*

This follows immediately from Theorem 4 because all posteriors are singleton, so the game has disjoint* posteriors.

5. Extension: Strategic Abstention

Results in Section 4 explicitly ruled out the possibility of abstention. However, Feddersen and Pesendorfer [1996, 1999] and Ghirardato and Katz [2006] show that strategic abstention may play an important role in voting games. This section extends the previous analysis so that voters can choose to abstain. The main finding is that some games still do not satisfy FIE.

Define an *ambiguous voting game with abstention (AVGA)* to be exactly as an ambiguous Poisson voting game before but $C = \{A, B, \emptyset\}$, where the action $\emptyset$ is viewed as abstaining. For simplicity and tractability, consider only on the case where there are two signals and both sets of posteriors are derived from Bayesian updating of a common set of priors.[5]

Definition. An ambiguous Poisson game has *Bayesian posteriors* if there exists a closed, convex and non-empty set of probability measures Π with full support so that for every $t \in T$,

$$\Pi_t = \{\frac{r(t|a)\pi(a)}{r(t|a)\pi(a) + r(t|b)\pi(b)} : \pi \in \Pi\}.$$

Signals are symmetric if $r(1|A) = r(2|B)$.

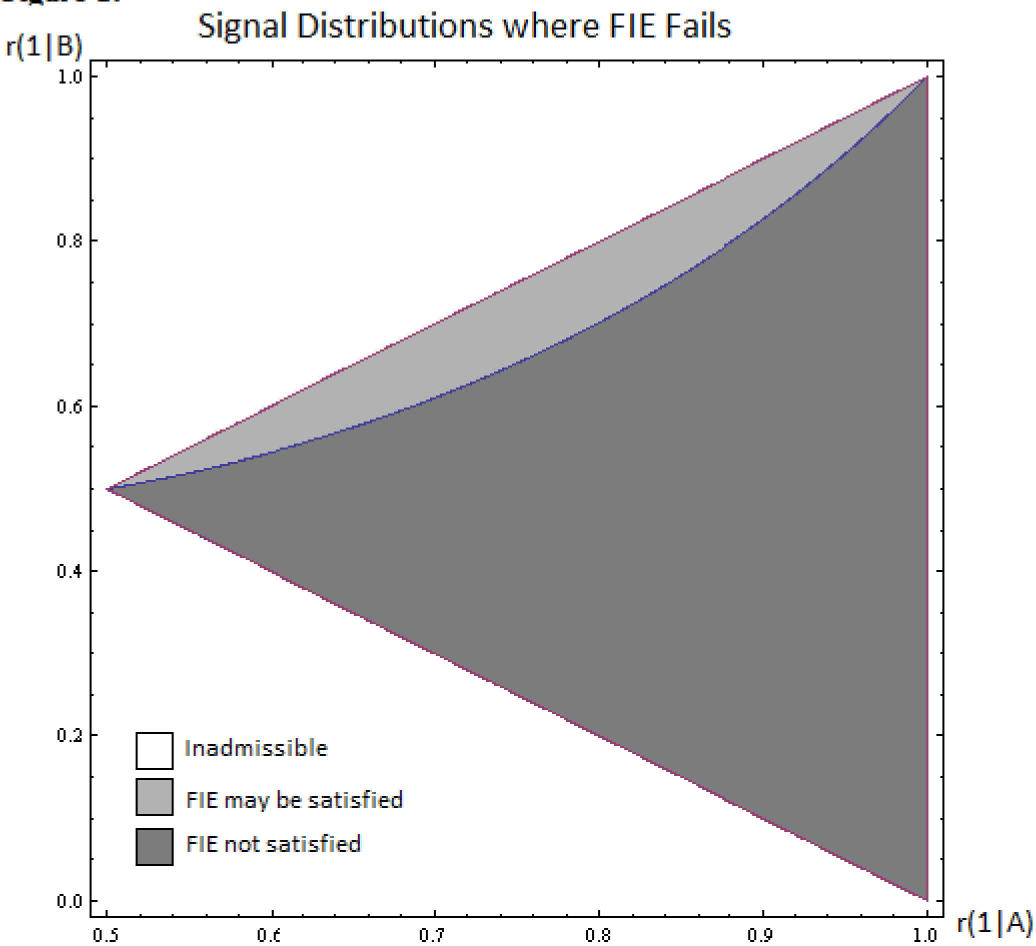
Theorem 6. *If $(\Gamma_n)_{n=1}^{\infty}$ is a sequence of AVGAs with voters who lack confidence, Bayesian posteriors and symmetric signals, then $(\Gamma_n)_{n=1}^{\infty}$ does not satisfy FIE.*

In the appendix, I relax symmetric signals substantially. Assuming WLOG that $r(1|a) + r(1|b) \geq r(2|a) + r(2|b)$ and $r(1|a) \geq r(1|b)$, the distributions of information that FIE will not hold for the area indicated by Figure 1.[6]

[5] Actually, I could assume instead that if $r(t|\omega) > r(t'|\omega)$ implies that $\min_{\pi \in \Pi_t} \pi(\omega) \geq \min_{\pi \in \Pi_{t'}} \pi(\omega)$ *and* $\max_{\pi \in \Pi_t} \pi(\omega) \leq \max_{\pi \in \Pi_{t'}} \pi(\omega)$.

[6] While the proof only shows that FIE fails in the dark gray region, I conjecture that FIE will fail in the light gray region as well. In fact, it's clear from the proof that FIE will fail for at least part of this region.

Figure 1:



With SEU voters, Feddersen and Pesendorfer [1996, 1999] and Bouton and Castanheira [2009] show that abstention leads to the "swing voter's curse." A less informed voter strictly prefers to abstain rather than vote for either alternative, but more informed voters do not abstain. This is positive for information aggregation because the percentage of votes cast by more informed voters is higher. An MEU swing voter perceives the probability of making a mistake with her vote to be larger than her SEU counterpart. As such, she is more likely to choose to abstain. In fact, the proof of Theorem 6 shows that with MEU voters who lack confidence, if a close election satisfies FIE, then *all voters prefer to abstain.* In contrast to SEU, abstention leads to no informed votes.

The equilibrium shown to exist in Proposition 1 requires voters to act as if flipping a coin. When allowing for abstention, this equilibrium still exists. However, there is another equilibrium in which all voters abstain. This is formalized by the following proposition.

Proposition 2. *If* Γ *is an AVGA that has voters who lack confidence, the strategy profile* σ^* *defined by* $\sigma^*(t)(\emptyset) = 1$ *for every* $t \in \{1, 2\}$ *is an equilibrium for* Γ*.*

This result is in stark contrast with Propositions 2 and 3 of Feddersen and Pesendorfer [1996] and Propositions 5 and 6 of Feddersen and Pesendorfer [1999]. In these papers, the fraction of voters who abstain remains bounded away from both zero and one along any sequence of equilibria. However, the equilibria given by Propositions 1 and 2 lead to the same distribution of outcomes in both states so the equilibria are the same from a payoff perspective.

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Appendix A. Details for Section 2

Following McLennan [1998], fix a finite set of alternatives $\mathcal{A}$, a finite set of players $I = \{0, 1, ..., n\}$ and a finite set of types T_i of types for each $i \in I$ and let $T := T_0 \times T_1 \times ... \times T_n$. Each player has the same preference over state-alternative pairs given by a function $u : T \times \mathcal{A} \to \mathbb{R}$. Suppose S_i is a finite set of pure strategies for each $i \in I$ and $S = S_0 \times S_1 \times ... \times S_n$. An aggregation rule $f : S \to \mathcal{A}$ maps the profile of actions to an alternative. Fix a non-empty, closed and convex set of common priors $\Pi \in \Delta T$. A *game of common interest with MEU players* is defined by the collection $(\mathcal{A}, I, T, u, S, f, \Pi)$.

For each i, let $\hat{S}_i : T_i \to \Delta S_i$ be a strategy for player i. This requires that the player's strategy be measurable with respect to her type. Let $\Sigma := \hat{S}_0 \times ... \times \hat{S}_n$ be the set of strategy profiles. As is convention, let σ_i denote player i's strategy and let σ_{-i} represents the vector of strategies chosen by players other than i. A strategy profile $\sigma^* \in \Sigma$ is an *equilibrium* for a game of common interest if

$$\sigma_i^*(t_i) \in \arg\max_{\sigma \in \Delta S_i} \min_{\pi \in \Pi} \mathbb{E}_\pi[\mathbb{E}_{\sigma^*_{-i}}[u((t_0, ..., t_n), f((s_0(t_0), ..., s_{i-1}(t_{i-1}), \sigma, s_{i+1}(t_{i+1}), ..., s_n(t_n))]|t_i]$$

for every $i \in I$.

To translate the example into this notation, let $n = 101$, $\mathcal{A} = \{A, B\}$ and $T_i = \{1, 2\}$ for $i > 0$ and $T_0 = \{a, b\}$. Define $u(\cdot)$ by

$$u((t_0, ..., t_{101}), c) \equiv u(t_0, c) = \begin{cases} 1 & t_0 = c \\ 0 & t_0 \neq c \end{cases}$$

for all T. Let $S_0 = \{\emptyset\}$ and $S_i = \{A, B\}$ for every $i > 0$. Set

$$f(s_0, ..., s_{101}) = \begin{cases} A & \text{if } \sum_{i=1}^{101} \chi_{s_i}(A) \geq 51 \\ B & otherwise \end{cases}$$

for all $(s_0, ..., s_{101}) \in S$, where $\chi_E(\cdot)$ is the indicator function of the set E. Define Π by

$$\Pi = \{\pi \in \Delta T : \pi(\{a\} \times T_1 \times ... \times T_{101}) \in [\underline{p}, \bar{p}] \text{ and}$$
$$\frac{\pi((a, t_1, ...t_{101}))}{\pi(\{a\} \times T_1 \times ... \times T_{101})} = \prod_{i=1}^{101} .6^{t_i}.4^{1-t_i} \text{ and}$$
$$\frac{\pi((b, t_1, ...t_{101}))}{\pi(\{b\} \times T_1 \times ... \times T_{101})} = \prod_{i=1}^{101} .6^{1-t_i}.4^{t_i}\}$$

which gives the desired form of priors. The remainder of the proof of Theorem 1 is given in the text.

Appendix B. Proof of Theorem 2

Proof. Define the set $\Lambda = \{l \in \mathbb{R}^{A\times\Omega} : \sum_{a\in A} l(a,\omega) = n\}$, noting that Λ is compact, and consider the correspondence $C_t : \Lambda \twoheadrightarrow \Delta A$ defined by

$$C_t(\lambda) = \arg \max_{\hat{\sigma}\in\Delta A} \min_{q\in\pi_t} \int_\omega \int_{Z(A)} \sum_{a\in A} \hat{\sigma}(a) U(x,t,a,\omega) dp(x|\lambda(\omega)) dq.$$

Define $C(\lambda) = \times_{t\in T} C_t(\lambda)$ and let $F : \Lambda \twoheadrightarrow \Lambda$ be defined by

$$F(\lambda) = \{n \sum_{t\in T} c_t(a) r(t|\omega) : c \in C(\lambda)\}.$$

If $\lambda \in F(\lambda)$ then (3.3) is satisfied, since then $\sigma^*(t) \in \arg\max_{\hat{\sigma}\in\Delta(A)} V_t(\hat{\sigma}, \sigma^*)$ for some σ^* that generates λ. Hence, existence of an equilibrium is equivalent to showing $F(\cdot)$ that has a fixed point. It remains to be shown that F is convex and closed. Since $F(\lambda)$ is an affine transformation of $C(\lambda)$, need to show that $C(\cdot)$ is convex and closed. Show first that all C_t are convex, compact and UHC.

[Convex:] Define $\phi : C(\Omega) \to \mathbb{R}$ by $\phi(f) = \min_{q\in\pi_t} \int f dq$, where $C(\Omega)$ is the set of continuous functions from Ω to the real numbers). Then ϕ and $p(x|\lambda(\omega))U(x,t,\cdot,\omega)$ are both concave. So $g : \Delta A \to \mathbb{R}$ defined by $g(\hat{\sigma}) = \phi(p(x|\lambda(\omega)) \sum_{a\in A} \hat{\sigma}(a) U(x,t,a,\omega))$ is also concave. Hence $g(x) = g(y) \implies g(\alpha x + (1-\alpha)y) \geq g(x) \forall \alpha \in [0,1]$ and $x, y \in C_t(\lambda) \implies \alpha x + (1-\alpha)y \in C_t(\lambda)$. Therefore $C_t(\lambda)$ is convex, from which it follows that $C(\cdot)$ is convex since a product of convex sets is convex. Since $C(\cdot)$ is convex, $F(\cdot)$ is convex.

[Closed:] ϕ is continuous by the Maximum Theorem (Theorem 17.31 of Aliprantis and Border [2006]; henceforth, AB). $p(x|\cdot)$ is continuous since it is a product of continuous functions. $U(x,t,\cdot,\omega)$ is continuous by assumption. So $\min_{q\in\pi_t} \int_\omega p(x|\lambda(\omega))[\sum_{a\in A} \hat{\sigma}(a) U(x,t,a,\omega)] dq$ is continuous. Hence $C_t(\lambda)$ is UHC and compact by the Maximum Theorem as the set of solutions to a maximization problem.

$C(\lambda)$ is compact for all λ by the Tychonoff product theorem (AB Theorem 2.61) because $C(\lambda)$ is a product of compact sets. By AB Theorem 17.20, it suffices to show that if $\lambda_n \to \lambda$, $x_n \in C(\lambda_n)$, and $x_n \to x$ then $x \in C(\lambda)$. Given such sequences, let $x_{n,t}$ be the t-th component of x_n and x_t the t-th component of x for any $t \in T$. By definition of the product topology, $x_n \to x \iff x_{n,t} \to x_t$ for all $t \in T$. By definition of $C(\cdot)$, $x_{n,t} \in C_t(\lambda_n)$ for each n. Because $C_t(\cdot)$ is UHC and compact, $x_t \in C_t(\lambda)$. Since t is arbitrary, $x_t \in C_t(\lambda)$ for all $t \in T$ and by definition of $C(\cdot)$, $x \in C(\lambda)$. Hence $C(\cdot)$ is UHC and compact. AB Theorem 17.10 establishes that $C(\cdot)$ is closed and thus $F(\cdot)$ is closed.

Since Λ is compact and $F(\cdot)$ is closed and convex, applying Kakutani's Fixed point theorem (AB Corollary 17.55) yields a λ^* such that $\lambda^* \in F(\lambda^*)$.

□

Appendix C. Preliminaries for the Remaining Proofs

Lemma 1 relies on two functions of the strategy profile.
Formally, if other votes unfold so that the realized action profile is in the event

$$Piv_A = \{x \in Z(C) : x(A) = x(B) \text{ or } x(A) = x(B) - 1\}, \tag{C.1}$$

then the voter is pivotal for candidate A; let Piv_B be the corresponding event for B. Each voter's best response depends on the relationship between her set of posteriors and the function $b : \Omega \times (\Delta C)^T \to [0, 1]$ given by $b(b, \sigma) =$

$$\frac{Pr(Piv_B|b,\sigma) + Pr(Piv_A|b,\sigma)}{Pr(Piv_B|b,\sigma) + Pr(Piv_B|a,\sigma) + Pr(Piv_A|b,\sigma) + Pr(Piv_A|a,\sigma)}. \tag{C.2}$$

and $b(a, \sigma) = 1 - b(b, \sigma)$. The probabilities in this function depend only on the strategy profile and not on an individual voter's type.
Another key equation is the *insurance strategy*, denoted $\hat{s}(\cdot, \sigma)$, is given by

$$\hat{s}(A,\sigma) = \frac{2(\mathbb{E}[U|b,\sigma] - \mathbb{E}[U|a,\sigma]) + Pr(Piv_B|b,\sigma) + Pr(Piv_B|a,\sigma)}{Pr(Piv_B|b,\sigma) + Pr(Piv_B|a,\sigma) + Pr(Piv_A|b,\sigma) + Pr(Piv_A|a,\sigma)}.$$

and $\hat{s}(B, \sigma) = 1 - \hat{s}(A, \sigma)$. This maps a strategy profile σ into the strategy a voter would play to ensure his expected utility is independent of the state if $\hat{s}(A, \sigma) \in [0, 1]$. Otherwise, no strategy equalizes a voters expected utilities between states.
Notice that expected utility in state ω if the voter abstained is given by

$$\mathbb{E}[U|a,\sigma] = \sum_{n=0}^{\infty} \frac{e^{-\lambda(a)(A)}\lambda(a)(A)^n}{n!}[\sum_{j=0}^{n-1} \frac{e^{-\lambda(a)(B)}\lambda(a)(B)^j}{j!} + \frac{1}{2}\frac{e^{-\lambda(a)(A)}\lambda(a)(A)^n}{n!}]$$

where $\lambda(\omega)(c) = \mathbb{E}[x(c)|\omega, \sigma]$ as in equation (3.3). Define $\mathbb{E}[U|b, \sigma]$ analogously. This expression is precisely the probability that candidate ω wins in state ω. The expected utility of voting for candidate c in state ω when others play strategy profile σ is

$$\mathbb{E}[U|\omega, v_c, \sigma] = \mathbb{E}[U|\omega, \sigma] + [\chi_{\{\omega\}}(c) - \frac{1}{2}]Pr(Piv_c|\omega).$$

It is more convenient to focus on to consider properties of strategy profiles rather than games. Hence, reformulate FIE in terms of the strategy profile.

Definition. For a sequence of ambiguous voting games $(\Gamma_n)_{n=1}^{\infty}$, the sequence of strategy profiles $(\sigma_n)_{n=1}^{\infty}$ satisfies *FIE for* $(\Gamma_n)_{n=1}^{\infty}$ if σ_n is an equilibrium for Γ_n and for any $\epsilon > 0$ there exists an N so $n > N$ implies the correct candidate is elected in each state with probability higher than $1 - \epsilon$ if σ_n is played.[7]

Clearly, a sequence of ambiguous voting games (Γ_n) satisfies FIE if and only if there is a sequence of strategy profiles (σ_n) that satisfies FIE for (Γ_n). Additionally, let $\tau : C \times \Omega \times$

[7]The term "Full Information Equivalence" originally comes from Feddersen and Pesendorfer [1997, pp. 1041-2]. The above definition adapts it to the current setting.

$\Delta C^T \to [0,1]$ be the expected vote share for a candidate in a state given a strategy profile. Formally,

$$\tau(c|\omega,\sigma) = \sum_{t\in T} r(t|\omega)\sigma(t)(c).$$

Note that this does not depend on the number of voters. Further, if σ_n satisfies FIE it is necessary that there exists an N so that $\tau(A|a,\sigma_n) > \frac{1}{2}$ and $\tau(B|b,\sigma_n) > \frac{1}{2}$ for all $n > N$ for some σ_n that is an equilibrium for Γ_n.

Appendix D. Proofs from Section 4

Lemma 1 establishes the form of a voter's best response correspondence. This will be used to prove both Theorem 3 and Theorem 4.

Lemma 1. *For any σ^*, σ^* is an equilibrium if $\sigma^*_t(A) \in BR_t(\sigma^*)(A)$ where*

$$BR_t(\sigma)(A) = \begin{cases} \{0\} & \text{if } \mathbb{E}[U|a,v_B,\sigma] \geq \mathbb{E}[U|b,v_B,\sigma] \;\&\; b(b,\sigma) > p_t \\ & \quad \text{or } \mathbb{E}[U|b,v_A,\sigma] \geq \mathbb{E}[U|a,v_A,\sigma] \;\&\; b(b,\sigma) > q_t \\ [0,1] & \text{if } \mathbb{E}[U|a,v_B,\sigma] \geq \mathbb{E}[U|b,v_B,\sigma] \;\&\; b(b,\sigma) = p_t \\ & \quad \text{or } \mathbb{E}[U|b,v_A,\sigma] \geq \mathbb{E}[U|a,v_A,\sigma] \;\&\; b(b,\sigma) = q_t \\ \{1\} & \text{if } \mathbb{E}[U|a,v_B,\sigma] \geq \mathbb{E}[U|b,v_B,\sigma] \;\&\; b(b,\sigma) < p_t \\ & \quad \text{or } \mathbb{E}[U|b,v_A,\sigma] \geq \mathbb{E}[U|a,v_A,\sigma] \;\&\; b(b,\sigma) < q_t \\ \hat{BR}_t(\sigma)(A) & \text{otherwise} \end{cases}$$

and

$$\hat{BR}_t(\sigma)(A) = \begin{cases} \{0\} & \text{if } b(b,\sigma) > q_t \\ [0,\hat{s}(A,\sigma)] & \text{if } b(b,\sigma) = q_t \\ \{\hat{s}(A,\sigma)\} & \text{if } q_t > b(b,\sigma) > p_t \\ [\hat{s}(A,\sigma),1] & \text{if } b(b,\sigma) = p_t \\ \{1\} & \text{if } b(b,\sigma) < p_t \end{cases}$$

where $p_t = \min_{\rho\in\Pi_t} \rho(a)$ and $q_t = \max_{\rho\in\Pi_t} \rho(a)$. If $BR_t(\sigma) = \hat{BR}_t(\sigma)$ then $\hat{s}(A,\sigma,n) \in [0,1]$.

Proof. Throughout, a strategy is indexed solely by the probability of playing A. This is WLOG since ΔC is one dimensional. Let p_t and q_t be as in the statement of the Lemma. A player of type t has a best response to σ of playing A with probability s if s maximizes

$$V_t(s,\sigma) = \min_{\rho\in\Pi_t} \mathbb{E}_\rho\Big[\int [sU(t,A,\omega,x) + (1-s)U(t,B,\omega,x)]p(dx|\lambda(\omega))\Big].$$

This function is not in general differentiable everywhere. Since $V_t(\cdot,\sigma)$ is concave as a minimum of a set of linear functions, the super-differential exists everywhere. By definition and adapted to this setting, the super-differential is given by

$$\partial V_t(s,\sigma) = \{x \in \mathbb{R}^\Omega : V_t(y,\sigma) \leq V_t(s,\sigma) + \sum_\omega [(y(\omega) - s(\omega))x(\omega)] \forall y \in \Delta A\}.$$

The best response correspondence is the set of all s s.t. $0 \in \partial V_t(s,\sigma)$ where $\partial V_t(s,\sigma)$ is the super-differential of $V_t(\cdot,\sigma)$ at s. This follows from the dual to Aliprantis and Border [2006, Lem 7.10], which states that s is a maximum of $V_t(\cdot,\sigma)$ if and only if $0 \in \partial V_t(s,\sigma)$.
Consider the case where $\mathbb{E}[U|a, v_B, \sigma] \geq \mathbb{E}[U|b, v_B, \sigma]$. Note that

$$\begin{aligned} V_t(s,\sigma) &= \min_{p\in\Pi_t(A)} \{p[s\frac{1}{2}Pr(Piv_A|a) - (1-s)\frac{1}{2}Pr(Piv_B|a) + \mathbb{E}[U|a,\sigma]] + \\ &\quad +(1-p)[(1-s)\frac{1}{2}Pr(Piv_B|b) - s\frac{1}{2}Pr(Piv_A|b) + \mathbb{E}[U|b,\sigma]]\} \\ &= p_t[s\frac{1}{2}Pr(Piv_A|a) - (1-s)\frac{1}{2}Pr(Piv_B|a) + \mathbb{E}[U|a,\sigma]] + \\ &\quad +(1-p_t)[(1-s)\frac{1}{2}Pr(Piv_B|b) - s\frac{1}{2}Pr(Piv_A|b) + \mathbb{E}[U|b,\sigma]] \end{aligned}$$

because for every s

$$sPr(Piv_A|a)-(1-s)Pr(Piv_B|a)+2\mathbb{E}[U|a,\sigma] \geq (1-s)Pr(Piv_B|b)-sPr(Piv_A|b)+2\mathbb{E}[U|b,\sigma].$$

This occurs because the RHS reaches its minimum at $s=0$ and the LHS reaches its maximum at $s=0$. At $s=0$ the RHS equals $\mathbb{E}[U|a,v_B,\sigma]$ and the LHS equals $\mathbb{E}[U|b,v_B,\sigma]$. By hypothesis, $\mathbb{E}[U|a,v_B,\sigma] \geq \mathbb{E}[U|b,v_B,\sigma]$ so for every s the RHS is larger than the LHS. Thus, $V_t(s,\sigma)$ is differentiable in $s \in (0,1)$. By Aliprantis and Border [2006, Cor 7.17] $\partial V_t(s,\sigma)$ is singleton and coincides with the Gateaux derivative when it exists. Hence,

$$\partial V_t(s,\sigma) = \{p_t[\frac{1}{2}Pr(Piv_A|a) + \frac{1}{2}Pr(Piv_B|a)] - (1-p_t)[\frac{1}{2}Pr(Piv_A|b) + \frac{1}{2}Pr(Piv_B|b)]\}$$

and $0 \in \partial V_t(s,\sigma)$ only if $b(b,\sigma) = p_t$. If $b(b,\sigma) < p_t$ this is positive and if $b(b,\sigma) < p_t$ this is negative and hence no $s \in (0,1)$ is a maxima.
If $s = 1$ then the derivative is not defined since $V_t(1+\epsilon,\sigma)$ for any $\epsilon > 0$ does not exist. The super-differential does exist:

$$\partial V_t(1,\sigma) = \{x \in \mathbb{R} : V_t(y,\sigma) - V_t(1,\sigma) \leq (y-1)x \forall y \in [0,1]\}.$$

Since

$$V_t(y,\sigma)-V_t(1,\sigma) = (y-1)\frac{1}{2}(p_t[Pr(Piv_A|a)+Pr(Piv_B|a)]-(1-p_t)[Pr(Piv_A|b)+Pr(Piv_B|b)])$$

$0 \in \partial V_t(1,\sigma)$ if and only if

$$p_t[Pr(Piv_A|a) + Pr(Piv_B|a)] - (1-p_t)[Pr(Piv_A|b) + Pr(Piv_B|b) > 0.$$

As noted above, $b(b,\sigma) < p_t$ implies this is positive.
Additionally, if $s = 0$ the derivative is not defined since $V_t(0-\epsilon,\sigma)$ for any $\epsilon > 0$ does not exist. The super-differential does exist:

$$\partial V_t(0,\sigma) = \{x \in \mathbb{R} : V_t(y,\sigma) - V_t(0,\sigma) \leq y \cdot x \forall y \in [0,1]\}.$$

Since $V_t(y,\sigma) - V_t(0,\sigma) =$

$$y\frac{1}{2}(p_t[Pr(Piv_A|a) + Pr(Piv_B|a)] - (1-p_t)[Pr(Piv_A|n) + Pr(Piv_B|n)])$$

$0 \in \partial V_t(0,\sigma)$ if and only if

$$p_t[Pr(Piv_A|a) + Pr(Piv_B|a)] - (1-p_t)[Pr(Piv_A|b) + Pr(Piv_B|b)] < 0.$$

As noted above, $b(B,\sigma) > p_t$ implies this is negative.

The above observations show that if $\mathbb{E}[U|a, v_B, \sigma] \geq \mathbb{E}[U|b, v_B, \sigma]$, then the set of maximizers of $V_t(\cdot, \sigma)$ is

$$\arg\max_{s\in[0,1]} V_t(s,\sigma) = \begin{cases} \{1\} & \text{if } b(b,\sigma) > p_t \\ [0,1] & \text{if } b(b,\sigma) = p_t. \\ \{0\} & \text{if } b(b,\sigma) < p_t \end{cases}$$

If $\mathbb{E}[U|a, v_B, \sigma] \leq \mathbb{E}[U|b, v_B, \sigma]$, similar arguments show the same form of BR correspondence with the probability assigned to a equal to q_t instead of p_t.
Now, suppose that neither of the above inequalities hold. Then there exists an $\bar{s} \in (0, 1)$ so that the conditional expected utilities in A and B are equal. Further, if $s > \bar{s}$ the conditional expected utility in state a is larger than that in state b and if $s < \bar{s}$ then the expected utility in state B is larger than that in state A. Algebra shows that

$$\bar{s} = \frac{2(\mathbb{E}[U|b,\sigma] - \mathbb{E}[U|a,\sigma]) + Pr(Piv_B|b) + Pr(Piv_B|a)}{Pr(Piv_B|a) + Pr(Piv_A|a) + Pr(Piv_B|b) + Pr(Piv_A|b)}$$

which is $\hat{s}(A, \sigma)$.
Since for all $s \in (0, \bar{s})$ and every $s \in (\bar{s}, 1)$ the minimizer is unique, the Gateaux derivative exists whenever $s \notin \{0, \bar{s}, 1\}$. If $s \in (\bar{s}, 1)$ then

$$\partial V_t(s,\sigma) = \{\frac{\partial}{\partial s}V_t(s,\sigma)\} = \{p_t\frac{1}{2}[Pr(Piv_A|a)+Pr(Piv_B|a)]-(1-p_t)\frac{1}{2}[Pr(Piv_A|b)+Pr(Piv_B|b)]\}.$$

If $s' \in (0, \bar{s})$ then

$$\partial V_t(s',\sigma) = \{\frac{\partial}{\partial s}V_t(s,\sigma)\} = \{q_t\frac{1}{2}[Pr(Piv_A|a)+Pr(Piv_B|a)]-(1-q_t)\frac{1}{2}[Pr(Piv_A|b)+Pr(Piv_B|b)]\}.$$

Thus any $s \in (\bar{s}, 1)$ is an optimum only if

$$p_t[Pr(Piv_A|a) + Pr(Piv_B|a)] - (1-p_t)[Pr(Piv_A|b) + Pr(Piv_B|b)] = 0,$$

which happens when $p_t = b(b, \sigma)$. Similarly, any $s \in (0, \bar{s})$ is an optimum when $q_t = b(b, \sigma)$. Otherwise there cannot be an optimum in $(0, 1)\backslash\{\bar{s}\}$.
As above, when $s = 1$ then the derivative is not defined since $V_t(1 + \epsilon, \sigma)$ for any $\epsilon > 0$ does not exist. The super-differential does exist:

$$\partial V_t(1,\sigma) = \{x \in \mathbb{R} : V_t(y,\sigma) - V_t(1,\sigma) \leq (y-1)x \forall y \leq 1\}.$$

Since $V_t(y, \sigma) - V_t(1, \sigma)$ is equal to

$$(y-1)(p_t\frac{1}{2}[Pr(Piv_A|a) + Pr(Piv_A|b)] - (1-p_t)\frac{1}{2}[Pr(Piv_B|a) + Pr(Piv_B|b)]),$$

$0 \in \partial V_t(1, \sigma)$ if and only if

$$V_t(y,\sigma) - V_t(1,\sigma) \leq 0 \iff b(b,\sigma) \leq p_t.$$

Hence $s = 1$ is optimal only if $b(b, \sigma) \geq p_t$. Similar arguments show then $0 \in \partial V_t(0, \sigma) \iff b(b, \sigma) \geq q_t$.
By the above, we have covered the cases where $b(b, \sigma) \geq q_t$ and $b(b, \sigma) \leq p_t$. Suppose $p_t < b(b, \sigma) < q_t$. In this case,

$$q_t[Pr(Piv_A|a) + Pr(Piv_B|a)] > (1-q_t)[Pr(Piv_A|b) + Pr(Piv_B|b)]$$

and

$$p_t[Pr(Piv_A|a) + Pr(Piv_B|a)] < (1-p_t)[Pr(Piv_A|b) + Pr(Piv_B|b)].$$

So for $s > \bar{s}$,

$$\partial V_t(s,\sigma) = \{p_t \frac{1}{2}[Pr(Piv_A|a) + Pr(Piv_B|a)] - (1-p_t)\frac{1}{2}[Pr(Piv_A|b) + Pr(Piv_B|b)]\}$$

is a singleton strictly smaller than zero. For $s' < \bar{s}$,

$$\partial V_t(s',\sigma) = \{q_t \frac{1}{2}[Pr(Piv_A|a) + Pr(Piv_B|a)] - (1-q_t)\frac{1}{2}[Pr(Piv_A|b) + Pr(Piv_B|b)]\}$$

is a singleton strictly larger than zero. However, for $s = \bar{s}$

$$\partial V_t(\bar{s},\sigma) = \{p(a)\frac{1}{2}[Pr(Piv_A|a) + Pr(Piv_B|a)] - p(b)\frac{1}{2}[Pr(Piv_A|b) + Pr(Piv_B|b)] : p \in \Pi_t\}$$

Since $q_t > \rho(A) > p_t$, $\exists \rho \in \Pi_t$ s.t.

$$\frac{\rho(a)}{1-\rho(a)} = \frac{Pr(Piv_A|b) + Pr(Piv_B|b)}{Pr(Piv_A|a) + Pr(Piv_B|a)}$$

implying that $0 \in \partial V_t(\bar{s},\sigma)$ and $\bar{s}$ is the only maximizer when $q_t > b(B,\sigma) > p_t$. Combining the above results yields the desired form of the best response function.

□

In order to prove Theorem 3, two more preliminary results are necessary. Lemma 2 and Lemma 3 allow characterization of the worst case scenario. The proof of Theorem 3 will use both these facts to show that no equilibrium exists where a voter thinks the worst case scenario is independent of her vote.

Lemma 2. *For any $n \geq 1$, $\mathbb{E}[U|a,\sigma_n] \geq \mathbb{E}[U|B,\sigma_n] \iff \tau(A|a,\sigma_n) \geq \tau(A|b,\sigma_n)$.*

Proof. Let $f(x,\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$, the probability mass function of the Poisson distribution with mean λ, and $F(x,\lambda)$ its CDF. The CDF of the Poisson distribution has the form $\frac{\Gamma([x+1],\lambda)}{[x]!}$ where $[z]$ is the greatest integer less than or equal to z and $\Gamma(z,y)$ is the generalized incomplete gamma function:

$$\Gamma(z,y) = \int_y^\infty e^{-t}t^{z-1}dt.$$

We can write

$$\mathbb{E}[U|a,\sigma_n] = Q(\tau(A|a,\sigma_n)n) + \frac{1}{2}\sum_{j=0}^{\infty} f(j,\tau(A|a,\sigma_n)n)f(j,\tau(B|a,\sigma_n)n)$$

where $Q(\cdot)$ is given by

$$Q(\lambda) = \sum_{j=0}^{\infty} f(j,\lambda)F(j-1,n-\lambda).$$

Observe that

$$\frac{\partial Q}{\partial \lambda} = \sum_{x=1}^{\infty}[\frac{\partial f(j,\lambda)}{\partial \lambda}F(j-1,n-\lambda) + f(j,\lambda)\frac{\partial F(j-1,n-\lambda)}{\partial \lambda}].$$

By the fundamental theorem of calculus, $\frac{\partial F(x,\lambda)}{\partial \lambda} = -\frac{e^{-\lambda}\lambda^x}{x!}$ and $\frac{\partial f(x,\lambda)}{\partial \lambda} = \frac{e^{-\lambda}\lambda^{x-1}(x-\lambda)}{x!}$ whenever x is an integer. Given this, the above sum can be written as

$$\begin{aligned}
\frac{\partial Q}{\partial \lambda} &= \sum_{x=1}^{\infty}[\frac{\partial f(x,\lambda)}{\partial \lambda}F(x-1,n-\lambda)+f(x,\lambda)\frac{\partial F(x-1,n-\lambda)}{\partial \lambda}] \\
&= \sum_{x=1}^{\infty}[\frac{e^{-\lambda}\lambda^{x-1}(x-\lambda)}{x!}F(x-1,n-\lambda)+\frac{e^{-\lambda}\lambda^{x}}{x!}\frac{e^{-n+\lambda}(n-\lambda)^{x-1}}{x-1!}] \\
&= \sum_{x=1}^{\infty}[-\frac{\lambda^{x}e^{-\lambda}}{x!}F(x-1,n-\lambda)+\frac{x\lambda^{x-1}e^{-\lambda}}{x!}F(x-1,n-\lambda)+\frac{e^{-\lambda}\lambda^{x}}{x!}\frac{e^{-n+\lambda}(n-\lambda)^{x-1}}{x-1!}] \\
&= \sum_{x=1}^{\infty}\frac{\lambda^{x-1}e^{-\lambda}}{x-1!}F(x-1,n-\lambda)-\sum_{x=1}^{\infty}\frac{\lambda^{x}e^{-\lambda}}{x!}F(x-1,n-\lambda)+\sum_{x=1}^{\infty}\frac{e^{-\lambda}\lambda^{x}}{x!}\frac{e^{-n+\lambda}(n-\lambda)^{x-1}}{x-1!} \\
&= \sum_{x=1}^{\infty}\frac{\lambda^{x-1}e^{-\lambda}}{x-1!}[F(x-2,n-\lambda)+f(x-1,n-\lambda)]+\sum_{x=1}^{\infty}[\frac{e^{-\lambda}\lambda^{x}}{x!}\frac{e^{-n+\lambda}(n-\lambda)^{x-1}}{x-1!}-\frac{\lambda^{x}e^{-\lambda}}{x!}F(x-1,n-\lambda)] \\
&= \sum_{y=0}^{\infty}\frac{\lambda^{y}e^{-\lambda}}{y!}F(y-1,n-\lambda)+\sum_{x=1}^{\infty}[\frac{e^{-\lambda}\lambda^{x-1}}{x-1!}\frac{e^{-n+\lambda}(n-\lambda)^{x-1}}{x-1!}+\frac{e^{-\lambda}\lambda^{x}}{x!}\frac{e^{-n+\lambda}(n-\lambda)^{x-1}}{x-1!}-\frac{\lambda^{x}e^{-\lambda}}{x!}F(x-1,n-\lambda)] \\
&= \sum_{x=1}^{\infty}\frac{e^{-\lambda}\lambda^{x}}{x!}\frac{e^{-n+\lambda}(n-\lambda)^{x-1}}{x-1!}+\sum_{x=0}^{\infty}\frac{\lambda^{x}e^{-\lambda}}{x!}\frac{(n-\lambda)^{x}e^{-n+\lambda}}{x!} \\
&= e^{-n}[\sum_{x=1}^{\infty}\frac{\lambda^{x}}{x!}\frac{(n-\lambda)^{x-1}}{x-1!}+\sum_{x=0}^{\infty}\frac{\lambda^{x}}{x!}\frac{(n-\lambda)^{x}}{x!}
\end{aligned}$$

Now, we must deal with the second term.

$$\begin{aligned}
\frac{\partial}{\partial \lambda}\sum_{j=0}^{\infty}f(j,\lambda)f(j,n-\lambda) &= \sum_{x=0}^{\infty}\frac{\partial}{\partial \lambda}e^{-n}\frac{\lambda^{x}(n-\lambda)^{x}}{x!x!}] \\
&= \sum_{x=1}^{\infty}e^{-n}[\frac{x\lambda^{x-1}(n-\lambda)^{x}}{x!x!}-\frac{x\lambda^{x}(n-\lambda)^{x-1}}{x!x!}] \\
&= \sum_{x=1}^{\infty}e^{-n}[\frac{\lambda^{x-1}(n-\lambda)^{x}}{x!(x-1)!}-\frac{\lambda^{x}(n-\lambda)^{x-1}}{x!(x-1)!}]
\end{aligned}$$

Adding together shows that $\frac{\partial}{\partial \lambda}\mathbb{E}[U|a,\sigma_n,n]$ is equal to

$$e^{-n}[\sum_{x=0}^{\infty}\frac{\lambda^{x}}{x!}\frac{(n-\lambda)^{x}}{x!}+\frac{1}{2}\sum_{x=1}^{\infty}(\frac{\lambda^{x}}{x!}\frac{(n-\lambda)^{x-1}}{x-1!}+\frac{\lambda^{x-1}(n-\lambda)^{x}}{x!(x-1)!})].$$

Clearly, this term is positive. Recall that $\lambda=\tau(A|a,\sigma_n)n$ so that $\frac{\partial\mathbb{E}[U|a,\sigma_n]}{\partial\tau(A|a,\sigma_n)}=\frac{\partial\mathbb{E}[U|a,\sigma_n]}{\partial\lambda}\frac{\partial\lambda}{\partial\tau(A|A,\sigma_n)}=n\frac{\partial\mathbb{E}[U|a,\sigma_n]}{\partial\lambda}$. Since $\frac{\partial\mathbb{E}[U|a,\sigma_n]}{\partial\lambda}\geq 0$, so is $\frac{\partial\mathbb{E}[U|a,\sigma_n]}{\partial\tau(A|a,\sigma_n)}$.

Since $\frac{\partial\mathbb{E}[U|a,\sigma_n]}{\partial\tau(A|a,\sigma_n)}\geq 0$, as $\tau(A|a,\sigma_n)$ increases, $\mathbb{E}[U|a,\sigma_n]$ increases. Similarly for $\tau(A|b,\sigma_n)$ and $\mathbb{E}[U|b,\sigma_n]$. Since the expected number of voters in each state is equal, the terms $\mathbb{E}[U|a,\sigma_n]$ and $\mathbb{E}[U|b,\sigma_n]$ are equal whenever $\tau(A|a,\sigma_n)$ and $\tau(B|b,\sigma_n)$ are equal. This establishes the claim.

□

Lemma 3. *If $\frac{1}{2}<\tau(B|b,\sigma_n)<\tau(A|a,\sigma_n)$ then $\hat{s}(A,\sigma_n)<\frac{1}{2}$. In particular, when the expected winner in each state is correct, $\hat{s}(A,\sigma_n)<\frac{1}{2}\iff b(b,\sigma_n)>\frac{1}{2}$.*

Proof. Suppose $\frac{1}{2}<\tau(B|b,\sigma_n)<\tau(A|a,\sigma_n)$. By Lemma 2, $\mathbb{E}[U|b,\sigma_n]<\mathbb{E}[U|a,\sigma_n]$. Consider the numerator of $\hat{s}(A,\sigma_n)$. Recall that it is

$$\phi(\sigma_n)=2(\mathbb{E}[u|b,\sigma_n]-\mathbb{E}[u|a,\sigma_n])+Pr(Piv_B|b,\sigma_n)+Pr(Piv_B|a,\sigma_n).$$

The fraction is less than $\frac{1}{2}$ if and only if

$$2\phi(\sigma_n)<[Pr(Piv_B|b,\sigma_n)+Pr(Piv_B|a,\sigma_n)+Pr(Piv_A|b,\sigma_n)+Pr(Piv_A|a,\sigma_n)].$$

Equivalently, this holds if and only if

$$4(\mathbb{E}[u|b,\sigma_n]-\mathbb{E}[u|a,\sigma_n])+Pr(Piv_B|b,\sigma_n)+Pr(Piv_B|a,\sigma_n)-Pr(Piv_A|b,\sigma_n)-Pr(Piv_A|a,\sigma_n)<0.$$

We can rewrite

$$\gamma=Pr(Piv_B|b,\sigma_n)+Pr(Piv_B|a,\sigma_n)-Pr(Piv_A|b,\sigma_n)-Pr(Piv_A|a,\sigma_n)$$

as a function only of $\tau(A|a,\sigma_n)$ and $\tau(B|b,\sigma_n)$. Set $t=\tau(B|b,\sigma_n)$ and $s=\tau(A|a,\sigma_n)$ for convenience. Expanding and writing in terms of t and s,

$$\begin{aligned}\gamma &= e^{-n}\sum_{j=0}^{\infty} n^{2j}[\frac{t^j(1-t)^j}{j!j!}+n\frac{t^j(1-t)^{j+1}}{j!j+1!}+\frac{s^j(1-s)^j}{j!j!}+n\frac{s^{j+1}(1-s)^j}{j!j+1!}]- \\ &\quad -e^{-n}\sum_{j=0}^{\infty} n^{2j}[\frac{t^j(1-t)^j}{j!j!}+n\frac{t^{j+1}(1-t)^j}{j!j+1!}+\frac{s^j(1-s)^j}{j!j!}+n\frac{s^j(1-s)^{j+1}}{j!j+1!}] \\ &= \sum_{j=0}^{\infty} e^{-n}n^{2j+1}[\frac{t^j(1-t)^{j+1}}{j!j+1!}+\frac{s^{j+1}(1-s)^j}{j!j+1!}-\frac{t^{j+1}(1-t)^j}{j!j+1!}-\frac{s^j(1-s)^{j+1}}{j!j+1!}].\end{aligned}$$

Recall that

$$\begin{aligned}\mathbb{E}[U|b,\sigma_n] &= \sum_{j=0}^{\infty} f(j;tn)F(j-1;(1-t)n)+\frac{1}{2}\sum_{j=0}^{\infty} f(j;tn)f(j;(1-t)n) \\ &:= \hat{\psi}_n(\tau(B|b,\sigma_n))\end{aligned}$$

and similarly $\mathbb{E}[U|a,\sigma_n]=\hat{\psi}_n(\tau(A|a,\sigma_n))$. Setting

$$\theta_n(t)=\sum_{j=0}^{\infty} e^{-n}n^{2j+1}[\frac{t^j(1-t)^{j+1}}{j!j+1!}-\frac{t^{j+1}(1-t)^j}{j!j+1!}]$$

and

$$\psi_n(x)=4\hat{\psi}_n(x)+\theta_n(x).$$

gives that

$$\hat{s}(A,\sigma_n)<\frac{1}{2}\iff \psi_n(t)-\psi_n(s)<0.$$

From Lemma 2 and writing $\lambda=nt$, we have that

$$\frac{\partial\hat{\psi}_n}{\partial\lambda} = e^{-n}[\sum_{x=0}^{\infty}\frac{\lambda^x}{x!}\frac{(n-\lambda)^x}{x!}+\frac{1}{2}\sum_{x=1}^{\infty}(\frac{\lambda^x}{x!}\frac{(n-\lambda)^{x-1}}{x-1!}+\frac{\lambda^{x-1}(n-\lambda)^x}{x!(x-1)!})].$$

which is positive. Now,

$$\theta_n(\lambda)=\sum_{j=0}^{\infty} e^{-n}\frac{\lambda^j(n-\lambda)^{j+1}-\lambda^{j+1}(n-\lambda)^j}{j!(j+1)!}$$

so that

$$\begin{aligned}\frac{\partial\theta_n}{\partial\lambda} &= \sum_{j=0}^{\infty} e^{-n}\frac{\partial}{\partial\lambda}\frac{\lambda^j(n-\lambda)^{j+1}-\lambda^{j+1}(n-\lambda)^j}{j!(j+1)!} \\ &= \sum_{j=0}^{\infty} e^{-n}\frac{\lambda^{j-1}(n-\lambda)^{j+1}}{(j-1)!(j+1)!}-\frac{\lambda^j(n-\lambda)^j}{j!j!}-\frac{\lambda^j(n-\lambda)^j}{j!j!}+\frac{\lambda^{j+1}(n-\lambda)^{j-1}}{(j-1)!(j+1)!} \\ &= \sum_{j=1}^{\infty} e^{-n}\frac{\lambda^{j-1}(n-\lambda)^{j+1}+\lambda^{j+1}(n-\lambda)^{j-1}}{(j-1)!(j+1)!}-2\sum_{j=0}^{\infty} e^{-n}\frac{\lambda^j(n-\lambda)^j}{j!j!}\end{aligned}$$

Combining,

$$\begin{aligned}\frac{\partial\psi_n}{\partial t} &= [4\frac{\partial\hat{\psi}_n}{\partial\lambda}+\frac{\partial\theta_n}{\partial\lambda}]\frac{\partial\lambda}{\partial t}\\ &= n[2\sum_{x=0}^{\infty}e^{-n}\frac{\lambda^x(n-\lambda)^x}{x!^2}+3\sum_{j=1}^{\infty}e^{-n}\frac{\lambda^{j-1}(n-\lambda)^{j+1}+\lambda^{j+1}(n-\lambda)^{j-1}}{(j-1)!(j+1)!}]\end{aligned}$$

which is clearly greater than 0.
To show that $\psi_n(\tau(B|b,\sigma_n))-\psi_n(\tau(A|a,\sigma_n))<0$, recall that we can write this as $\int_s^t \frac{\partial\psi_n(x)}{\partial x}dx$ which is negative because the integrand is positive but $\tau(A|a,\sigma_n)>\tau(B|b,\sigma_n)$. Therefore, whenever $\tau(A|a,\sigma_n)>\tau(B|b,\sigma_n)$ it must be that $\hat{s}(A,\sigma_n)<\frac{1}{2}$.
To complete the second part of the Lemma, note the following.

Claim 1. $b(b,\sigma_n,n)>\frac{1}{2} \iff |\tau(A|a,\sigma_n)-\frac{1}{2}|>|\tau(B|b,\sigma_n)-\frac{1}{2}|$.

Proof. $b(b,\sigma_n,n)>\frac{1}{2} \iff Pr(Piv_A|b)+Pr(Piv_B|b)>Pr(Piv_A|a)+Pr(Piv_B|a)$
Let $t=\tau(A|a,\sigma_n)$ so that $Pr(Piv_A|a)+Pr(Piv_B|a)$ equals

$$2\sum_{j=0}^{\infty}p(2j)\begin{pmatrix}2j\\ j\end{pmatrix}t^j(1-t)^j+\sum_{j=0}^{\infty}p(2j+1)\begin{pmatrix}2j+1\\ j+1\end{pmatrix}[t^j(1-t)^{j+1}+t^{j+1}(1-t)^j]$$

where $p(x)=\frac{e^{-n}n^x}{x!}$. Take the derivative with respect to t to get

$$(1-2t)[2\sum_{j=0}^{\infty}j\{p(2j)\begin{pmatrix}2j\\ j\end{pmatrix}t^{j-1}(1-t)^{j-1}+\sum_{j=0}^{\infty}p(2j+1)\begin{pmatrix}2j+1\\ j+1\end{pmatrix}t^{j-1}(1-t)^{j-1}\}]$$

which is positive whenever $t<.5$ and negative whenever $t>.5$. Similarly for $Pr(Piv_A|b)+Pr(Piv_B|b)$. Given the symmetry of $Pr(Piv_A|a)+Pr(Piv_B|b)$ with respect to $\tau(A|a,\sigma_n)$ and $Pr(Piv_A|b)+Pr(Piv_B|b)$ with respect to $\tau(B|b,\sigma_n)$, the claim follows immediately.

□

From Claim 1, whenever $b(b,\sigma_n)>\frac{1}{2}$, $|\tau(A|a,\sigma_n)-\frac{1}{2}|>|\tau(B|a,\sigma_n)-\frac{1}{2}|$. Further, if the expected winners are correct, it must be that both $\tau(A|a,\sigma_n)>\frac{1}{2}$ and $\tau(B|b,\sigma_n)>\frac{1}{2}$. It follows that $\tau(A|a,\sigma_n)>\tau(B|b,\sigma_n)$, so $\hat{s}(A,\sigma_n)<\frac{1}{2}$. Similarly, suppose that $\hat{s}(A,\sigma_n)<\frac{1}{2}$ and the expected winners are correct. From the above, $\tau(A|a,\sigma_n)>\tau(B|b,\sigma_n)>\frac{1}{2}$, so by Claim 1 $b(b,\sigma_n)>\frac{1}{2}$.

□

Proof of Theorem 3:

Proof. First, note that if there is no t so that $r(t|a)\neq r(t|b)$, no strategy profile can satisfy FIE, completing the proof. Therefore, assume that for some t, $r(t|a)\neq r(t|b)$.
Suppose, for the sake of contradiction, that (σ_n) is a sequence of equilibria to (Γ_n) that satisfies FIE.

Claim 2. $BR_t(\sigma_n)=\hat{BR}_t(\sigma_n)$ for all t when n is sufficiently large.

Proof. There are two cases.
In the first case, all types play pure strategies for all n high enough. In this case, by Myerson [2000] Theorem 1 and Lemma 1 it must be that the magnitudes of the pivotal probabilities are equal when each type plays a pure strategy. This only holds when there is a partition of the type space $\{T_A, T_B\}$ where $t \in T_A$ implies that $\sigma_n(t)(A) = 1$ and $t \in T_B$ implies that $\sigma_n(t)(B) = 1$ and $r(T_A|a) = r(T_B|b)$. If this is the case, then

$$Pr(Piv_A|a) = Pr(Piv_B|b) > 0$$

and

$$Pr(Piv_B|a) = Pr(Piv_A|b) > 0$$

since the expected vote shares in each state are equal. Further, Lemma 2 shows that $\mathbb{E}[U|a, \sigma_n] = \mathbb{E}[U|b, \sigma_n]$. This shows that $b(b, \sigma_n) = \frac{1}{2}$ and $\hat{s}(A, \sigma_n) = \frac{1}{2}$. In particular, $BR_t(\sigma_n) = \hat{BR}_t(\sigma_n)$.
In the second case, some type $\hat{t}$ plays a mixed strategy for all n high enough. If $BR_t(\sigma_n) \neq \hat{BR}_t(\sigma_n)$ then either

$$\mathbb{E}[U|a, \sigma_n] \geq \mathbb{E}[U|b, \sigma_n] + \frac{1}{2}(Pr(Piv_B|b) + Pr(Piv_B|a))$$

or

$$\mathbb{E}[U|b, \sigma_n] \geq \mathbb{E}[U|a, \sigma_n] + \frac{1}{2}(Pr(Piv_A|b) + Pr(Piv_A|a)).$$

Consider first $\mathbb{E}[U|a, \sigma_n] \geq \mathbb{E}[U|b, \sigma_n] + \frac{1}{2}(Pr(Piv_B|b) + Pr(Piv_B|a))$. In this case, because σ_n is an equilibrium, Lemma 1 implies that $b(b, \sigma_n) = p_{\hat{t}}$. By assumption, $p_{\hat{t}} < \frac{1}{2}$ which implies that

$$|\tau(A|a, \sigma_n) - \frac{1}{2}| < |\tau(B|b, \sigma_n) - \frac{1}{2}$$

$|$ by Claim 1. By FIE, $\tau(A|a, \sigma_n), \tau(B|b, \sigma_n) > \frac{1}{2}$ for n high enough. Since $\tau(A|a, \sigma_n) < \tau(B|b, \sigma_n)$, Lemma 2 implies that $\mathbb{E}[U|b, \sigma_n] > \mathbb{E}[U|a, \sigma_n]$, a contradiction.
Similarly, if $\mathbb{E}[U|b, \sigma_n] \geq \mathbb{E}[U|a, \sigma_n] + \frac{1}{2}(Pr(Piv_A|b) + Pr(Piv_A|a))$ it must be that $b(n, \sigma_n) = q_{\hat{t}}$. By assumption, $q_{\hat{t}} > \frac{1}{2}$ which implies that

$$|\tau(A|a, \sigma_n) - \frac{1}{2}| > |\tau(B|b, \sigma_n) - \frac{1}{2}$$

$|$ by Claim 1. By FIE, $\tau(A|a, \sigma_n), \tau(B|b, \sigma_n) > \frac{1}{2}$ for n high enough. Since $\tau(A|a, \sigma_n) > \tau(B|b, \sigma_n)$, Lemma 2 implies that $\mathbb{E}[u|a, \sigma_n] > \mathbb{E}[U|b, \sigma_n]$, a contradiction.
Hence for n sufficiently high, if (σ_n) satisfies FIE then $BR_t(\sigma_n) = \hat{BR}_t(\sigma_n)$ for all t.

□

I now show that no type plays a pure strategy for n high enough. Note that $\tau(A|A, \sigma_n), \tau(B|B, \sigma_n) > \frac{1}{2}$ for n high enough since (σ_n) satisfies FIE.

Claim 3. If σ_n satisfies FIE, then $\sigma_n(t) \in (0, 1)$ for all t and all n high enough.

Proof. Suppose $\sigma_n(t)(A) \in \{0, 1\}$ for some t. WLOG, assume that either $\sigma_n(2)(A) = 1$ or $\sigma_n(2)(B) = 1$.

Assume the former. Then it must be that $1 \in BR_2(\sigma_n)(A)$ so $b(b, \sigma_n) \leq p_2 < \frac{1}{2}$ by Lemma 1. By Lemma 3 it must be that $\hat{s}(A, \sigma_n) > \frac{1}{2}$. Because (σ_n) satisfies FIE, some type of voter must vote for A with probability smaller than $\frac{1}{2}$. WLOG, assume this type is 1, so that $\sigma_n(1)(A) \leq \frac{1}{2} < \hat{s}(A, \sigma_n)$ for n high enough. Hence, it must be that $b(b, \sigma_n) \geq q_1$. Combining with $p_2 \geq b(b, \sigma_n)$ gives that $p_2 \geq q_1$, which is a contradiction of $p_2 < \frac{1}{2} < q_1$.
Now, assume the latter. It must be that $0 \in BR_2(\sigma_n)(A)$ so $b(B, \sigma_n) \geq q_2$ by Lemma 1. By Lemma 3 it must be that $\hat{s}(A, \sigma_n) < \frac{1}{2}$. Since (σ_n) satisfies FIE, some type of voter must vote for A with probability larger than $\frac{1}{2}$. WLOG, assume this type is 1, so that $\sigma_n(1)(A) \geq \frac{1}{2} > \hat{s}(A, \sigma_n)$ for n high enough. Lemma 1 implies that $b(B, \sigma_n) \leq p_1$. Combining with $b(B, \sigma_n) \geq q_2$ gives that $p_1 \geq q_2$, which is a contradiction of $p_1 < \frac{1}{2} < q_2$.

□

This claim shows that in any equilibrium with FIE, all types of voters must play a mixed strategy for n high enough. Setting $[\underline{p}, \bar{p}] = \cap_{t \in T}[p_t, q_t]$, $b(b, \sigma_n) \in [\underline{p}, \bar{p}]$ for all n high enough, since otherwise at least one type of voter plays a pure strategy by Lemma 1. Further, if $b(b, \sigma_n) \in (\underline{p}, \bar{p})$, the best response of all voters is to play $\sigma_n(t)(A) = \hat{s}(A, \sigma_n)$. Because of this, vote shares in each state are the same, contradicting FIE.

Claim 4. Suppose that $b(b, \sigma_n) = \underline{p}$ for some σ_n and n high enough so that the previous claims apply. Then the expected winner in state b is not B.

Proof. WLOG, assume that $\underline{p} = p_1$; in fact, $p_1 = \max_{t \in T} p_t$ so $q_t > b(b, \sigma_n) > p_t \forall t \neq 1$. For n high enough, $\sigma_n(1)(A) \geq \hat{s}(A, \sigma_n)$ and $\sigma_n(t)(A) = \hat{s}(A, \sigma_n)$ for all $t \neq 1$ by Lemma 1. Because $b(b, \sigma_n) = p_1 < \frac{1}{2}$, by Lemma 3 it must be that $\hat{s}(A, \sigma_n) > \frac{1}{2}$. Therefore $\sigma_n(t)(A) > \frac{1}{2}$ for all t. Therefore, $\tau(B|b, \sigma_n) < \frac{1}{2}$ and thus B is not the expected winner in state B.

□

Claim 5. Suppose there is some n so that $b(B, \sigma_n) = \bar{p}$ where n is high enough that the above claims apply. Then the expected winner in state A is not a.

Proof. WLOG, assume that $\bar{p} = q_1$. For n high enough, $\sigma_n(1)(A) \leq \hat{s}(A, \sigma_n)$ and $\sigma_n(t)(A) = \hat{s}(A, \sigma_n)$ for all $t \neq 1$ by Lemma 1. By Lemma 3 it must be that $\hat{s}(A, \sigma_n) < \frac{1}{2}$. Therefore $\sigma_n(t)(A) < \frac{1}{2}$ for all t. Therefore, $\tau(A|A, \sigma_n) < \frac{1}{2}$ and A is not the expected winner in state A.

□

Therefore, for n high enough there is no equilibrium where both $\tau(A|A, \sigma_n) > \frac{1}{2}$ and $\tau(B|B, \sigma_n) > \frac{1}{2}$. This contradicts FIE.

□

Proof of Proposition 1:

Proof. Suppose σ is played. Clearly, $\mathbb{E}[U|a,\sigma] = \mathbb{E}[U|b,\sigma] = \frac{1}{2}$. This implies that $BR_t(\sigma) = \hat{BR}_t(\sigma)$ for all t by Lemma 1. Further, note that $b(b,\sigma) = \frac{1}{2}$ since $Pr(Piv_c|a) = Pr(Piv_c|b)$ for $c \in \{A,B\}$ since the vote shares are equal in both states. Since $b(b,\sigma) \in [p_t,q_t]$, voters of type t are willing to play $\sigma(t)(A) = \hat{s}(A,\sigma) = \frac{1}{2}$. Therefore, σ is an equilibrium.

□

Proof of Theorem 4:

Proof. WLOG, $T = \{1,2,...,T\}$ so that $\min_{p\in\Pi_i} p(a) < \min_{p\in\Pi_{i+1}} p(a)$ for every $i \in \{1,2,...,T-1\}$. Denote $[h] = \max_{z\in\mathbb{Z}} z \leq h$ and $\sigma(h)$ for some $h \in [1,T]$ the strategy profile such that if h is an integer then $\sigma(t)(A) = 0$ if $t \leq h$ and $\sigma(t)(A) = 1$ if $t > h$. If h is not an integer then $\sigma(h)$ is such that $\sigma(t)(A) = 0$ if $t < [h]$ and $\sigma(t)(A) = 1$ if $t > h$ and $\sigma([h])(A) = h-[h]$. The proof will show that for all n high enough, there is an $h(n)$ so that $\sigma(h(n))$ is an equilibrium and that the expected winner in A is A and the expected winner in B is b.
Define functions $z : [1,T]\times\mathbb{N} \to [0,1]$ and $\beta : [1,T]\times\mathbb{N} \to [0,1]$ by the formulas

$$z(h,n) := \begin{cases} \hat{s}(A,\sigma(h),n) & \hat{s}(A,\sigma(h),n) \in [0,1] \\ 1 & \hat{s}(A,\sigma(h),n) > 1 \\ 0 & \hat{s}(A,\sigma(h),n) < 0 \end{cases}$$

and

$$\beta(h,n) := b(B,\sigma(h),n).$$

Note that if $z(h,n) \in (0,1)$ then $z(h,n)$ is $\hat{s}(A,\sigma(h),n)$ for A when $\sigma(h)$ is played and there are n expected players. Further, $\beta(h,n)$ is $b(B,\sigma(h),n)$ when $\sigma(h)$ is played and there are n expected players .
Let $q_t = \max_{p\in\Pi_t} p(a)$ and $p_t = \min_{p\in\Pi_t} p(a)$. If $\hat{s}(A,\sigma,n) < 0$ then

$$\mathbb{E}[U|b,\sigma] - \mathbb{E}[U|a,\sigma] + \frac{1}{2}(Pr(Piv_B|b) + Pr(Piv_B|a)) < 0$$

so that $\mathbb{E}[U|a,v_B,\sigma] > \mathbb{E}[U|b,v_B,\sigma]$. Hence

$$BR_t(\sigma) = \begin{cases} 1 & b(b,\sigma) > p_t \\ [0,1] & b(b,\sigma) = p_t \\ 0 & b(b,\sigma) < p_t \end{cases}$$

by Lemma 1. Similarly, if $\hat{s}(A,\sigma) > 1$ then $1-\hat{s}(A,\sigma) < 0$ which implies

$$\mathbb{E}[U|a,\sigma] - \mathbb{E}[U|b,\sigma] + \frac{1}{2}((Pr(Piv_A|a) + Pr(Piv_A|b))) < 0$$

and thus $\mathbb{E}[U|a,v_B,\sigma] > \mathbb{E}[U|b,v_B,\sigma]$. Hence

$$BR_t(\sigma) = \begin{cases} 1 & b(b,\sigma) > q_t \\ [0,1] & b(b,\sigma) = q_t \\ 0 & b(b,\sigma) < q_t \end{cases}$$

by Lemma 1. Otherwise, $BR_t(\sigma)(A) = \hat{BR}_t(\sigma)(A)$.
Given the above notes, Lemma 1 shows that σ_h is an equilibrium if $\beta(h,n) \in \eta(h,n)$

$$\eta(h,n) = \begin{cases} [q_h, p_{h+1}] & h \in \mathbb{Z} \\ q_{[h]} & h \in ([h]+z(h,n), [h]+1) \\ [p_{[h]}, q_{[h]}] & h = [h]+z(h,n) \\ p_{[h]} & h \in ([h], [h]+z(h,n)) \end{cases}.$$

It's clear that $\hat{s}(\cdot, \sigma_n)$ is continuous by construction. It follows that $z(\cdot, n)$ is continuous since it can be written as the minimum of two continuous functions. Therefore $\eta(\cdot, n)$ is UHC, convex valued and increasing, as in Myerson [1998].
There exists numbers $I(A) \neq I(B)$ so that $\tau(A|\omega, \sigma_{I(\omega)}) = \tau(B|\omega, \sigma_{I(\omega)})$ for each ω. For n high enough, $\exists h(n)$ so that $\beta(h(n), n) \in \eta(h(n), n)$ and $h(n) \in (I(A), I(B))$ (or $(I(B), I(A))$ if $I(B) < I(A)$). This follows from $\beta(I(A), n) \to 0$, $\beta(I(B), n) \to 1$ and $\beta(\cdot, n)$ is continuous. Since $h(n) \in (I(A), I(B))$, $\tau(A|a, \sigma_{h(n)}) > \tau(B|a, \sigma_{h(n)})$ and $\tau(B|b, \sigma_{h(n)}) > \tau(A|b, \sigma_{h(n)})$. Using the arguments of Myerson [1998, Thm 2], the sequence of equilibrium vote shares must converge to the same limit in Myerson [1998, Thm 2] and thus as in that theorem, the correct candidate is elected and FIE holds.

□

Appendix E. Proofs from Section 5

Theorem 6 follows from a special case of Theorems 7 and 8.

Theorem 7. *Suppose $(\Gamma_n)_{n=1}^{\infty}$ is a sequence of AVGAs with voters who lack confidence and Bayesian posteriors. If there is no equilibrium where the worst case scenario for all voters is independent of their vote, $(\Gamma_n)_{n=1}^{\infty}$ does not satisfy FIE.*

Proof. The proof will be by contradiction. Suppose there is a sequence of equilibria $(\sigma_n)_{n=1}^{\infty}$ that satisfy FIE and that no equilibria of the game for any $n > 0$ are such that the worst case scenario is independent of the strategy chosen.
Begin by deriving the best response correspondence for voters when the worst case scenario varies with the strategy played. For any strategy $s \in \Delta C$, represent s by the ordered pair $(\frac{s(A)}{1-s(\emptyset)}, s(\emptyset))$ if $s(\emptyset) < 1$ and $(0,1)$ otherwise. Note that there is a bijection between these ordered pairs corresponds and each strategy profile. Now, define a function $\hat{s} : \Omega \times [0,1) \times (\Delta C)^T \to \mathbb{R}$ by

$$\hat{s}(A; s, \sigma) = \frac{2(\mathbb{E}[U|b,\sigma] - \mathbb{E}[U|a,\sigma]) + (1-s)[Pr(Piv_B|a,\sigma) + Pr(Piv_B|b,\sigma)]}{(1-s)[Pr(Piv_B|a,\sigma) + Pr(Piv_B|b,\sigma) + Pr(Piv_A|a,\sigma) + Pr(Piv_A|b,\sigma)]}$$

and $\bar{s} : \sigma \to [0,1]$ implicitly by

$$\hat{s}(A; \bar{s}(\sigma), \sigma) = \begin{cases} 1 & if\ \mathbb{E}[U|b,\sigma] > \mathbb{E}[U|a,\sigma] \\ 0 & if\ \mathbb{E}[U|b,\sigma] < \mathbb{E}[U|a,\sigma] \end{cases}$$

and $\bar{s}(\sigma) = 1$ if $\mathbb{E}[U|b,\sigma] = \mathbb{E}[U|a,\sigma]$. Note that if $\sigma(t)(\emptyset) < \bar{s}(\sigma)$, the voter's worst case scenario still changes with her vote. In this case, playing the strategy defined by $\sigma(t)(A) = \hat{s}(A; \sigma(t)(\emptyset), \sigma)$ equalizes the voter's expected utilities across states. On the other hand, if $\sigma(t)(\emptyset) \geq \bar{a}(\sigma)$, the voter is abstaining enough that her vote will no longer affect the worst case scenario.

Lemma 4. *Suppose that the worst case scenario is not independent of the strategy picked given σ and that the expected winner is correct in each state. If σ is an equilibrium and $b(b,\sigma) \in (p_t, q_t)$, then $\sigma(t) \in BR_t(\sigma)$ where*

$$BR_t(\sigma) = \begin{cases} \{(0,1)\} & if\ \mathbb{E}[U|b,\sigma] = \mathbb{E}[U|a,\sigma] \\ = \begin{cases} \{(0,1)\} & if\ \frac{q_t}{1-q_t} < \frac{Pr(Piv_A|b,\sigma)}{Pr(Piv_A|a,\sigma)} \\ \{1\} \times [\bar{s}(\sigma), 1] & if\ \frac{q_t}{1-q_t} = \frac{Pr(Piv_A|b,\sigma)}{Pr(Piv_A|a,\sigma)} \\ \{(1, \bar{s}(\sigma))\} & if\ \frac{q_t}{1-q_t} > \frac{Pr(Piv_A|b,\sigma)}{Pr(Piv_A|a,\sigma)} \end{cases} & if\ \mathbb{E}[U|b,\sigma] > \mathbb{E}[U|a,\sigma] \\ = \begin{cases} \{(0,1)\} & if\ \frac{p_t}{1-p_t} > \frac{Pr(Piv_B|b,\sigma)}{Pr(Piv_B|a,\sigma)} \\ \{0\} \times [\bar{s}(\sigma), 1] & if\ \frac{p_t}{1-p_t} = \frac{Pr(Piv_B|b,\sigma)}{Pr(Piv_B|a,\sigma)} \\ \{(0, \bar{a}(\sigma))\} & if\ \frac{p_t}{1-p_t} < \frac{Pr(Piv_B|b,\sigma)}{Pr(Piv_B|a,\sigma)} \end{cases} & if\ \mathbb{E}[U|b,\sigma] < \mathbb{E}[U|a,\sigma] \end{cases}.$$

Proof. (I drop the subscript t for convenience).
Suppose $p < b(b,\sigma) < q$. If the voter plays strategy (s,θ), she gets

$$\begin{aligned} V_t(s,\theta;\sigma) &= \min_{\pi \in [p,q]} \pi\{\mathbb{E}[U|a,\sigma] + (1-\theta)[sPr(Piv_A|a,\sigma) - (1-s)Pr(Piv_B|a,\sigma)]\} + \\ &\quad +(1-\pi)\{\mathbb{E}[U|b,\sigma] + (1-\theta)[(1-s)Pr(Piv_B|b,\sigma) - sPr(Piv_A|b,\sigma)]\}. \end{aligned}$$

Given a fixed $\theta < \bar{s}(\sigma)$, consider $v_{\theta\sigma} : [0,1] \to \mathbb{R}$ define by $v_{a\sigma}(s) = V_t(s,\theta;\sigma)$. Note that

$$\partial v_{a\sigma}(s) = \begin{cases} \{(1-\theta)[p[Pr(Piv_A|a,\sigma) + Pr(Piv_B|a,\sigma)] - & if\ s > \hat{s}(A;\theta,\sigma) \\ \quad -(1-p)[Pr(Piv_B|b,\sigma) + Pr(Piv_A|b,\sigma)]]\} & \\ \{(1-a\theta)[\pi[Pr(Piv_A|a,\sigma) + Pr(Piv_B|a,\sigma)] - & if\ s = \hat{s}(A,\theta,\sigma) \\ \quad -(1-\pi)[(Pr(Piv_B|b,\sigma) + Pr(Piv_A|b,\sigma)]] : \pi \in [p,q]\} & \\ \{(1-\theta)[q[Pr(Piv_A|a,\sigma) + Pr(Piv_B|a,\sigma)] + & if\ s < \hat{s}(A,\theta,\sigma) \\ \quad +(1-q)[Pr(Piv_B|b,\sigma) + Pr(Piv_A|b,\sigma)]]\} & \end{cases}$$

As in Lemma 1, given $p < b(b,\sigma) < q$, $0 \in \partial v_{\theta\sigma}(s)$ only if $s = \hat{s}(A,\theta,\sigma)$. Given this, consider $v_\sigma : [0,1] \to \mathbb{R}$ defined by $v_\sigma(\theta) = V_t(\hat{s}(A,\theta,\sigma),\theta,\sigma)$. Write $pc\omega = Pr(Piv_c|\omega,\sigma)$. By construction

$$\mathbb{E}[U|a,\sigma] + (1-\theta)[\hat{s}Pr(Piv_A|a,\sigma) - (1-\hat{s})Pr(Piv_B|a,\sigma)]] = [\mathbb{E}[U|b,\sigma] + (1-\theta)[(1-\hat{s})Pr(Piv_B|b,\sigma) - \hat{s}Pr(Piv$$

when $\hat{s} = \hat{s}(A, \theta, \sigma)$. So if $\theta < \bar{s}(\sigma)$

$$\begin{aligned}
v_\sigma(\theta) &= \mathbb{E}[U|a,\sigma] + (1-\theta)[\hat{s}Pr(Piv_A|a,\sigma) - (1-\hat{s})Pr(Piv_B|a,\sigma)]] \\
\partial v_\sigma(\theta) &= \{\frac{\partial}{\partial\theta}[(1-\theta)\frac{2(\mathbb{E}[U|b,\sigma] - \mathbb{E}[U|a,\sigma]) + (1-\theta)[pBa + pBb]}{(1-\theta)[pAa + pBb + pBa + pAb]}pAa - \\
&\quad -(1-\theta)\frac{2(\mathbb{E}[U|a,\sigma] - \mathbb{E}[U|b,\sigma]) + (1-\theta)[pAa + pAb]}{(1-\theta)[pAa + pBb + pBa + pAb]}pBa]\} \\
&= \{\frac{\partial}{\partial\theta}[\frac{2(\mathbb{E}[U|b,\sigma] - \mathbb{E}[U|a,\sigma]) - 2(\mathbb{E}[U|a,\sigma] - \mathbb{E}[U|b,\sigma])}{pAa + pBb + pBa + pAb} + \\
&\quad + \frac{(1-\theta)[pBa + pBb]pAa - pBa[pAa + pAb]}{pAa + pBb + pBa + pAb}]\} \\
&= \{\frac{pBa[pAa + pAb] - [pBa + pBb]pAa}{pAa + pBb + pBa + pAb}\} \\
&= \{\frac{pBa(pAb) - pBb(pAa)}{pAa + pBb + pBa + pAb}\}
\end{aligned}$$

Since FIE implies that $\frac{Pr(Piv_A|A,\sigma)}{Pr(Piv_B|A,\sigma)} < \frac{Pr(Piv_A|B,\sigma)}{Pr(Piv_B|B,\sigma)}$, no $\sigma(t)(\emptyset) < \bar{a}(\sigma)$ is optimal. Therefore, the voter abstains enough that worst case scenario is independent of whether she votes for A or B when she votes.

We can think of her as a SEU voter that assigns either probability p to A (if $\mathbb{E}[U|b,\sigma] < \mathbb{E}[U|a,\sigma]$) or q to A (if $\mathbb{E}[U|b,\sigma] > \mathbb{E}[U|a,\sigma]$). In this case, because $p < b(b,\sigma) < q$, the voter votes for B (in the first case) or A (in the second case) for sure conditional on voting. In the first case, she abstains for sure if $\frac{p}{1-p} > \frac{Pr(Piv_B|b,\sigma)}{Pr(Piv_B|a,\sigma)}$, and abstains with probability $\bar{s}(\sigma)$ if $\frac{p}{1-p} < \frac{Pr(Piv_B|b,\sigma)}{Pr(Piv_B|a,\sigma)}$. She is willing to abstain with any probability between $[\bar{s}(\sigma), 1]$ if $\frac{p}{1-p} = \frac{Pr(Piv_B|b,\sigma)}{Pr(Piv_B|a,\sigma)}$. In the second case, she abstains for sure if $\frac{q}{1-q} < \frac{Pr(Piv_A|b,\sigma)}{Pr(Piv_A|a,\sigma)}$, and abstains with probability $\bar{a}(\sigma)$ if $\frac{q}{1-q} > \frac{Pr(Piv_A|b,\sigma)}{Pr(Piv_A|a,\sigma)}$. She is willing to abstain with any probability between $[\bar{a}(\sigma), 1]$ if $\frac{q}{1-q} = \frac{Pr(Piv_A|b,\sigma)}{Pr(Piv_A|a,\sigma)}$. This establishes the best response correspondence when $p < b(b,\sigma) < q$.

□

Arguments along the lines of Bouton and Castanheira [2009] Lemma 1 or Feddersen and Pesendorfer [1996] Proposition 1 establish that a voter of type t strictly prefers to abstain whenever $b(b,\sigma) = q_t$ or $b(b,\sigma) = p_t$.

Lemma 5. *Suppose that the worst case scenario is not independent of the strategy picked given σ and that the expected winner is correct in each state. If σ is an equilibrium and $b(b,\sigma) < p_t$, then $\sigma(t) \in BR_t(\sigma)$ where*

$$BR_t(\sigma) = \begin{cases} \{(1,0)\} & if\ \frac{p_t}{1-p_t} > \frac{Pr(Piv_A|b,\sigma)}{Pr(Piv_A|a,\sigma)} \\ \{1\} \times [0, \bar{a}(\sigma)] & if\ \frac{p_t}{1-p_t} = \frac{Pr(Piv_A|b,\sigma)}{Pr(Piv_A|a,\sigma)} \\ \widetilde{BR_{A,t}}(\sigma) & if\ \frac{p_t}{1-p_t} < \frac{Pr(Piv_A|b,\sigma)}{Pr(Piv_A|a,\sigma)} \end{cases}$$

and

$$\widetilde{BR_{A,t}}(\sigma) = \begin{cases} = \begin{cases} \{(0,1)\} & if\ \frac{q_t}{1-q_t} < \frac{Pr(Piv_A|b,\sigma)}{Pr(Piv_A|a,\sigma)} \\ \{1\} \times [\bar{s}(\sigma), 1] & if\ \frac{q_t}{1-q_t} = \frac{Pr(Piv_A|b,\sigma)}{Pr(Piv_A|a,\sigma)} \\ \{(1, \bar{s}(\sigma))\} & if\ \frac{q_t}{1-q_t} > \frac{Pr(Piv_A|b,\sigma)}{Pr(Piv_A|a,\sigma)} \end{cases} & if\ \mathbb{E}[U|b,\sigma] > \mathbb{E}[U|a,\sigma] \\ \{(0,1)\} & otherwise \end{cases}$$

Proof. She votes for A conditional on voting because $b(b,\sigma)$ is low enough relative to her priors. She never abstains if $\frac{p_t}{1-p_t} > \frac{Pr(Piv_A|b,\sigma)}{Pr(Piv_A|a,\sigma)}$. If $\frac{p_t}{1-p_t} = \frac{Pr(Piv_A|b,\sigma)}{Pr(Piv_A|a,\sigma)}$, she's indifferent between abstaining and voting for B and so is willing to play any mixture between voting and abstaining. She abstains at least enough that she can't affect the outcome with her vote if $\frac{p_t}{1-p_t} < \frac{Pr(Piv_A|b,\sigma)}{Pr(Piv_A|a,\sigma)}$. If she abstains more than $\bar{s}(\sigma)$, she acts as if she's an SEU voter who assigns probability p_t to a if $\mathbb{E}[U|a,\sigma] > \mathbb{E}[U|b,\sigma]$ and q_t to a if $\mathbb{E}[U|b,\sigma] > \mathbb{E}[U|a,\sigma]$. Her best response correspondence is exactly as in Bouton and Castanheira [2009], establishing the result.

□

Lemma 6. *Suppose that the worst case scenario is not independent of the strategy picked given σ and that the expected winner is correct in each state. If σ is an equilibrium and $b(B,\sigma) > q_t$, then $\sigma(t) \in BR_t(\sigma)$ where*

$$BR_t(\sigma) = \begin{cases} \{(0,0)\} & if\ \frac{q_t}{1-q_t} < \frac{Pr(Piv_B|b,\sigma)}{Pr(Piv_B|a,\sigma)} \\ \{0\} \times [0, \bar{s}(\sigma)] & if\ \frac{q_t}{1-q_t} = \frac{Pr(Piv_B|b,\sigma)}{Pr(Piv_B|a,\sigma)} \\ \widetilde{BR_{B,t}}(\sigma) & if\ \frac{q_t}{1-q_t} > \frac{Pr(Piv_B|b,\sigma)}{Pr(Piv_B|a,\sigma)} \end{cases}$$

and4

$$\widetilde{BR_{B,t}}(\sigma) = \begin{cases} = \begin{cases} \{(0,1)\} & if\ \frac{p_t}{1-p_t} > \frac{Pr(Piv_B|b,\sigma)}{Pr(Piv_B|a,\sigma)} \\ \{0\} \times [\bar{s}(\sigma), 1] & if\ \frac{p_t}{1-p_t} = \frac{Pr(Piv_B|b,\sigma)}{Pr(Piv_B|a,\sigma)} \\ \{(0, \bar{s}(\sigma))\} & if\ \frac{p_t}{1-p_t} < \frac{Pr(Piv_B|b,\sigma)}{Pr(Piv_B|a,\sigma)} \end{cases} & if\ \mathbb{E}[U|a,\sigma] > \mathbb{E}[U|b,\sigma] \\ \{(0,1)\} & otherwise \end{cases}$$

Proof. She votes for B conditional on voting because $b(b,\sigma)$ is high enough. She never abstains if $\frac{q_t}{1-q_t} < \frac{Pr(Piv_B|b,\sigma)}{Pr(Piv_B|a,\sigma)}$. If $\frac{q_t}{1-q_t} = \frac{Pr(Piv_B|b,\sigma)}{Pr(Piv_B|a,\sigma)}$, she's indifferent between abstaining and voting for B and so is willing to play any mixture between voting and abstaining. She abstains at least enough that she can't affect the outcome with her vote if $\frac{q_t}{1-q_t} > \frac{Pr(Piv_B|b,\sigma)}{Pr(Piv_B|a,\sigma)}$. If she abstains more than $\bar{s}(\sigma)$, she acts as if she's an SEU voter who assigns probability p_t to a if $\mathbb{E}[U|a,\sigma] > \mathbb{E}[U|b,\sigma]$ and q_t to a if $\mathbb{E}[U|b,\sigma] > \mathbb{E}[U|a,\sigma]$. Her best response correspondence is exactly as in Bouton and Castanheira [2009], establishing the result.

□

Now, focus on the specific conditions at equilibrium. Because of the quasi-Bayesian posteriors and the lack of confidence, $p_2 \leq p_1 < \frac{1}{2} < q_2 \leq q_1$. These values partition $[0,1]$ into regions where the best response correspondence of the voters has similar properties when $b(\cdot)$ is within that region. Proceed by analyzing these regions separately.

Suppose now that $b(B,\sigma_n) \in (p_2,p_1)$. Note that this implies that $\frac{\sigma_n(1)(A)}{\sigma_n(1)(A)+\sigma_n(1)(B)} = 1$ when $\sigma_n(1)(\emptyset) < 1$. If σ_n satisfies FIE, then $\sigma_n(1)(\emptyset) < 1$. Suppose first that $\sigma_n(1)(\emptyset) < \bar{a}(\sigma_n)$ so $\frac{p_1}{1-p_1} \geq \frac{Pr(Piv_A|b,\sigma)}{Pr(Piv_A|a,\sigma)}$. Additionally, it must be that $\frac{\sigma_n(2)(B)}{\sigma_n(2)(A)+\sigma_n(2)(B)} = 1$ and $\sigma_n(1)(\emptyset) < 1$ so $\mathbb{E}[U|B,|\sigma_n] > \mathbb{E}[U|A,\sigma_n]$ and $\frac{q_2}{1-q_2} \leq \frac{Pr(Piv_B|b,\sigma)}{Pr(Piv_B|a,\sigma)}$. Note that since $Pr(Piv_A|b,\sigma_n) > Pr(Piv_B|b,\sigma_n)$ and $Pr(Piv_A|a,\sigma) < Pr(Piv_B|a,\sigma)$, it follows from $p_1 < q_2$ that these are mutually impossible.

Now, if $\sigma_n(1)(\emptyset) \geq \bar{a}(\sigma_n)$ and FIE is satisfied, it must be that $\bar{s}(\sigma_n)n \to \infty$. Otherwise, there is a sub-sequence (n_k) so that $\bar{s}_{n_k}(\sigma_{n_k}) \to \bar{n}$ where $\bar{n}$is finite. Along this sub-sequence there is a probability approaching $e^{-\bar{n}}$ that no one votes. Therefore, each candidate is elected with a minimum probability tending to $\frac{1}{2}e^{-\bar{n}}$ in each state along this sub-sequence so FIE cannot be satisfied.

Since $\bar{s}(\sigma_n)n \to \infty$, the pivot probabilities and conditional expected utilities are exactly equivalent to the game with an expected number of voters $a_n(\sigma_n)n$ and with a strategy profile $\hat{\sigma}_n$ defined by

$$\hat{\sigma}_n(t)(a) = \frac{\sigma_n(t)(a)}{1-\bar{a}(\sigma_n)}.$$

By construction, this strategy profile and this game make the worst case scenario is independent of the strategy picked. However, since $a_n(\sigma_n)n \to \infty$ the hypothesis that there is no sequence of equilibria so that the worst case scenario is independent of the strategy picked, which contradicts the existence of such a strategy profile.

Now suppose that $b(b,\sigma_n) \in (q_2,q_1)$. Note that this implies that $\frac{\sigma_n(2)(B)}{\sigma_n(2)(A)+\sigma_n(2)(B)} = 1$ when $\sigma_n(2)(\emptyset) < 1$. If σ_n satisfies FIE, then $\sigma_n(2)(\emptyset) < 1$. If additionally $\sigma_n(2)(\emptyset) < \bar{s}(\sigma_n)$ then $\frac{q_2}{1-q_2} \leq \frac{Pr(Piv_B|b,\sigma)}{Pr(Piv_B|a,\sigma)}$. Additionally, it must be that $\frac{\sigma_n(1)(A)}{\sigma_n(1)(A)+\sigma_n(1)(B)} = 1$ and $\sigma_n(1)(\emptyset) < 1$ so $\mathbb{E}[U|a,\sigma_n] > \mathbb{E}[U|b,\sigma_n]$ and $\frac{p_1}{1-p_1} \geq \frac{Pr(Piv_A|b,\sigma_n)}{Pr(Piv_A|a,\sigma_n)}$. Note that since

$$Pr(Piv_A|b,\sigma_n) > Pr(Piv_B|b,\sigma_n)$$

and

$$Pr(Piv_A|a,\sigma) < Pr(Piv_B|a,\sigma),$$

it follows that

$$\frac{Pr(Piv_B|b,\sigma)}{Pr(Piv_B|a,\sigma)} < \frac{Pr(Piv_A|b,\sigma_n)}{Pr(Piv_A|a,\sigma_n)}$$

from $p_1 < q_2$ that these are mutually impossible. If $\sigma_n(2)(\emptyset) \geq \bar{s}(\sigma_n)$ the same logic as when $b(B,\sigma_n) \in (p_2,p_1)$ leads to the same contradiction.

If $b(b,\sigma_n) \in [0,p_1] \cup [p_2,q_1] \cup [q_2,1]$ it follows from Lemmas 4-6 that all voters will vote for the same candidate whenever they do not abstain, which contradicts FIE.

Therefore, there is no sequence of equilibria that satisfies FIE.

□

Assume WLOG that $r(1|a) + r(1|a) \geq 1$ and that $r(1|a) \geq r(1|b)$ (otherwise, relabel candidates and types).

Define $\hat{\tau} : \{A, B\} \times \Omega \to [0, 1]$ by

$$\hat{\tau}(A|a) = (\frac{\sqrt{r(2|a)} + \sqrt{r(2|b)}}{\sqrt{r(1|a)} + \sqrt{r(1|b)}})^2 r(1|a)$$
$$\hat{\tau}(B|a) = r(2|a)$$
$$\hat{\tau}(A|b) = (\frac{\sqrt{r(2|a)} + \sqrt{r(2|b)}}{\sqrt{r(1|a)} + \sqrt{r(1|b)}})^2 r(1|b)$$
$$\hat{\tau}(B|b) = r(2|b)$$

which would be the limiting vote shares for each candidate in each state if voters were expected utility.

Theorem 8. *Fix any sequence $(\Gamma_n)_{n=1}^{\infty}$ of AVGAs with voters who lack confidence and quasi-Bayesian posteriors. If the inequalities*

$$2 + \sqrt{\frac{\hat{\tau}(B|b)}{\hat{\tau}(A|b)}} + \sqrt{\frac{\hat{\tau}(B|a)}{\hat{\tau}(A|a)}} > 2(\frac{\hat{\tau}(A|b)\hat{\tau}(B|b)}{\hat{\tau}(A|a)\hat{\tau}(B|a)})^{\frac{1}{4}} - 1)\frac{\sqrt{\frac{\hat{\tau}(A|b)}{\hat{\tau}(B|b)}}}{1 - \sqrt{\frac{\hat{\tau}(A|b)}{\hat{\tau}(B|b)}}} \tag{E.1}$$

and

$$2 + \sqrt{\frac{\hat{\tau}(B|b)}{\hat{\tau}(A|b)}} + \sqrt{\frac{\hat{\tau}(B|a)}{\hat{\tau}(A|a)}} > \frac{\sqrt{\frac{\hat{\tau}(A|b)}{\hat{\tau}(B|b)}}}{1 - \sqrt{\frac{\hat{\tau}(A|b)}{\hat{\tau}(B|b)}}} - \frac{\frac{\hat{\tau}(B|a)}{\sqrt{\hat{\tau}(A|b)\hat{\tau}(B|b)}}}{1 - \frac{\hat{\tau}(B|a)}{\sqrt{\hat{\tau}(A|b)\hat{\tau}(B|b)}}} \tag{E.2}$$

both hold, then for n sufficiently high, the worst case scenario for all voters is independent of their vote in any equilibrium.

Proof. The proof will be by contradiction. The first step characterizes what such an equilibrium would look like if it existed.

Lemma 7. *Suppose that (σ_n^*) is a sequence of equilibrium strategy profiles to an SEU Poisson voting game with $T = \{1, 2\}$ and $r(1|a)+r(1|b) \geq r(2|a)+r(2|b)$ that satisfies FIE. If $q_1(a) > q_2(a)$ and $\sigma_n^* \to \sigma$, then $\sigma(1)(B) = 0$, $\sigma(2)(B) = 1$, $\mu(a, \sigma) = \mu(b, \sigma)$, $\tau(A|a, \sigma) < \tau(B|b, \sigma)$ and $\frac{\tau(A|b,\sigma)}{\tau(B|b,\sigma)} > \frac{\tau(B|a,\sigma)}{\tau(A|a,\sigma)}$.*

Proof. The beginning claim follows from Bouton and Castanheira [2009] Lemma 1 and Theorem 1. At the limit

$$\mu(a) = \mu(b) \iff (\sqrt{\tau(A|a)} - \sqrt{\tau(B|a)})^2 = (\sqrt{\tau(B|b)} - \sqrt{\tau(A|b)})^2.$$

Rewriting,

$$\sqrt{(1 - \bar{a})r(1|a)} - \sqrt{r(2|a)} = \sqrt{r(2|b)} - \sqrt{(1 - \bar{a})r(1|b)}$$

where $\bar{a} = \sigma(1)(\emptyset)$. Solving for $\bar{a}$ yields

$$1 - (\frac{\sqrt{1 - r(1|a)} + \sqrt{1 - r(1|b)}}{\sqrt{r(1|a)} + \sqrt{r(1|b)}})^2.$$

The remaining results follow from algebra.

□

The worst case scenarios is independent of the strategy chosen given σ is played if and only if either

$$\mathbb{E}[U|b,\sigma,n] - \frac{1}{2}Pr(Piv_A|b,\sigma,n) \geq \mathbb{E}[U|a,\sigma,n] + \frac{1}{2}Pr(Piv_A|a,\sigma,n) \tag{E.3}$$

or

$$\mathbb{E}[U|a,\sigma,n] - \frac{1}{2}Pr(Piv_B|a,\sigma,n) \geq \mathbb{E}[U|b,\sigma,n] + \frac{1}{2}Pr(Piv_B|b,\sigma,n) \tag{E.4}$$

as in Lemma 1. Consider the limiting equilibrium strategy profile. At this strategy profile, neither of these equations holds for n large enough.

Lemma 8. *Suppose $r(1|a) + r(1|b) \geq r(2|a) + r(2|b)$. Let σ be the limiting strategy profile from Lemma 7. If*

$$2 + \sqrt{\frac{\tau(B|b)}{\tau(A|b)}} + \sqrt{\frac{\tau(B|a)}{\tau(A|a)}} > 2(\frac{\tau(A|b)\tau(B|b)}{\tau(A|a)\tau(B|a)})^{\frac{1}{4}} - 1)\frac{\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}}{1-\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}}$$

then $\mathbb{E}[U|b,\sigma,n] - \mathbb{E}[U|a,\sigma,n] < \frac{1}{2}(Pr(Piv_A|a,\sigma,n) + Pr(Piv_A|b,\sigma,n))$ for n large enough.

Proof. Lemma 7 shows that

$$\tau(B|b)\tau(A|b) > \tau(A|a)\tau(B|a) \tag{E.5}$$

and

$$\frac{\tau(A|b)}{\tau(B|b)} > \frac{\tau(B|a)}{\tau(A|a)} \tag{E.6}$$

whenever the above conditions are satisfied. Set

$$\mu(\omega) = -\tau(A|\omega) - \tau(B|\omega) + 2\sqrt{\tau(B|\omega)\tau(A|\omega)}$$

noting that $\mu(A) = \mu(B) = \mu \in [-1, 0]$.
Since

$$\mathbb{E}[U|a,\sigma] = 1 - e^{-(\tau(A|a)+\tau(B|a))n}(\sum_{k=1}^{\infty}(\frac{\tau(B|a)}{\tau(A|a)})^{\frac{k}{2}}I_k(2n\sqrt{\tau(A|a)\tau(B|a)}) - \frac{1}{2}I_0(2n\sqrt{\tau(A|a)\tau(B|a)})$$

and

$$\mathbb{E}[U|b,\sigma] = 1 - e^{-(\tau(A|b)+\tau(B|b))n}(\sum_{k=1}^{\infty}\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}^{k}I_k(2n\sqrt{\tau(A|b)\tau(B|b)}) - \frac{1}{2}I_0(2n\sqrt{\tau(A|b)\tau(B|b)})$$

(where $I_k(\cdot)$ is a modified Bessel function of the first kind (see Myerson [2000], p. 27)), the conclusion is equivalent to
(E.7)

$$e^{-(\tau(A|a)+\tau(B|a))n}\sum_{k=1}^{\infty}\sqrt{\frac{\tau(B|a)}{\tau(A|a)}}^{k}I_k(2n\sqrt{\tau(A|a)\tau(B|a)}) - e^{-(\tau(A|b)+\tau(B|b))n}\sum_{k=1}^{\infty}\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}^{k}I_k(2n\sqrt{\tau(A|b)\tau(B|b)})$$

is less than

$$\frac{1}{2}(Pr(Piv_A|b) + Pr(Piv_A|a) + I_0(2n\sqrt{\tau(A|b)\tau(B|b)}) - I_0(2n\sqrt{\tau(A|a)\tau(B|a)})).$$

Let $\phi(n)$ be the value of (E.7).

By Baricz [2010] equation (2.6) we have that if $y > x > 0$ and $k > 0$ is an integer then

$$I_k(x) < e^{x-y}(\frac{y}{x})^{\frac{1}{2}} I_k(y). \tag{E.8}$$

Using equations (E.5) and (E.8), we have that

$$e^{-(\tau(A|a)+\tau(B|a))n}(\sum_{k=1}^{\infty} \sqrt{\frac{\tau(B|a)}{\tau(A|a)}}^k I_k(2n\sqrt{\tau(A|a)\tau(B|a)})$$

$$< e^{(\mu-2\sqrt{\tau(B|b)\tau(B|b)})n}(\frac{\tau(A|b)\tau(B|b)}{\tau(A|a)\tau(B|a)})^{\frac{1}{4}} \sum_{k=1}^{\infty} \sqrt{\frac{\tau(B|a)}{\tau(A|a)}}^k I_k(2n\sqrt{\tau(A|b)\tau(B|b)})$$

so we find that

$$\begin{aligned}\frac{\phi(n)}{e^{-2\sqrt{\tau(B|A)\tau(B|B)})n}} &< e^{\mu n}(\frac{\tau(A|b)\tau(B|b)}{\tau(A|a)\tau(B|a)})^{\frac{1}{4}} \sum_{k=1}^{\infty} \sqrt{\frac{\tau(B|a)}{\tau(A|a)}}^k I_k(2n\sqrt{\tau(A|b)\tau(B|b)}) - \\ &\quad -e^{\mu n} \sum_{k=1}^{\infty} \sqrt{\frac{\tau(A|b)}{\tau(B|b)}}^k I_k(2n\sqrt{\tau(A|b)\tau(B|b)}) \\ &< (\frac{\tau(A|b)\tau(B|b)}{\tau(A|a)\tau(B|a)})^{\frac{1}{4}} \sum_{k=1}^{\infty} \sqrt{\frac{\tau(B|b)}{\tau(A|b)}}^k I_k(2n\sqrt{\tau(A|b)\tau(B|b)}) - \\ &\quad -e^{\mu n} \sum_{k=1}^{\infty} \sqrt{\frac{\tau(A|b)}{\tau(B|b)}}^k I_k(2n\sqrt{\tau(A|b)\tau(B|b)})\end{aligned}$$

since $\frac{\tau(B|a)}{\tau(A|a)} < \frac{\tau(A|b)}{\tau(B|b)}$. Setting

$$\bar{K}(n) = \sum_{k=1}^{\infty} \sqrt{\frac{\tau(A|b)}{\tau(B|b)}}^k I_k(2n\sqrt{\tau(A|b)\tau(B|b)}) > 0$$

and

$$\theta = (\frac{\tau(A|b)\tau(B|b)}{\tau(A|a)\tau(B|a)})^{\frac{1}{4}} - 1 > 0$$

yields that

$$\mathbb{E}[U|b,\sigma] - \mathbb{E}[U|a,\sigma] < \theta e^{\mu n} \bar{K}(n) e^{-2n\sqrt{\tau(A|b)\tau(B|b)}}$$

Note that

$$\begin{aligned}
\bar{K}(n) &= \sum_{k=1}^{\infty} \sqrt{\frac{\tau(A|b)}{\tau(B|b)}}^{k} I_k(2n\sqrt{\tau(A|b)\tau(B|b)}) \\
&< \sum_{k=1}^{\infty} \sqrt{\frac{\tau(A|b)}{\tau(B|b)}}^{k} I_0(2n\sqrt{\tau(A|b)\tau(B|b)}) \\
&\approx \sum_{k=1}^{\infty} \sqrt{\frac{\tau(A|b)}{\tau(B|b)}}^{k} \frac{e^{\sqrt{(2n\sqrt{\tau(A|b)\tau(B|b)})^2}}}{\sqrt{2\pi\sqrt{(2n\sqrt{\tau(A|b)\tau(B|b)})^2}}} \\
&= \frac{\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}}{1-\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}} \frac{e^{2n\sqrt{\tau(A|b)\tau(B|b)}}}{2\sqrt{\pi n\sqrt{\tau(A|b)\tau(B|b)}}}
\end{aligned}$$

by Abramowitz and Stegun [1965] equations (9.7.1) and (9.7.7) and that when $k \geq 0$ it follows that $I_k(x) > I_{k+1}(x)$. Therefore, for n large enough

$$\phi(n) < \frac{\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}}{1-\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}} \theta e^{\mu n} I_0(2n\sqrt{\tau(A|b)\tau(B|b)}) e^{-2n\sqrt{\tau(A|B)\tau(B|B)}}$$

Since

$$\begin{aligned}
&\frac{1}{2}[Pr(Piv_A|b) + Pr(Piv_A|a) + e^{-(\tau(A|b)+\tau(B|b))n} I_0(2n\sqrt{\tau(A|b)\tau(B|b)}) - \\
&\quad -e^{-(\tau(A|a)+\tau(B|a))n} I_0(2n\sqrt{\tau(A|a)\tau(B|a)})] \\
= &\ \frac{1}{2}[e^{-(\tau(A|b)+\tau(B|b))n}(2I_0(2n\sqrt{\tau(A|b)\tau(B|b)}) + I_1(2n\sqrt{\tau(A|b)\tau(B|b)})\sqrt{\frac{\tau(B|b)}{\tau(A|b)}}) + \\
&\quad +e^{-(\tau(A|a)+\tau(B|a))n} I_1(2n\sqrt{\tau(A|a)\tau(B|a)})\sqrt{\frac{\tau(B|a)}{\tau(A|a)}}] \\
\approx &\ \frac{e^{\mu n}}{4\sqrt{\pi n}}\left(\frac{\sqrt{\frac{\tau(B|a)}{\tau(A|a)}}}{(\tau(A|a)\tau(B|a))^{\frac{1}{4}}} + \frac{2+\sqrt{\frac{\tau(B|b)}{\tau(A|b)}}}{(\tau(A|b)\tau(B|b))^{\frac{1}{4}}}\right),
\end{aligned}$$

it suffices to show that

$$\left[\frac{\sqrt{\frac{\tau(B|a)}{\tau(A|a)}}}{(\tau(A|a)\tau(B|a))^{\frac{1}{4}}} + \frac{2+\sqrt{\frac{\tau(B|b)}{\tau(A|b)}}}{(\tau(A|b)\tau(B|b))^{\frac{1}{4}}}\right] > \frac{\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}}{1-\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}} \frac{2(\frac{\tau(A|b)\tau(B|b)}{\tau(A|a)\tau(B|a)})^{\frac{1}{4}} - 1)}{(\tau(A|b)\tau(B|b))^{\frac{1}{4}}}.$$

Note that

$$\frac{\sqrt{\frac{\tau(B|a)}{\tau(A|a)}}}{(\tau(A|a)\tau(B|a))^{\frac{1}{4}}} + \frac{2+\sqrt{\frac{\tau(B|b)}{\tau(A|b)}}}{(\tau(A|b)\tau(B|b))^{\frac{1}{4}}} > \frac{2+\sqrt{\frac{\tau(B|b)}{\tau(A|b)}}+\sqrt{\frac{\tau(B|a)}{\tau(A|a)}}}{(\tau(A|b)\tau(B|b))^{\frac{1}{4}}}$$

because $\tau(B|b)\tau(A|b) > \tau(B|a)\tau(A|a)$. Therefore, if

$$2+\sqrt{\frac{\tau(B|b)}{\tau(A|b)}}+\sqrt{\frac{\tau(B|a)}{\tau(A|a)}} > 2(\frac{\tau(A|b)\tau(B|b)}{\tau(A|a)\tau(B|a)})^{\frac{1}{4}}-1)\frac{\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}}{1-\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}}$$

then the claim holds.

□

Lemma 9. *Suppose $r(1|a)+r(1|b) \geq r(2|a)+r(2|b)$. Let σ be the limiting strategy profile from Lemma 7. If*

$$2+\sqrt{\frac{\tau(B|b)}{\tau(A|b)}}+\sqrt{\frac{\tau(B|a)}{\tau(A|a)}} > \frac{\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}}{1-\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}}-\frac{\frac{\tau(B|a)}{\sqrt{\tau(A|b)\tau(B|b)}}}{1-\frac{\tau(B|a)}{\sqrt{\tau(A|b)\tau(B|b)}}}$$

then $\mathbb{E}[U|a,\sigma,n]-\mathbb{E}[U|b,\sigma,n] < \frac{1}{2}Pr(Piv_B|a,\sigma)+\frac{1}{2}Pr(Piv_B|b,\sigma)$ for n large enough.

Proof. As in Lemma 8, we can write the claim as

$$e^{-(\tau(A|b)+\tau(B|b))n}\sum_{k=1}^{\infty}\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}^{k} I_k(2n\sqrt{\tau(A|b)\tau(B|b)})- \\ - e^{-(\tau(A|a)+\tau(B|a))n}\sum_{k=1}^{\infty}\sqrt{\frac{\tau(B|a)}{\tau(A|a)}}^{k} I_k(2n\sqrt{\tau(A|a)\tau(B|a)}) \tag{E.9}$$

is less than

$$\frac{1}{2}(Pr(Piv_A|b)+Pr(Piv_A|a)-I_0(2n\sqrt{\tau(A|b)\tau(B|b)})+I_0(2n\sqrt{\tau(A|a)\tau(B|a)})).$$

Write $\phi(n)$ to be the value of (E.9). Note that

$$e^{-(\tau(A|a)+\tau(B|a))n}\sum_{k=1}^{\infty}\sqrt{\frac{\tau(B|a)}{\tau(A|a)}}^{k} I_k(2n\sqrt{\tau(A|a)\tau(B|a)})$$

$$> e^{-(\tau(A|a)+\tau(B|a))n}\sum_{k=1}^{\infty}\sqrt{\frac{\tau(B|a)}{\tau(A|a)}}^{k}\sqrt{\frac{\tau(A|a)\tau(B|a)}{\tau(A|b)\tau(B|b)}}^{k} I_k(2n\sqrt{\tau(A|b)\tau(B|b)})e^{2n\sqrt{\tau(A|a)\tau(B|a)}-2n\sqrt{\tau(A|b)\tau(B|b)}}$$

$$= e^{\mu n-2n\sqrt{\tau(A|b)\tau(B|b)}}\sum_{k=1}^{\infty}\frac{\tau(B|a)}{\sqrt{\tau(A|b)\tau(B|b)}}^{k} I_k(2n\sqrt{\tau(A|b)\tau(B|b)})$$

since whenever $k > \frac{1}{2}$ and $y > x$ we have

$$I_k(x) > (\frac{x}{y})^k e^{x-y} I_k(y) \tag{E.10}$$

by equation (2.2) of Baricz [2010].

We have that

$$\begin{aligned}
\phi(n) &< e^{-(\tau(A|b)+\tau(B|b))n}\sum_{k=1}^{\infty}\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}^{k} I_k(2n\sqrt{\tau(A|b)\tau(B|b)}) - \\
&\quad - e^{\mu n - 2n\sqrt{\tau(A|b)\tau(B|b)}}\sum_{k=1}^{\infty}\frac{\tau(B|a)}{\sqrt{\tau(A|b)\tau(B|b)}}^{k} I_k(2n\sqrt{\tau(A|b)\tau(B|b)}) \\
&= \frac{e^{\mu n}\sum_{k=1}^{\infty}(\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}^{k} - \frac{\tau(B|a)}{\sqrt{\tau(A|b)\tau(B|b)}}^{k})I_k(2n\sqrt{\tau(A|b)\tau(B|b)})}{e^{2n\sqrt{\tau(A|b)\tau(B|b)}}} \\
&< \frac{e^{\mu n}I_0(2n\sqrt{\tau(A|b)\tau(B|b)}\sum_{k=1}^{\infty}(\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}^{k} - \frac{\tau(B|a)}{\sqrt{\tau(A|b)\tau(B|b)}}^{k})}{e^{2n\sqrt{\tau(A|b)\tau(B|b)}}} \\
&\approx \frac{e^{\mu n}\sum_{k=1}^{\infty}(\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}^{k} - \frac{\tau(B|a)}{\sqrt{\tau(A|b)\tau(B|b)}}^{k})}{2\sqrt{\pi n\sqrt{\tau(A|b)\tau(B|b)}}} \\
&= \frac{e^{\mu n}(\frac{\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}}{1-\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}} - \frac{\frac{\tau(B|a)}{\sqrt{\tau(A|b)\tau(B|b)}}}{1-\frac{\tau(B|a)}{\sqrt{\tau(A|b)\tau(B|b)}}})}{2\sqrt{\pi n\sqrt{\tau(A|b)\tau(B|b)}}}
\end{aligned}$$

so it suffices to show that

$$2+\sqrt{\frac{\tau(B|b)}{\tau(A|b)}}+\sqrt{\frac{\tau(B|a)}{\tau(A|a)}} > \frac{\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}}{1-\sqrt{\frac{\tau(A|b)}{\tau(B|b)}}} - \frac{\frac{\tau(B|a)}{\sqrt{\tau(A|b)\tau(B|b)}}}{1-\frac{\tau(B|a)}{\sqrt{\tau(A|b)\tau(B|b)}}}$$

which is the hypothesis.

□

Now, consider the specific conditions at equilibrium. Suppose that σ is an equilibrium. If the election is not close, then it must be that either

$$\mathbb{E}[U|a,\sigma]-\mathbb{E}[U|b,\sigma] > \frac{1}{2}(Pr(Piv_B|b,\sigma)+Pr(Piv_B|a,\sigma))$$

or

$$\mathbb{E}[U|b,\sigma]-\mathbb{E}[U|a,\sigma] > \frac{1}{2}(Pr(Piv_A|b,\sigma)+Pr(Piv_A|a,\sigma)).$$

By Bouton and Castanheira [2009] Lemma 1, restrict attention to profiles indexed by $\theta \in [0,1]$ defined by $1-\sigma_\theta(1)(A) = \sigma_\theta(1)(\emptyset) = \theta$ and $\sigma_\theta(2)(B) = 1$. Let $\bar{a}$ be defined as in Lemma 1.

For n high enough, if σ_θ is an equilibrium then $\theta \in (0,1)$. Therefore, it must be the case that either

(E.11) $$p_1(Pr(Piv_A|a,\sigma_\theta)) = (1-p_1)Pr(Piv_A|b,\sigma_\theta),$$

(E.12) $$\frac{p_2}{1-p_2} < \frac{Pr(Piv_A|b,\sigma_\theta) + Pr(Piv_B|b,\sigma_\theta)}{Pr(Piv_A|a,\sigma_\theta) + Pr(Piv_B|a,\sigma_\theta)} < \frac{p_1}{1-p_1},$$

and (E.3) all hold or

(E.13) $$q_1(Pr(Piv_A|a,\sigma_\theta)) = (1-q_1)Pr(Piv_A|b,\sigma_\theta),$$

(E.14) $$\frac{q_2}{1-q_2} < \frac{Pr(Piv_A|b,\sigma_\theta) + Pr(Piv_B|b,\sigma_\theta)}{Pr(Piv_A|a,\sigma_\theta) + Pr(Piv_B|a,\sigma_\theta)} < \frac{q_1}{1-q_1},$$

and (E.4) all hold.

By Lemmas 8 and 9 above neither (E.4) nor (E.3) holds at $\sigma_{\bar{\theta}}$. It's easy to verify the following inequalities hold when when the expected winner is correct in both states:

- $\frac{\partial \frac{Pr(Piv_A|b,\sigma_\theta)+Pr(Piv_B|b,\sigma_\theta)}{Pr(Piv_A|a,\sigma_\theta)+Pr(Piv_B|a,\sigma_\theta)}}{\partial\theta} < 0$
- $\frac{\partial \frac{Pr(Piv_A|b,\sigma_\theta)}{Pr(Piv_A|a,\sigma_\theta)}}{\partial\theta} < 0$
- $\frac{\partial}{\partial\theta}(\mathbb{E}[U|b,\sigma_\theta] - \frac{1}{2}Pr(Piv_A|b,\sigma_\theta)) > 0$
- $\frac{\partial}{\partial\theta}(\mathbb{E}[U|a,\sigma_\theta] + \frac{1}{2}Pr(Piv_A|a,\sigma_\theta)) < 0$
- $\frac{\partial}{\partial\theta}(\mathbb{E}[U|b,\sigma_\theta] + \frac{1}{2}Pr(Piv_B|b,\sigma_\theta)) > 0$
- $\frac{\partial}{\partial\theta}(\mathbb{E}[U|a,\sigma_\theta] - \frac{1}{2}Pr(Piv_B|a,\sigma_\theta)) < 0$

Suppose that equations (E.11), (E.12) and (E.4) all hold for some σ_θ. It is the case that

$$\frac{Pr(Piv_A|b,\sigma_{\bar{\theta}})}{Pr(Piv_A|a,\sigma_{\bar{\theta}})} > 1$$

for n large enough (using standard formulas for pivot probabilities). Since (E.11) holds and $\frac{p_1}{1-p_2} < 1$, it must be that $\theta > \bar{\theta}$ because $\frac{\partial \frac{Pr(Piv_A|B,\sigma_{\bar{\theta}})}{Pr(Piv_A|A,\sigma_{\bar{\theta}})}}{\partial\theta} < 0$. However, this implies that

$$\mathbb{E}[U|a,\sigma_\theta] - \frac{1}{2}Pr(Piv_A|a,\sigma_\theta)) < \mathbb{E}[U|a,\sigma_{\bar{\theta}}] - \frac{1}{2}Pr(Piv_A|a,\sigma_{\bar{\theta}}))$$

and

$$\mathbb{E}[U|b,\sigma_\theta] + \frac{1}{2}Pr(Piv_B|b,\sigma_\theta) > \mathbb{E}[U|b,\sigma_{\bar{\theta}}] + \frac{1}{2}Pr(Piv_B|b,\sigma_{\bar{\theta}}).$$

Note therefore that

$$\mathbb{E}[U|b,\sigma_\theta] + \frac{1}{2}Pr(Piv_B|b,\sigma_\theta) > \mathbb{E}[U|a,\sigma_\theta] - \frac{1}{2}Pr(Piv_A|a,\sigma_\theta))$$

which means that (E.4) cannot hold.

Now, suppose that equations (E.13), (E.14) and (E.3) all hold for some σ_a. It can be verified that

$$\frac{Pr(Piv_A|b,\sigma_{\bar{\theta}}) + Pr(Piv_B|b,\sigma_{\bar{\theta}})}{Pr(Piv_A|a,\sigma_{\bar{\theta}}) + Pr(Piv_B|a,\sigma_{\bar{\theta}})} \leq 1$$

for n large enough using Myerson [2000] Equation 5.5, with equality holding only if $r(1|a) = r(2|b)$. Since (E.14) holds and $\frac{q_2}{1-q_2} > 1$, since $\frac{\partial \frac{Pr(Piv_A|B,\sigma_\theta)+Pr(Piv_B|B,\sigma_\theta)}{Pr(Piv_A|A,\sigma_\theta)+Pr(Piv_B|A,\sigma_\theta)}}{\partial\theta} < 0$ it must be that $\theta < \bar{\theta}$ for n large enough. However, this implies that

$$\mathbb{E}[U|a,\sigma_\theta] + \frac{1}{2}Pr(Piv_A|a,\sigma_\theta)) > \mathbb{E}[U|a,\sigma_{\bar{\theta}}] + \frac{1}{2}Pr(Piv_A|a,\sigma_{\bar{\theta}}))$$

and

$$\mathbb{E}[U|b,\sigma_\theta] - \frac{1}{2}Pr(Piv_A|b,\sigma_\theta) < \mathbb{E}[U|b,\sigma_{\bar{\theta}}] - \frac{1}{2}Pr(Piv_A|b,\sigma_{\bar{\theta}}).$$

Note therefore that

$$\mathbb{E}[U|b,\sigma_\theta] - \frac{1}{2}Pr(Piv_A|b,\sigma_\theta) < \mathbb{E}[U|a,\sigma_\theta] + \frac{1}{2}Pr(Piv_A|a,\sigma_\theta))$$

which means that (E.3) cannot hold. Therefore, σ_θ is not an equilibrium, which is a contradiction and completes the proof.

□

Proof of Proposition 1:

Proof. Note that $Pr(Piv_c|\omega,\sigma^*) = 1$ for all ω, c and $\mathbb{E}[U|a,\sigma^*] = \mathbb{E}[U|b,\sigma^*]$ since no one votes. From there, the logic in Proposition 1 shows that a fixed voter would prefer to randomize with equal probability between A and B rather than play any other strategy that mixes between voting for A and B. On the other hand, since the tie breaking rule is a coin-flip, she can induce the same distribution over outcomes by abstaining. Hence, she weakly prefers to abstain rather than flip a coin and is thus willing to play her strategy profile.

□