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Cost-Benefit Analysis with Adaptive Preferences

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Draft 19 April 2010

Abstract

In traditional welfare economics preferences of individuals are the measuring rod for the performance of economic policy. Preferences were assumed to be fixed. Can we maintain an individualistic welfare economics, if we realistically assume that preferences are influenced by the goods we consume? Can preferences remain the measuring rod, if there is a feedback from the objects to be measured onto the measuring rod? I present a theory which provides a positive answer, if preferences are "adaptive". Furthermore I develop the empirical hypothesis that the empirical and experimental results of behavioural economics conform to the assumption of adaptive preferences. It is further shown how normative partial equilibrium analysis (i.e. cost-benefit analysis) can be done under adaptive preferences.

1 Introduction

Behavioural economics and happiness research have led to substantial criticism of the traditional neoclassical homo oeconomicus model. One of the reasons for the long persistence of this model was its great usefulness for an individualistic quasi-utilitarian approach in normative economics, i.e. its usefulness for welfare economics. Given the empirical evidence against the neoclassical model with an exogenously given ordinal utility function behavioural economics had the difficult task to develop a normative theory – i.e. a theory of policy advice, which takes the results of behavioural economics into account. There are by now several papers which deal with this task[1]; and they make productive suggestions how to go about finding a solution for this problem.

In this paper I do not deal with these suggestions that are made in the literature. Not surprisingly they have a certain "paternalistic" touch, of which the authors are fully aware.

[1] Richard Thaler and Cass Sunstein, Nudge, New Haven and London, 2008; Botond Köszegi and Matthew Rabin, Choices, situations, and happiness, Journal of Public Economics, 2008, 92. p.1821-1832; B. Douglas Bernheim and Antonio Rangel, Beyond Revealed Preference: choice-theoretic Foundations for Behavioral Welfare Economics, Quarterly Journal of Economics 2009, p.51- 103; George Loewenstein and Emily Haisley, the Economist as Therapist: methodological Ramifications of ´Light` Paternalism, in A. Caplin and A. Schotter (Eds.), Perspectives on the Future of Economics: Positive and Normative Economics, Volume 1 of Handbook of Economic Methodologies, Oxford University Press 2009, as well as the literature cited in these articles and books.

But to my taste the policy proposals, although interesting for solving specific problems, so far do not go deep enough into the conceptual issues, (or philosophical issues?) which arise for normative economics out of the potential invalidity of the neoclassical model.

The major problem which I see is the fact that we cannot realistically maintain the position that preferences are exogenous. We have to face the problem that the measuring rod for economic performance, i.e. preferences of individuals are themselves informed by the actual events going on in the economy. How can you measure the length of a table, if your measuring rod changes with the length of the table? You may get the result that all tables have a length of two meters.

In this paper I deal with this particular problem. I believe I have an answer. So far it is not an answer, which works all the time. But then further research may help to enlarge or to modify the answer. I believe my answer relies on assumptions which are reasonably realistic. My approach led me to think about the function of the concept of "preferences" in economics. After all it is a concept, which plays a central role in economics, whereas other social sciences do not really make this clear distinction between one group of causes of human decisions, i.e. constraints, and another group of causes, i.e. preferences. In the following section 2 I start with this conceptual issue.

In sections 3 to 9 I then develop a model and a theory of adaptive preferences. In section 10 I try to analyse the meaning of the results derived in the model, and thereby offer a new interpretation of the meaning of the neoclassical approach. In section 11 I argue that global optimisation is something which we have to forego in the case of adaptive preferences. Section 12 uses the concept of adaptive preferences to discuss the behaviour of organisations and of political bodies. Section 13 argues that the findings of behavioural economics are in line with the hypothesis of adaptive preferences. Section 14 shows that decentralised decision making in the spirit of cost-benefit analysis remains possible with adaptive preferences instead of fixed preferences. It also argues that this possibility makes the massive growth of wealth in the recent centuries another "proof" of the realism of the hypothesis of adaptive preferences.

2 The Concept of Preferences in Normative Economics

What is the meaning of the concept of preferences? These days we observe that neurobiology catches the attention of the public. We understand more than in the past, how our brain works and how our actions and emotions are related to the electric currents and the blood flows going on all the time in our brain. Even economists have started to do “neuro-economics”. In a sense, people are then looked at like machines. If the research programme of this mechanical, scientific way of looking at humans is successful to the end, we can abandon the concept of “preferences”. And on its way it is a task of this research programme to minimise the degree to which one has to refer to “preferences” as an explanation of human behaviour. Explaining behaviour by reference to preferences in this programme is an admission of a lack of knowledge about the real causes for the observed behaviour. The baby wants to drink from the breast of his mother – not because he prefers doing so rather than not doing so, but because he is hungry.

But beyond the scientific programme of explaining human behaviour, economics always had another function which we may call “social philosophy” or more specifically: understanding, how a society of free people works, could work and should work. In this research programme freedom of the will is not in doubt, but rather assumed to exist, in order to understand the concept of civil liberty in a society. In this programme we ask the question: how is civil liberty possible, how can freedom of people in society be maximised? Here then we may define: freedom of action in a society is a situation, in which the agent does not have to justify to society, what he or she is doing. To the extent that society or government forces an agent to justify his/her actions, these actions are no longer free actions. As an example: free elections are characterised by the fact that the votes of the voters count independent of whether the agent (= the voter) can give a good reason or can give any reason for voting as he/she did. The secret ballot serves the very function of separating the vote from a need to justify the choice taken by the voter.

The way economists or social philosophers model free actions (i.e. actions which do not have to be justified) is by reference to the preferences of the agent. "Preferences" in the model of the economist are then the placeholder of the concept of freedom. The on-looking theorist wants to understand the interaction of free persons in society. For that purpose the theorist needs to specify what people are doing without treating them in the model like automata or machines. "Preferences" are then the determinant of the actions of the individual agent. But then, maximisation of freedom means maximisation of the influence of preferences on the social outcome of actions.

Social Choice as a field of economics arose precisely for the reason that economists were shocked by Arrow's impossibility theorem. But this theorem only holds, if we want to have a very wide array of possible preferences of society members. It is known that by limiting the set of possible preferences we can get “possibility theorems”, like the one by Black on single peakedness of preferences. So the Arrow impossibility theorem is a theorem about the limits of the principle that as much as possible in society should be just the reflection of individual preferences. Not "anything goes" tells us Arrow's theorem.

Here I do not want to speculate on the limits of maximisation of the influence of preferences on the social outcome, or for that matter, on the limits of individual freedom. Obviously there are limits. Apart from Arrow's impossibility theorem there exists the problem of “compossibility” of freedoms or of “compossibility” of individual rights, or of “compossibility” of individual consumption baskets[2]. Thus, constraints of actions have to be taken into account, if we want to understand a free society. But it would be a mistake to explain everything by constraints, as for example long ago Becker and Stigler announced as their research principle[3]. This latter research strategy corresponds to the modern mechanistic research programme of neuro-economics, which essentially has no place for the concept of freedom.

Thus there are two (or more?) legitimate research programmes in economics: 1. The scientific research programme, which tries to explain human behaviour, and which hopes eventually to disband the concept of preferences. 2. The search for improvement in a society of free men and women, which wants to give preferences a central place in its models of social life.

But the more you invoke preferences as a cause of outcomes, the greater the role of preferences is in your approach, the more you need to accept that preferences are not fixed, but are endogenously determined. Thus, I hope that my contribution here is a step forward in this second research programme, as it liberates it from the constraint to assume that preferences are fixed.

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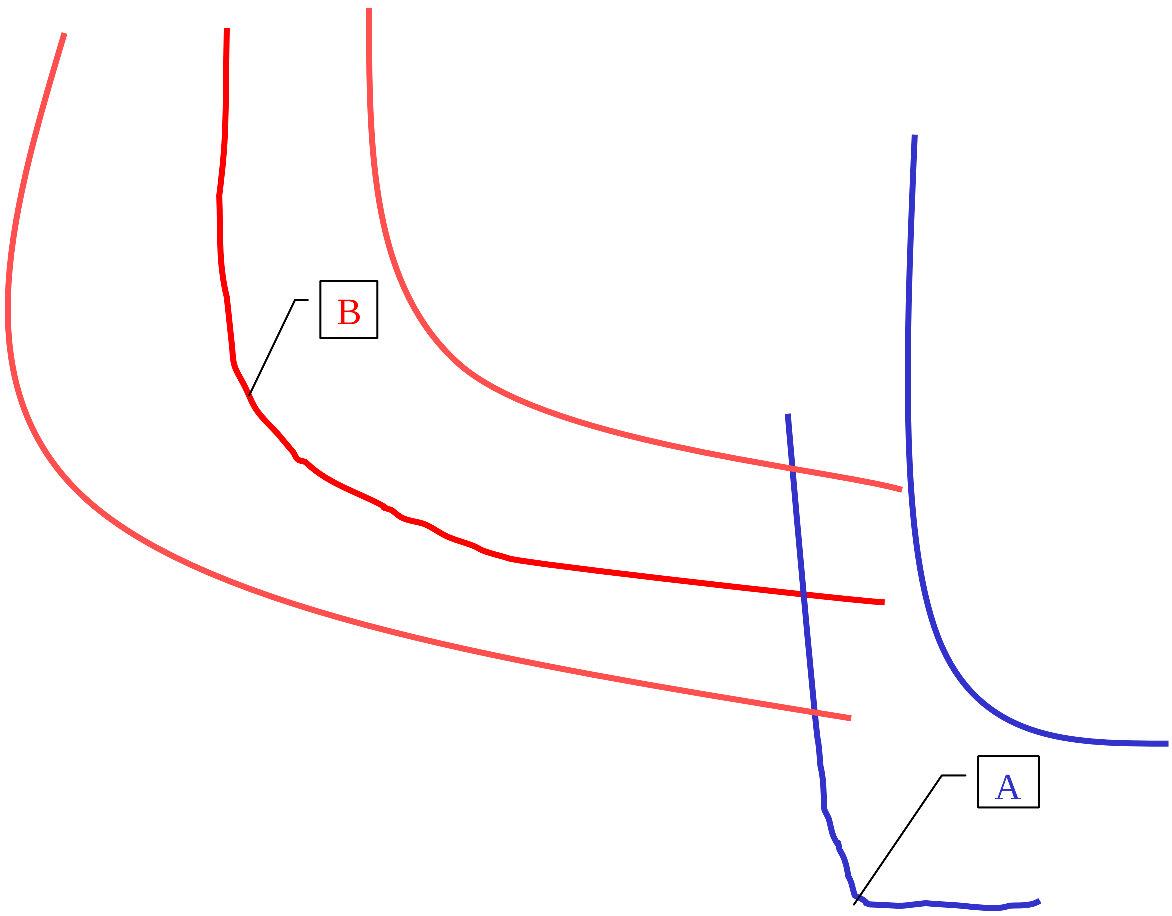
[2] Steiner, H. “The structure of a set of compossible rights” in Journal of Philosophy 74, 1977, p. 767-775
[3] Stigler, George and Gary Becker, De gustibus non est disputandum, American Economic Review, 1977

I concentrate on a particular class of endogenously changing preferences, which I call adaptive preferences. Adaptive preferences are a form of preference formation so that baskets, which we consume in the actual state of the world, are more strongly preferred over other baskets than they would in a different state of the world, in which another basket is consumed. The following picture represents this idea. The red indifference curves represent preferences which correspond to the actual consumption basket B. The blue indifference curves represent preferences which correspond to the actual consumption basket A. As I have drawn the indifference curves we see that with preferences corresponding to the A basket A is preferred over basket B, and with preferences corresponding to the B basket B is preferred over basket A. Provided, we have the task to decide between A and B, what can we do? If we start at A, with preferences corresponding to A, it appears that B is inferior to this starting point. If we start at B, with preferences corresponding to B, it appears that A is inferior to this starting point B. But then it appears that preferences induce a very conservative, status-quo oriented policy, something which is not in the tradition of economics, which has a reform oriented tradition.

Graph 1: Endogenously determined preferences



But there is another possibility: gradual change. Could it be that, starting with preferences corresponding to A, a gradual movement from A to B is seen as an improvement in each small step, because preferences gradually change along with the actual movement? And could it be that, starting with preferences corresponding to B, a gradual improving movement from B to A does not exist? If this were the case we might see B to be in some sense superior to A, because, given the choice between A and B, in the long run we end up in B, even if we have A as a starting point with preferences corresponding to A.

This idea of gradualism is at the core of my attempt to incorporate endogenously changing tastes into a coherent welfare economics[4]. I believe this to be an important task – and I do hope that others will join me in developing this theory. The central concept of the positive answer then is the concept of an improving path, i.e. a path of small, gradual improvements.

[4] It was this idea of gradual improvement, which informed my first attempt to deal with the welfare economics of endogenous preferences in Carl Christian von Weizsäcker, Notes on Endogenous Change of Tastes, Journal of Economic Theory, 1971, Vol 3, p. 345- 372.

3 The Model Set-Up

I now set up a formal model.

There is one person who is a consumer of a commodity basket x in the the n-dimensional Euclidean space R^n. We work in a continuous time model. The utility function $U(x)$ of the person depends itself on past experience as expressed by an N-dimensional vector q which reflects past consumption. N can be smaller or larger than n; or we can have N = n. We formalise this dependence on past consumption in the following way

$$\frac{dq}{dt} \equiv \dot{q} = \alpha Qx - \alpha q = \alpha(Qx - q) \tag{1}$$

where Q is an N times n matrix and α is N times N matrix with zero values outside the main diagonal and positive values on the main diagonal. That N can be substantially larger than n implies the possibility to approximate any complicated structure of influences of past consumption on present preferences. But the reader should be aware that in the present paper I do not make full use of the potentially very high dimensionality of the "preference space": The image of the commodity space in the preference space generated by the linear mapping Qx is of course of dimension n and thus, if $N > n$, is a subspace of the preference space not of higher dimension than the commodity space. If the initial value q also is in this subspace then only preferences become relevant which are elements of the n-dimensional image space of the commodity space. Thus a higher dimensionality of the preference space than the commodity space is a somewhat spurious generalisation from simply assuming that q is a weighted average of past consumption vectors. Nevertheless it is worthwhile to make this distinction in dimensionality of preference space and commodity space, because, I believe, many of the results of this paper can be generalised to the cases in which the "dynamics" of preference formation is more general than that of a constant-coefficient linear differential equation.

What is ruled out (but later work surely could change this) is that age of the person or other demographic factors which change with calendar time have a specific influence on present preferences. In a sense this is equivalent to assuming that the individual has an infinite life time.

In this paper I exclude the issues of inter-temporal allocation of individual consumption. If we were to include this inter-temporal allocation of consumption we would have to deal with an overlap of two inter-temporal processes: the inter-temporal optimisation of consumption for given preferences and the inter-temporal change in preferences, which then would also lead to revisions of the inter-temporal plan. Thus the result would be an inter-temporal consumption path which is characterised by time inconsistency, like, of course, in the real world. This overlap would make notation much more complicated than it is now in this paper. I therefore have not presented it in this paper. But modelling this overlap can be done and the results are basically the same. One method to model this overlap is to assume that inter-temporal optimisation always takes place with the preferences prevailing at the time this optimisation takes place. Another one is to also model the awareness of the person that her preferences will change as a consequence of her present and future consumption, without her knowing in detail, how these preferences will look like in the future. The "philosophy" of my approach is not the one taken by some other economists, who work with "meta-preferences", or "extended preferences" which optimally determine the choice of actual preferences[5]. Such an approach essentially is equivalent to the assumption of fixed preferences. Thus my approach is to say: time consistency of consumption paths is not a relevant criterion for the welfare economics of perhaps wholly or at least partly unanticipated endogenously changing preferences.

We want to compare in a normative sense different time paths $x(t)$ of consumption. But at the beginning we have to concentrate on any given time path $x(t)$. So we conceptually have to distinguish between the general structure of preference formation as expressed in differential equation (1) and any given path $x(t)$ of consumption through time. I hope that the following definitions make this distinction clear.

I introduce the following definition:

Definition 1: A preference structure is or the laws of motion of preferences are a dynamic system of endogenous preference changes as described in differential equation (1). (End of definition).

[5] Cf. for example Gary S. Becker, Accounting for Tastes, Harvard University Press, Cambridge, Mass, 1996

If consumption $\bar{x}$ remains constant over time, then by differential equation (1) q converges towards a particular value, namely $Q\bar{x}$. If the initial q already was equal to $Q\bar{x}$ then preferences do not change as long as x remains constant. Let $\hat{q}(x)$ be defined by

$$\hat{q}(x) \equiv Qx$$

It is a mapping from the n-dimensional commodity space into the N-dimensional "preference space" which specifies the preferences that remain stable as long as x remains constant through time. I introduce the following

Definition 2: $\hat{q}(x) = Qx$ are called the preferences corresponding to x or the x-corresponding preferences (End of definition).

The utility function can be written

$$U = U(x;q)$$

If we give the utility function a strictly ordinal meaning then strictly speaking it does not make sense to compare utility values derived from different values of q, because they then stand for different preferences. Nevertheless I will make heavy use of comparisons of U-values derived from different q-values. For the moment the reader may want to give U a cardinal interpretation, like, for example "happiness". But it turns out that our results allow us also to switch back to a strictly ordinal story. On this see Section 8 below.

Assumption 1: U is continuously differentiable with respect to x and q.

This assumption of course implies that U is continuous in x and q. The economic meaning of continuity of the utility function $U(x)$ is well understood: it implies that for two baskets x^0 and x^1 such that x^1 is preferred over x^0 there exist neighbourhoods of x^0 and x^1 such that this preference relation is kept intact for any pair of vectors coming from these two neighbourhoods. Continuity of the utility function with respect to q essentially means that small changes in past consumption will not change utility of any given consumption vector x by a large amount. Or to put it in ordinal terms: if for a given q the basket x^1 is preferred over x^0 then for a sufficiently small change in q it is still the case that basket x^1 is preferred over x^0.

We now consider consumption paths $x(t)$ through time. Among paths $x(t)$ which are piecewise continuous we define a subset which we call "improving paths". This description is motivated by an axiom, which allows a minimum of comparability between different preferences.

Consider a path of vector x through time such that $x(0) = x^0$ is the starting point, $x(T) = x^T$ is the end point and there is a set J with a finite number of moments of time, called jump points, $t_1, t_2, ...t_H$ such that $0 < t_1 < t_2 < .... < t_H \leq T$. Let I_T be the interval $[0,T]$ of real numbers. Thus, the number of jump points is designated by the integer H . This integer H could be zero. Then we consider paths $x(t)$ such that $x(t)$ is continuous in $I_T - J$ and such that $x(t)$ is right- side-continuous at t, if $t \in J$. In other words: we look at paths $x(t)$ for which holds $\lim_{t \downarrow \bar{t}} x(t) = x(\bar{t})$ for any $\bar{t} \in I_T$ and $\lim_{t \to \bar{t}} x(t) = x(\bar{t})$ for any $\bar{t} \in I_T - J$.

For such paths $q(t)$ is well defined by the differential equation (1), if $q(0) = q^0$ is given. Indeed we find the unique solution

$$q(t) = e^{-\alpha t}\left[q^0 + \alpha \int_0^t e^{\alpha z} Qx(z)dz \right]$$

The integral is well defined for piecewise continuous functions $x(z)$.

For any point where the time derivative of x, i.e. vector $\dot{x}$ exists we can define the following expression

$$\hat{U} \equiv \sum_{i=1}^{n} \frac{\partial U}{\partial x_i} \dot{x}_i$$

Note that this is different from the time derivative of U which is

$$\dot{U} \equiv \sum_{i=1}^{n} \frac{\partial U}{\partial x_i} \dot{x}_i + \sum_{i=1}^{n} \frac{\partial U}{\partial q_i} \dot{q}_i$$

The expression $\hat{U}$ has an economic meaning. If $\hat{U}$ is positive it means that "real income" increases. For example, if x comes about by maximisation of U with respect to x subject to a budget constraint then $\hat{U} > 0$ implies that either the budget rises or the price index (in terms of the Divisia index) falls, i.e. that real income rises. More generally $\hat{U}$ is the change of utility which is due to changes in x, keeping preferences the same. Therefore it is plausible to talk about an improvement if $\hat{U} > 0$.

Definition 3: If $x(t)$ is differentiable at $\bar{t}$ and if $\hat{U} > 0$ at $\bar{t}$ we then also say that real income rises at $\bar{t}$.

To cope with piecewise continuous paths we introduce the following definition. For a given path $x(t)$ we consider a time point $\bar{t}$.

Definition 4: For a given path $x(t)$ the point $\bar{t}$ is an improvement point, if there exists a non-empty interval $K(\bar{t}) = [\hat{t}, \bar{t}]$ such that for $t \in K(\bar{t})$ we have $U(x(t); q(\bar{t})) \leq U(x(\bar{t}); q(\bar{t}))$.
(End of Definition).

Thus an improvement point $\bar{t}$ is characterised by utility evaluated at preferences corresponding to this point such that for slightly smaller t utility is not larger than utility at $\bar{t}$. Obviously, if $x(t)$ is differentiable at $\bar{t}$ and if $\bar{t}$ is an improving point then $\hat{U}(\bar{t}) \geq 0$. If $\bar{t}$ is one of the H jump points then it only can be an improving point, if the "utility jump" at $\bar{t}$ is an "upward jump".

Note the asymmetric treatment of past and future in this definition. The utility comparison is only made between present and past, not between present and future.

This consideration motivates the following definitions of an improving and a weakly improving path.

We use the following notation for piecewise continuous paths $x(t)$. By $\{x(t); q^0; T\}$ we mean a path of the piecewise continuous consumption vector $x(t)$ in the time interval $[0,T]$ such that preferences are determined by $q(t) = e^{-\alpha t}\left[q^0 + \alpha \int_0^t e^{\alpha z} Qx(z)dz\right]$ with the initial value q^0.

<u>Definition 5.</u> Let $\{x(t); q^0; T\}$ be a piecewise differentiable path of consumption vector x with H jump points $J = \{0 < t_1 < t_2 < .... t_H \leq T\}$ on the time interval I_T. $H = 0$ is admitted. The path is called a <u>weakly improving path</u>, if

1. for any $\bar{t} \in I_T$ -0 the point $\bar{t}$ is an improving point and
2. There exists $\varepsilon > 0$ such that for $0 < t < \varepsilon$ we have $U(x(t); q(t)) \geq U(x(0); q(t))$

(End of Definition).

The definition of an improving point only involves weak inequalities. Thus a constant path $x(t) = \bar{x}$ for $t \in I_T$ is a path to which Definition 5 also applies: note that $\hat{U} = 0$ whenever $\dot{x} = 0$. Therefore we talk of <u>weakly</u> improving paths. Note also that $t = 0$ need not be an improving point. We want to make the definition independent of the path $x(t)$ outside of the interval. But condition 2 takes care of $t = 0$.

We come from weakly improving paths to (strictly) improving paths if an event is involved which to a measurable degree involves a strict improvement. Such an event we call an improving event.

<u>Definition 6:</u> By an <u>improving event</u> we mean:

<u>either</u> an improving jump at some $\bar{t}$, i.e. $\lim_{t \uparrow \bar{t}} U(x(t); q(t)) < U(x(\bar{t}); q(\bar{t}))$

<u>or</u> for some time interval (t_0, t_1) with $t_0 < t_1$ $x(t)$ is differentiable and real income rises for all $t \in (t_0, t_1)$, i.e. $\hat{U}$ exists and $\hat{U} > 0$ for all $t \in (t_0, t_1)$. (End of Definition)

We then define an improving path in the following way:

Definition 7: Let $\{x(t); q^0; T\}$ be a piecewise continuous path of consumption vector $x(t)$ with H jump points $J = \{0 < t_1 < t_2 < ....t_H \leq T\}$ on the time interval I_T. $H = 0$ is admitted.

The path is called an improving path if

1. it is a weakly improving path as defined in Definition 5
2. it contains at least one improving event as defined in Definition 6.

(End of Definition).

We are particularly interested in a specific class of improving paths which we call "balanced" improving paths.

Definition 8: An improving path $\{x(t); q^0; T\}$ is a balanced improving path, if the initial preferences q^0 correspond to the initial consumption basket $x(0)$, i.e. if

$q^0 = Qx(0) = \hat{q}(x(0))$ (End of Definition).

For a balanced improving path we also give the three characteristics $\{x(t); q^0; T\}$ of an improving path in the form $\{x(t); q^0 = \hat{q}(x^0); T\}$ to make clear that in that case the initial value of x, i.e. $x(0) = x^0$ already determines the initial preferences $\hat{q}(x^0)$.

Thus a balanced improving path is an improving path which "breaks away" from a formerly stationary state. The following "improvement axiom" makes it clear why balanced improving paths are of particular interest.

I now introduce the improvement axiom. If preferences endogenously change through time welfare economics would become impossible unless we had some way of normatively comparing consumption paths that do not have the same preferences. We need some kind of "meta-preferences". But I want to restrict meta-preferences to a minimum. The meta-preferences in our case are encapsulated in the following axiom.

Improvement Axiom. Let $\{x^*(t); q^{*0}; T\}$ be a balanced improving path. Let $\{x(t); q^0; T\}$ be a path in which consumption remains constant. Thus $x(t) = x(0)$ for $0 \leq t \leq T$ Moreover preferences correspond to the constant basket, i.e. $q^0 = Qx(0) = \hat{q}(x(0))$. If $x^*(0) = x(0)$ and if moreover $x^*(T) \neq x(0)$ then the consumer prefers $\{x^*(t); q^{*0}; T\}$ over $\{x(t); q^0; T\}$.

The Improvement Axiom is highly plausible: starting from the same tastes (as represented by $q^{*0} = q^0 = Qx^*(0) = Qx(0)$) corresponding to initial consumption and starting with the same consumption basket the consumer prefers improvement over constant consumption, i.e. over stagnation, even if he or she is aware that preferences later are influenced by the evolving situation through time. But an additional condition is of course that the improvement path does not come back to the starting point.

If we accept the Improvement Axiom we can, as will be shown, maintain the concept of progress such that it is consistent with welfare economics – even with endogenously changing preferences.

Note that the Improvement Axiom is far away from a complete meta- preference pre- ordering over different preferences. In particular I want to emphasise that – beyond a general awareness that her/his preferences may change – the person may be unable to predict, even in a probabilistic sense, her/his preferences five or ten or twenty years from now.

I now introduce the concept of adaptive preferences.

Definition 9: The laws of motion of preferences are called adaptive preferences, if they fulfil the following inequality: for any basket x in commodity space
$U(x; \hat{q}(x)) \equiv U(x; Qx) \geq U(x; q)$ for all q in preference space. (End of Definition).

In other words: we speak of adaptive preferences, if the utility of a given basket x is highest under preferences which correspond to this basket x. Adaptive preferences imply that in a stationary economy preferences converge to those preferences which make the utility of the prevailing stable basket as high as possible among all preferences. In section 8 below I give a purely ordinal interpretation of this concept of adaptive preferences.

As will be discussed later in more detail adaptive preferences are a kind of preference conservatism combined with a kind preference flexibility.

In the following I also make use of a subclass of adaptive preferences which I call smooth adaptive preferences. Consider any basket x in commodity space and the preferences $\hat{q}(x)$ corresponding to it. As we already observed, as long as x remains constant through time q converges to $\hat{q}(x)$. Since, by adaptive preferences, q converges to the utility maximising point $\hat{q}(x)$, "on average" utility rises as long as x remains constant. The assumption of smooth adaptive preferences then is that utility rises at every moment as long as x remains constant. The differential of utility with respect to time is given by

$$\dot{U} \equiv \sum_{i=1}^{n} \frac{\partial U}{\partial x_i} \dot{x}_i + \sum_{i=1}^{n} \frac{\partial U}{\partial q_i} \dot{q}_i$$

In case that x does not change through time this boils down to

$$\dot{U} \equiv \sum_{i=1}^{n} \frac{\partial U}{\partial q_i} \dot{q}_i = \frac{\partial U}{\partial q} (\alpha(Qx - q)) = \frac{\partial U}{\partial q} \alpha(\hat{q}(x) - q))$$

Here $\frac{\partial U}{\partial q}$ is the vector of partial derivatives of U with respect to the different N components of q; and $\alpha(\hat{q}(x) - q)$ is the vector of time derivatives of the different N components of q. After this preparation I introduce the concept of smooth adaptive preferences by the following definition.

Definition 10: Smooth adaptive preferences are adaptive preferences which, for every x and every q, conform to the inequality $\frac{\partial U}{\partial q} \alpha(\hat{q}(x) - q) \geq 0$. (End of definition)

Smooth adaptive preferences essentially mean that $\hat{q}(x)$ is the only utility peak in preference space. It is a kind of "single peakedness" assumption for the preference space.

A further definition is non-circularity. As discussed in the introduction we would consider it spurious progress, if a balanced improvement path would turn back to its origin, i.e. would be circular. This motivates the following definition.

Definition 11: An improvement path $\{x(t); q^0; T\}$ is called non-circular, if $x(T) \neq x(0)$. (End of definition).

As we shall see there is a close connection between non-circularity of balanced improvement paths and adaptive preferences.

The following notation will be used: $x^*(>)_q x$ means that x^* is preferred over x with preferences q. It is equivalent to the expression $U(x^*;q) > U(x;q)$.

I now introduce an assumption, which has been frequently used in traditional preference theory, the assumption of non-satiation.

<u>Assumption 2:</u> The utility function $U(x;q)$ reflects non-satiation: If $x^1 \geq x^0$ and $x^1 \neq x^0$ then $U(x^1;q) > U(x^0;q)$

4 A simple model with two goods

Assume the utility function of a person to be $U = \frac{1}{1-\gamma} g x_1^{1-\gamma} + \frac{1}{1-\gamma}(1-g) x_2^{1-\gamma}$. Here x_1 and x_2 are the two quantities of the two goods consumed and $0 < g < 1$ is a weight parameter of the two goods, and $\gamma > 1$ is a substitution parameter of the two goods. $1/\gamma$ is the elasticity of substitution. Given the prices of the two goods the ratio z in which the goods are consumed can be computed to be $z = \frac{g}{1-g} p^{\frac{1}{\gamma}}$ with $p = \frac{p_2}{p_1}$ the price ratio of the two goods. Now I introduce the influence of past consumption on present tastes. In this simple model I can assume that the weight factor $\frac{g}{1-g}$ is influenced by a weighted average q of the past values of z. We may write $\frac{g}{1-g} = aq^{\mu}$, where $a > 0$ is a constant weight parameter of the two goods and μ is a parameter, which indicates the strength of the influence of past consumption on present tastes. We assume $0 \leq \mu < 1$. The case $\mu = 0$ is the case of fixed preferences. The assumption $\mu < 1$ is related to the property of "adaptiveness" of tastes. So the demand function now reads $z = aq^{\mu} p^{\frac{1}{\gamma}}$.

Past consumption q is modelled as an exponentially weighted average of former levels of z. We then get the linear differential equation $\dot{q} = \alpha(z - q)$. Here the real number $\alpha > 0$ is a "speed" parameter for the adaptation of tastes to any given level of consumption z. The solution of the differential equation then is

$$q = e^{-\alpha t}(q_0 + \alpha \int_0^t e^{\alpha\tau} z(\tau) d\tau)$$

We now ask: is there a long run demand function, if prices remain constant? The answer is yes. We can compute it by solving the differential equation keeping p constant and by looking at the limit as time goes to infinity, or by observing that constant prices in the long run will lead to a situation of constant quantities, hence a constant level of z, which again implies that the weighted average of past consumption converges to the actual level of consumption. Thus a stationary level of z and q will be characterised by $q = z$. Using this equation for the computation of the long run value of z by means of the demand function yields the equation

$$z = aq^{\mu} p^{\frac{1}{\gamma}} = az^{\mu} p^{\frac{1}{\gamma}} \text{ from which follows } z = a^{\frac{1}{1-\mu}} p^{\frac{1}{\gamma(1-\mu)}}$$

The long run demand function thus is of a similar kind as the short run demand function; but the elasticity of substitution is higher than in the short run case. Thus, for example, if the short run elasticity of substitution is one half (corresponding to $\gamma = 2$) and the influence parameter μ of past consumption is also one half, then the long run elasticity of substitution is equal to 1, which corresponds to a logarithmic utility function. Indeed, in this specific model the long run demand function always has the property that there exists a utility function which would generate the long run demand function. So the question arises: what is the economic interpretation of this long run utility function? In sections 6 and 7 I address this question in a more general model.

5 Consumption Constraints and Improving Paths

In economics we are used to work with the fundamental distinction between constraints and preferences which jointly determine human behaviour. So far I have only talked about preferences. I now introduce constraints. As an example we can use the textbook budget constraint. Let p be the price vector of the n commodities. Assume the budget to be unity. Thus we have the budget constraint $px \leq 1$. Due to the non-satiation assumption (Assumption 2 above) we know that utility maximisation implies $px = 1$. Thus assume that the person maximises utility subject to the constraint $px = 1$. Assume further that the person simply maximises current utility without regard to utility in the future. I come back to this assumption below. Thus, as long as prices remain constant through time, changes in the basket x are only due to changes in preferences which are themselves induced by current consumption according to equation (1). In this way we obtain a differential equation for x through time and thus a path $x(t)$. Let us now observe that marginal utility in the utility maximum is proportional to prices. Thus, with an appropriate choice of $\lambda > 0$ we can write

$$\hat{U} \equiv \sum_{i=1}^{n} \frac{\partial U}{\partial x_i} \dot{x}_i = \lambda \sum_{i=0}^{n} p_i \dot{x} = 0$$

It follows that the path $x(t)$ is a weakly improving path.

Assume now that the budget set gradually becomes more favourable: prices change according to the following inequality

$$\sum_{i=1}^{n} x_i(p,q) \frac{dp_i}{dt} \equiv \sum_{i=1}^{n} x_i(p;q) \dot{p}_i < 0$$

We then obtain from the budget constraint

$$0 = \sum_{i=1}^{n} p_i \dot{x}_i + x_i \dot{p}_i$$

which together with the inequality above implies for some appropriate choice of $\lambda > 0$ that

$$\hat{U} \equiv \sum_{i=1}^{n} \frac{\partial U}{\partial x_i} \dot{x}_i = \lambda \sum_{i=1}^{n} p_i \dot{x}_i > 0$$

Thus the path $x(t)$ is an improving path.

We thus see that the definition of improvement conforms to our intuition about improvement for changing budget constraints. It also makes clear that improvement essentially is a purely ordinal concept.

6 Smoothly Adaptive Preferences Imply Non-Circularity of Balanced Improvement Paths

In this section I prove

Theorem 1: Assumptions: Preferences are smoothly adaptive and the piecewise differentiable path $\{x(t); q^0; T\}$ is a balanced improving path. Proposition: Then $x(T) \neq x(0)$, i. e. the path is non-circular. Proof: Where $x(t)$ is differentiable we differentiate utility with respect to time: $\dot{U} = \hat{U} + \frac{\partial U}{\partial q}\alpha(\hat{q}(x) - q)$. Because the path is an improving path we have $\hat{U} \geq 0$. Because preferences are smoothly adaptive we have $\frac{\partial U}{\partial q}\alpha(\hat{q}(x) - q) \geq 0$. Thus $\dot{U} \geq 0$. At a jump point the jump must be an improving jump, because otherwise the path would not be weakly improving and thus not improving. Hence we know by integration over the time interval $[0;T]$ that $U(x(T);q(T)) \geq U(x(0),q(0))$. Moreover, because the path is strictly improving we have at least one improving event. If it is a jump point utility strictly rises at this point and hence it is higher at T than it is at time zero. If it is an interval in which $\hat{U}$ is strictly positive it is again the case that $U(x(T);q(T)) > U(x(0),q(0))$. Now, observe that because preferences are adaptive and the improvement path is balanced we have $U(x(0);q(0)) \geq U(x(0);q)$ for any q. Thus the inequality $U(x(T);q(T)) > U(x(0),q(0))$ can only hold, if $x(T) \neq x(0)$. This proves the theorem.

7 Fixed "Quasi-Preferences" and Their Economic Meaning

We can define a function $V(x)$ from the commodity space to the real numbers, i.e. to "utility space" by using the preferences which correspond to x. Thus, we define

$$V(x) = U(x;\hat{q}(x)) = U(x;Qx)$$

I now assume preferences to be adaptive, which implies

$$V(x) \geq U(x;q) \text{ for any } q$$

On the other hand, due to the non-satiation assumption (Assumption 2) we know that for $x^1 \geq x^0$ and $x^1 \neq x^0$ we obtain

$$V(x^1) = U(x^1;\hat{q}(x^1)) \geq U(x^1;\hat{q}(x^0)) > U(x^0;\hat{q}(x^0)) = V(x^0)$$

The function $V(x)$ has the formal structure of a utility function. Preference changes seem to have disappeared, due to the fact that we have associated certain preferences to each consumption basket. But, of course, preference changes are behind the scenes, because for each x different preferences apply. The last inequality, which is due to adaptiveness of preferences and due to the non-satiation assumption, indicates that "utility" $V(x)$ rises monotonically with rising commodity baskets. An indifference surface corresponding to a higher "utility" then lies above an indifference surface corresponding to a lower "utility".

Does the "quasi utility function" $V(x)$ have an economic meaning? The answer is "yes". It is an indicator for the existence or non-existence of a balanced improving path between any two baskets. This is the content of Theorem 2. Before I state the theorem I introduce the following definitions.

Definition 12: The set of baskets reachable by a balanced improving path. For any x^0 let $A(x^0)$ be the set of baskets which can be reached from x^0 by means of a balanced improving path. End of definition.

Remark 1: Provided there is non-circularity of balanced improving paths, x^0 is not an element of $A(x^0)$.

Definition 13: The lower bound of the set $A(x^0)$. Let $\overline{A}(x^0)$ be the lower bound of $A(x^0)$. Formally the set $\overline{A}(x^0)$ is defined as the set of baskets, such that for $\overline{x} \in \overline{A}(x^0)$ we have: for each neighbourhood $M(\overline{x})$ there exists $x \in M(\overline{x})$ with $x \in A(x^0)$ and there exists $z \in M(\overline{x})$ with $z \notin M(\overline{x})$. End of definition.

Remark 2: Due to the non-satiation assumption we know that, if $x^1 \in A(x^0)$ and if $x^2 \geq x^1$ then $x^2 \in A(x^0)$. For, either $x^2 = x^1$, then obviously $x^2 \in A(x^0)$ or $x^2 \neq x^1$ then by the non-satiation assumption $U(x^2;q) > U(x^1;q)$ for any q. Thus we simply have to "prolong" the balanced improving path leading in finite time from x^0 to x^1 by adding a jump from x^1 to x^2, which by the utility inequality is an improving jump.

Definition 14: The set $\overline{A}(x^0)$ is smooth, if for any $\overline{x} \in \overline{A}(x^0)$ there exists a unique $p(\overline{x}) \geq 0$ of dimension n, with $\sum_{i=1}^{n} p_i(\overline{x}) = 1$ with the following property: Let $f(x)$ be any differentiable function on the commodity space such that $f(x) = \bar{f}$ for $x \in \overline{A}(x^0)$, and $f(x) > \bar{f}$ for $x \in A(x^0)$. Then $\sum_{i=1}^{n} \frac{\partial f(\overline{x})}{\partial x_i} dx_i = 0$, whenever $\sum_{i=1}^{n} p_i(\overline{x}) dx_i = 0$. End of definition

In other words: The set $\overline{A}(x^0)$ is smooth, if for each $\overline{x} \in \overline{A}(x^0)$ we find a unique hyperplane, which is tangential to $\overline{A}(x^0)$ in point $\overline{x}$.

I now show that the function $V(x)$ is a complete indicator for pairs of baskets (x^0, x^1) such that x^1 can be reached from x^0 by a balanced improving path.

Theorem 2: Assumptions: 1. Preferences are adaptive; 2. Balanced improving paths are non-circular. 3. For any basket x^0 the lower bound $\overline{A}(x^0)$ is smooth, as defined in Definition 14.

Proposition: If and only if $V(x^1) > V(x^0)$ we have $x^1 \in A(x^0)$.

Proof: The proof is in a series of three lemmas.

Lemma 1: Consider any point $\overline{x} \in \overline{A}(x^0)$. For any $\tilde{x}$ in R^n the following holds: If $U(\tilde{x};\hat{q}(\overline{x})) > V(\overline{x})$ then $\tilde{x} \in A(x^0)$ In words: If $\tilde{x}$ has a higher utility than $\overline{x}$ under preferences corresponding to $\overline{x}$ then $\tilde{x}$ is contained in $A(x^0)$. Proof: Because the utility function is continuous, and because $\hat{q}(x) = Qx$ depends continuously on x we can find a neighbourhood $M(\overline{x})$ of $\overline{x}$ such that for $x^1 \in M(\overline{x})$ we have $U(\tilde{x};\hat{q}(x^1)) > U(x^1;\hat{q}(x^1))$. Moreover, because of $\overline{x} \in \overline{A}(x^0)$ the intersection $\hat{M}(\overline{x}) \equiv M(\overline{x}) \cap A(x^0)$ is not empty. We then choose x^1 to be in $\hat{M}(\overline{x})$. Thus there exists a balanced improving path $\{x(t); q^0 = \hat{q}(x^0); T\}$ starting at x^0 and ending at some time T at $x^1 \in \hat{M}(\overline{x})$. We now "prolong" this improving path beyond T by keeping $x(t)$ constant, i.e. $x(t) = x^1 \; for T \leq t < T_0$ for an appropriately chosen T_0. Indeed, since $x(t) = x^1$ preferences gradually converge to $\hat{q}(x^1)$. Due to continuity of the utility function we can find a neighbourhood $N^*(\hat{q}(x^1))$ of $\hat{q}(x^1)$ such that for $\tilde{q}^1 \in N^*(\hat{q}(x^1))$ we have $U(\tilde{x};\tilde{q}^1) > U(x^1;\tilde{q}^1)$. Let $q[t]$ be the value of q on the prolonged improvement path at time t. Because of the convergence of $q[t]$ towards $\hat{q}(x^1)$ we can find a finite T_0 such that $q[T_0] \in N^*(\hat{q}(x^1)$ and thus $U(\tilde{x}; q[T_0]) > U(x^1; q[T_0])$. We then take a utility improving jump of x at time T_0 from x^1 to $\tilde{x}$, which again is consistent with $\{x(t); \hat{q}(x^0); T_0\}$ being a balanced improving path starting at x^0 and finishing at $\tilde{x}$. This proves the lemma.

Lemma 2: The Theorem is correct, if $n = 2$.

In words: if there are only two distinct goods then the Theorem is correct.

Proof: Consider any basket $\bar{x} \in \bar{A}(x^0)$. Because $\bar{A}(x^0)$ is smooth we can find a real number $\pi(\bar{x})$ with $0 < \pi(\bar{x}) < 1$ such that for any differentiable function $f(x)$ defined on the commodity space with $f(x) = \bar{f}$ for $x \in \bar{A}(x^0)$ and $\frac{\partial f}{\partial x_1} = \lambda \pi(\bar{x})$, $\frac{\partial f}{\partial x_2} = \lambda(1-\pi(\bar{x}))$ for some appropriately chosen λ. From this follows that the marginal "trade-off" between x_1 and x_2 to stay within $\bar{A}(x^0)$ is given by the differential equation $\pi(\bar{x})dx_1 + (1-\pi(\bar{x}))dx_2 = 0$. We thus obtain a differential equation for the definition of $\bar{A}(x^0)$. I now introduce the following notation. Let $U_1(x,q) = \frac{\partial U(x;q)}{\partial x_1}$ and $U_2(x,q) = \frac{\partial U(x;q)}{\partial x_2}$ be the marginal utilities of the two goods, keeping preferences unchanged. Due to Lemma 1 we then obtain the equations $\pi(\bar{x}) = \lambda U_1(\bar{x};\hat{q}(\bar{x}))$ and $1-\pi(\bar{x}) = \lambda U_2(\bar{x};\hat{q}(\bar{x}))$ for some appropriately chosen $\lambda > 0$. That is, the curve defining $\bar{A}(x^0)$ at $\bar{x}$ and the indifference curve through $\bar{x}$ corresponding to $U(\bar{x};\hat{q}(\bar{x}))$ must have the same slope: otherwise we could find baskets x near $\bar{x}$ which are below $\bar{A}(x^0)$ and at the same time have utility $U(x;\hat{q}(\bar{x})) > U(\bar{x};\hat{q}(\bar{x}))$, contrary to Lemma1. Remember that $\frac{\partial U(\bar{x};\hat{q}(\bar{x}))}{\partial q_j} = 0$ for $j = 1,2,...N$, due to adaptive preferences, which means that U is maximised with respect to q at $\hat{q}(\bar{x})$. We then have $\frac{\partial V(\bar{x})}{\partial x_i} = U_i(\bar{x};\hat{q}(\bar{x})) + \sum_{j=1}^{N} \frac{\partial U}{\partial q_j}\frac{\partial \hat{q}_j}{\partial x_i} = U_i(\bar{x};\hat{q}(\bar{x}))$ for $i = 1,2$. Thus, we obtain the differential equation $\frac{\partial V(\bar{x})}{\partial x_1}dx_1 + \frac{\partial V(\bar{x})}{\partial x_2}dx_2 = 0$ defining $\bar{A}(x^0)$, i.e. $V(\bar{x})$ remains constant as long as $\bar{x}$ remains in $\bar{A}(x^0)$. Moreover we know that x^0 itself is in $\bar{A}(x^0)$: any neighbourhood of x^0 contains baskets x such that $U(x;\hat{q}(x^0)) > U(x^0;\hat{q}(x^0)$ and thus $x \in A(x^0)$.And, due to non-circularity, $x^0 \notin A(x^0)$; thus any neighbourhood of x^0 contains baskets z (slightly below x^0) which have the property $U(z;q) < U(x^0;q)$ for any q so that, if $z \in A(x^0)$, also $x^0 \in A(x^0)$, in contradiction to the non-circularity assumption. Thus $z \notin A(x^0)$.Then the differential equation defining $\bar{A}(x^0)$ also goes through x^0, which implies that $\bar{A}(x^0)$ is defined by the constant value $V(x^0)$. Since, due to non-satiation, $A(x^0)$ lies above $\bar{A}(x^0)$ and since due to non-satiation $V(x)$ rises with rising x we see that $x \in A(x^0)$ if and only if $V(x) > V(x^0)$. This proves Lemma 2.

Lemma 3: Generalisation of Lemma 2 to n goods.

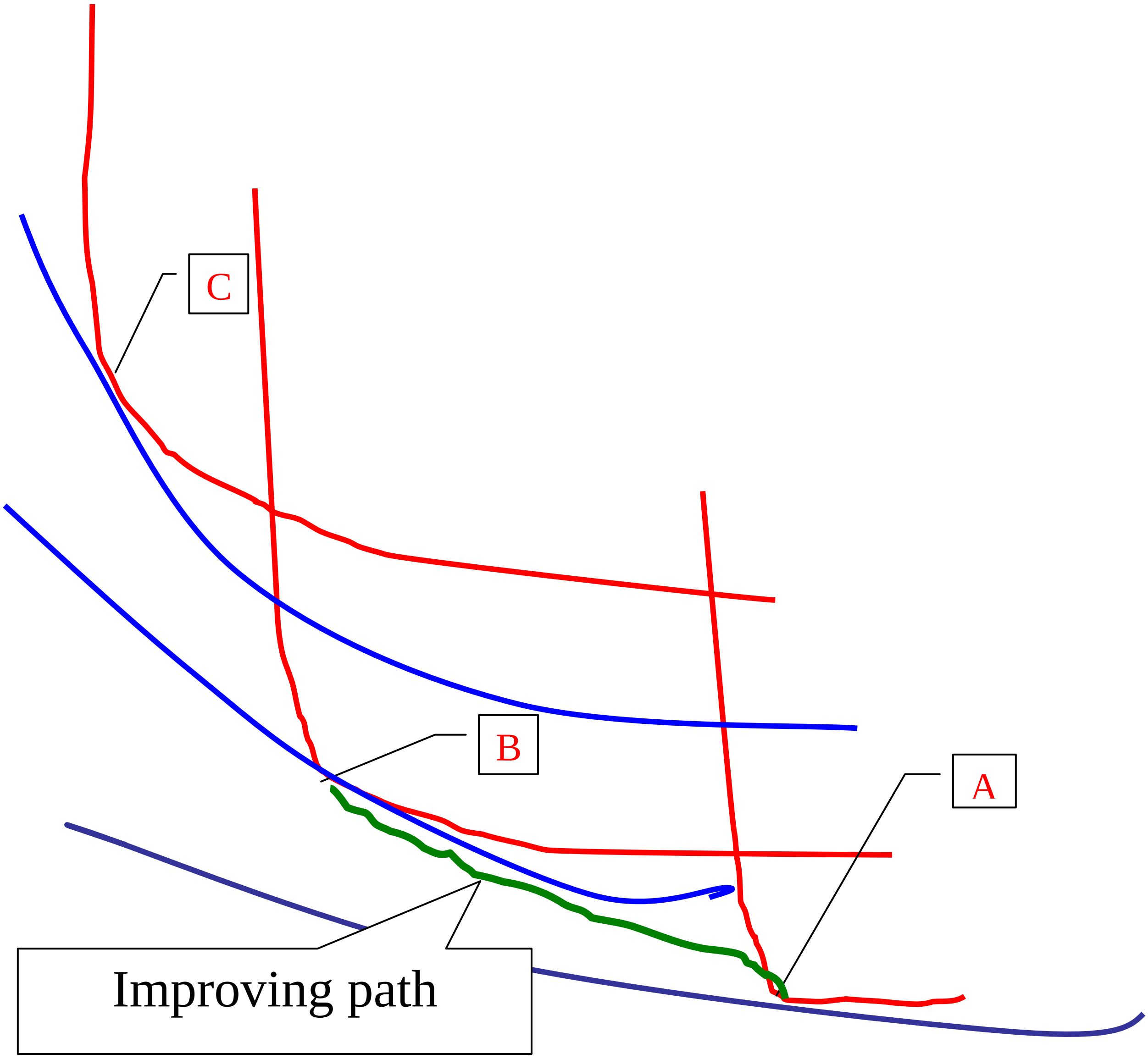
Proof: We go back to R^n. Take any $\bar{x} \in \bar{A}(x^0)$. We then form the triangle of the points 0, x^0 and $\bar{x}$. We look at the 2-dimensional subspace spanned by these three points. In this space S^2 we select two linearly independent vectors y and z. We may then express every element $x \in S^2$ as a linear combination of y and z. Thus, in particular, we may have $x^0 = \lambda^0 y + \mu^0 z$ and $\bar{x} = \bar{\lambda} y + \bar{\mu} z$. We now form the intersections of $A(x^0)$ with S^2 and of $\bar{A}(x^0)$ with S^2. Let $A^*(x^0) \equiv A(x^0) \cap S^2$ and $\bar{A}^*(x^0) \equiv \bar{A}(x^0) \cap S^2$. If the assumptions of the theorem are fulfilled for $A(x^0)$ and for $\bar{A}(x^0)$ they are also fulfilled for the two-dimensional sets $A^*(x^0)$ and $\bar{A}^*(x^0)$. Thus, we can apply Lemma 2 for the restricted space S^2. But this implies that $x \in A^*(x^0)$ if and only if $V(x) > V(x^0)$. We then have derived Lemma 3.

We thereby have given a proof of Theorem 2.

Remark 3: I have made the assumption that $\bar{A}(x^0)$ is smooth. I believe it should be possible to derive smoothness of $\bar{A}(x^0)$ from the other assumptions, but I so far have not succeeded to find a proof for this conjecture. Earlier[6] I have shown a somewhat similar theorem for the case that the utility function implies a unique long run demand basket for any given budget, an assumption which I do not make here for reasons explained below in Section 11. In that earlier theorem I do not rely on calculus, but I do rely on the Samuelson-Houthakker theorem on revealed preference. Because I there do not use calculus I do not need the assumption that $\bar{A}(x^0)$ is smooth.

Remark 4: I have made heavy use of the non-satiation assumption. Nevertheless, it is my conjecture that it should be possible to prove a theorem similar to Theorem 2 without the non-satiation assumption. What is essential for the theorem is the continuity of the utility function. What is also essential is the prefect divisibility of goods. As can be seen in the proof of Lemma 1, for the construction of a balanced improving path we need not to be restricted in the amount of time the improvement path takes, because the steps taken by $x(t)$ may have to be very small.

[6] C Christian von Weizsäcker, The Welfare Economics of Adaptive Preferences, Preprints of the Max Planck Institute for Research on Collective Goods, 2005/11
http://www.coll.mpg.de/biblio/type/129/issue/2005/sort/issue



Graph 2: Theorem 2 : The red indifference curves are part of preferences corresponding to A resp B resp C. The blue indifference curves represent V(x). The green curve is a balanced improving path from A to B.

Remark 5: Theorem 1 derived non-circularity of balanced improving paths from the assumption of smooth adaptive preferences. I was not able to derive non-circularity from adaptive preferences without the additional assumption of smoothness. In Theorem 2 I could, of course, have assumed smooth adaptive preferences; then by Theorem 1 non-circularity would have been guaranteed. I did not go that way because adaptiveness is the more fundamental concept than smooth adaptiveness.

8 From Cardinal to Ordinal Utility With Adaptive Preferences

The definition of an improving path only involves the "ordinal" side of utility. $\hat{U} > 0$ essentially means that real income rises. The same is true of improving jumps. Is it possible to develop a purely ordinal theory of adaptive preferences and their relation to balanced improving paths? The answer is yes.

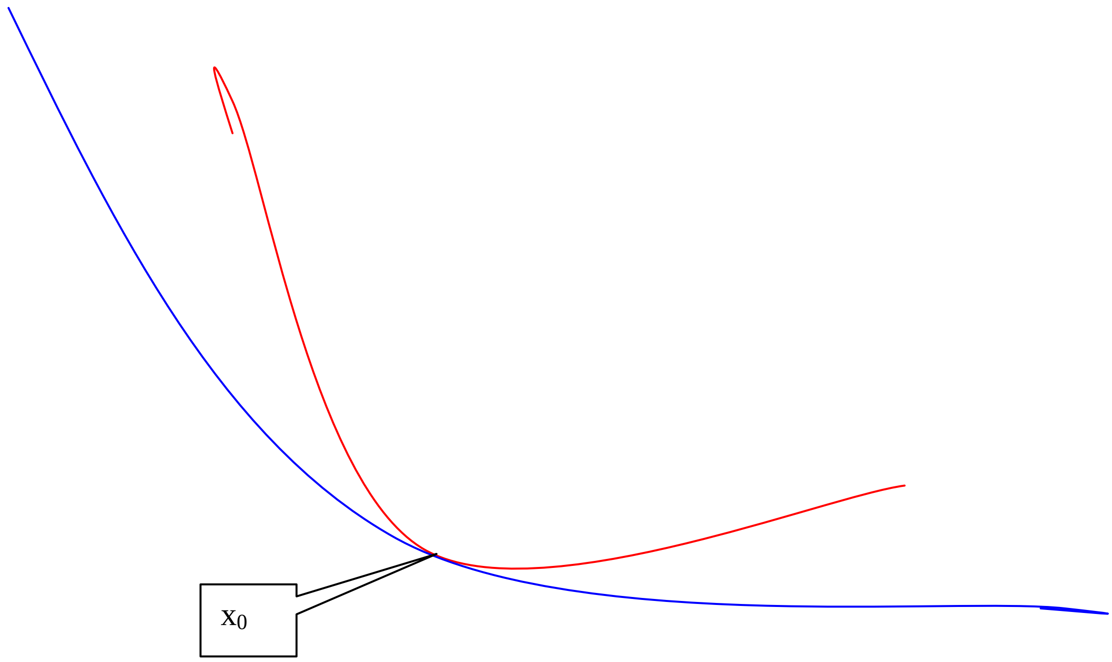
Consider an arbitrary complete pre-ordering of the commodity space as represented by an ordinal differentiable quasi-utility function $V(x)$. We assume $V(x)$ to conform to a quasi-non-satiation condition: If $x' \geq x$ and $x' \neq x$ then $V(x') > V(x)$. Given this quasi-utility function (wherever it comes from) and given the actual preference structure (or "laws of motion" of preferences) of the person we can define a system of ordinal utility functions $U(x;q)$ conforming to these preferences in the following way.

We remember the notation: $x'(>)_q x$ means x' is preferred over x, given preferences q. And $x'(=)_q x$ means: the consumer is indifferent between x' and x, given preferences q. Let $I(\bar{x};q) = \{x : x(=)_q \bar{x}\}$ be the set of baskets x such that the consumer is indifferent between x and $\bar{x}$, given preferences q. Now I define: $U(\bar{x};q) = Min V(x) : x \in I(\bar{x};q)$. In words: (ordinal) utility of $\bar{x}$ under preferences q is the minimum value of $V(x)$ obtained within the set $I(\bar{x};q)$ of baskets that are indifferent to $\bar{x}$. Obviously this is a legitimate utility function representing the preferences q: If $\hat{x}(>)_q \bar{x}$ then, due to non-satiation, obviously $I(\hat{x};q)$ lies "above" $I(\bar{x};q)$ and thus by "quasi-non-satiation" of $V(x)$ and by our construction $U(\hat{x};q) > U(\bar{x};q)$, which shows that preferences q are reflected in this ordinal utility function.

I now define the following:

Definition 15: Preferences are defined to be ordinal-adaptive with respect to $V(x)$, if, for any x_0 we have $U(x_0;\hat{q}(x_0)) = V(x_0)$. (End of definition).

Obviously it is generally the case that $U(x_0;q) = Min V(x) : x \in I(x_0;q) \leq V(x_0)$, and thus, with ordinal-adaptive preferences we have the same inequality $U(x_0;\hat{q}(x_0)) \geq U(x_0;q)$ as with the "cardinal" adaptive preferences defined earlier. Ordinal-adaptive preferences imply that the $n-1$-dimensional indifference "curve" going through x_0 with preferences $\hat{q}(x_0)$ lies above the $n-1$-dimensional quasi-indifference curve corresponding to $V(x)$ and going through x_0.



Graph 3: Red: Indifference curve of ordinal utility corresponding to x^0. Blue: Indifference curve of quasi utility function $V(x)$ through x^0.

Now, in the real world a random choice of a differentiable quasi-utility function $V(x)$ is very unlikely to exhibit ordinal-adaptive preferences with respect to $V(x)$. The relevant empirical question is this one: is there any differentiable quasi-utility function $V(x)$ such that the empirically valid "laws of motion" of preferences correspond to the characteristics of ordinal-adaptive preferences with respect to that quasi-utility function $V(x)$? Obviously, by Theorem 2 the answer is "yes", if and only if the space of commodity baskets has a complete pre-ordering in terms of balanced improving paths, i.e. if there exists a quasi-utility function $V(x)$ which is an indicator for pairs of baskets (x, x') such that there exists a balanced improving path from x to x'.

Corollary 1: Assume, like in Theorem 2, that preferences are cardinal-adaptive, that balanced improving paths are non-circular and that $\overline{A}(x^0)$ is smooth for all x^0. Then preferences are also ordinal- adaptive. Proof: If cardinal- adaptive preferences prevail in the real world then by Theorem 2 we can find $V(x)$ which represent the pre-ordering of the commodity space in terms of balanced improvement paths, and thus, as derived above in this section, corresponding ordinal- adaptive preferences can be defined and prevail in the real world. End of proof.

The difference between the two concepts is this: in the cardinal case we start from the utility function $U(x;q)$ and then derive the corresponding quasi-utility function $V(x)$; in the ordinal case we start from the quasi-utility function $V(x)$ and then derive the corresponding ordinal utility function $U(x;q)$.

One implication of ordinal- adaptive preferences may make it easier to test the hypothesis that preferences are adaptive. It is the following proposition or corollary:

Corollary 2: Assume preferences to be adaptive. If $x(>)_{\hat{q}(\bar{x})}\bar{x}$ then also $x(>)_{\hat{q}(x)}\bar{x}$. Proof : Adaptive preferences imply $U(x;\hat{q}(x)) \geq U(x;\hat{q}(\bar{x}))$ and $U(\bar{x};\hat{q}(\bar{x})) \geq U(\bar{x};\hat{q}(x))$. And the "if statement" implies $U(x;\hat{q}(\bar{x})) > U(\bar{x};\hat{q}(\bar{x}))$. Thus it follows that $U(x;\hat{q}(x)) \geq U(x;\hat{q}(\bar{x}) > U(\bar{x};\hat{q}(\bar{x})) \geq U(\bar{x};\hat{q}(x))$. Hence $x(>)_{\hat{q}(x)}\bar{x}$, which finishes the proof.

Thus, it suffices to falsify the statement: "If $x(>)_{\hat{q}(\bar{x})}\bar{x}$ then – a fortiori – we also have $x(>)_{\hat{q}(x)}\bar{x}$." We then have falsified the hypothesis of adaptive preferences, both, of the cardinal and the ordinal variety.

The uninitiated reader may have easier access to an understanding of adaptive preferences by understanding this result encapsulated in Corollary 2: If some basket is preferred over another basket even with preferences corresponding to this other basket then a fortiori it is preferred over the other basket with preferences corresponding to itself. Therefore I introduce the following definition.

Definition 16: The heuristics of adaptive preferences is fulfilled, if for any pair of baskets x and $\bar{x}$ we have: if $x(>)_{\hat{q}(\bar{x})}\bar{x}$ then also $x(>)_{\hat{q}(x)}\bar{x}$.

9 Interpersonal Influences on Preferences

People imitate each other. Imitation is one of the important social processes. Assuming fixed preferences imitation has to be explained by learning under conditions of incomplete information. In my approach of adaptive preferences I do not have to decide whether imitation is a phenomenon due to learning processes or due to adaptive preferences. In this section I treat imitation as a form of interpersonal preference adaptation.

Imitation then can be seen as an adaptation of preferences to the observed conduct of others, be they other members of the family, like parents or siblings, be they neighbours, teachers, friends, co-members of clubs, colleagues at the work place or persons only known by viewing films or advertisements.

Imitation as an adaptation of preferences is one example of interpersonal influences on preferences. I believe it to be the most important one. But there may be others, which we cannot easily frame into something which we might call "imitation". Obviously the welfare economics of interpersonal influences on preferences is a difficult and so far under- explored field of analysis. In this section I only make a small beginning in this area of research using my concept of adaptive preferences. I concentrate on imitation.

The way I treated intra-personal adaptive preferences was that I defined certain preferences which correspond to any given consumption basket. Then I defined adaptive preferences to prevail if the preference structure had the feature that utility of any given basket x was higher (or at least not lower) with preferences corresponding to this basket than with any other preferences. The corresponding preferences were those to which preferences would converge in the long run, if the consumption basket was kept constant. Can we develop a similar procedure, if, due to imitative preferences, they are influenced by the consumption baskets of other people?

To find such a procedure is not straightforward. I have done some exploratory work. But I am confident that much more can be done. In this paper I take the specific case that preferences of individuals are influenced by the macro-vector of consumption. In this context I have found it useful to switch from utility functions over commodity baskets to indirect utility functions defined in price space. For a given budget $y = 1$ let the indirect utility function $\tilde{U}$ be defined by $\tilde{U}(p;q,X)$ where the preferences are determined by q and the macro-vector of consumption X. For any price vector p and any macro-vector X, assumed to be constant through time, we can expect that demand x and thus q converge to certain values $\hat{x}(p;X)$ and $\hat{q}(\hat{x}(p;X)) = Q\hat{x}(p;X)$. We may assume a certain distribution of income, which together with the price vector p and given preferences determines X. Provided that p remains constant through time, we can expect X to converge to a certain value $\hat{X}(p)$. Thus we may define $\hat{X}(p)$ to be the macro-vector which corresponds to p. Then preferences $\hat{q}(\hat{x}(p;\hat{X}(p))) = Q\hat{x}(p;\hat{X}(p))$ can be called the preferences corresponding to p.

In the intra-personal case of endogenous preferences we defined adaptive preferences as a preference structure such that the vector x provided the highest utility with preferences corresponding to x. We try to the same here: Thus we define adaptive preferences to be characterised by $\tilde{U}(p;\hat{q}(\hat{x}(p;\hat{X}(p))),\hat{X}(p)) \geq \tilde{U}(p;q,X)$.

We may then go further and define smooth adaptive preferences as a preference structure such that for a given constant p the time path of q and X is such that the time derivative of $\tilde{U}$ is non-negative. Under such conditions we then can have a theorem corresponding to Theorem 1.

Theorem 1a: Assumption: Preferences are smoothly adaptive and the piecewise differentiable path $\{p(t);q^0;T\}$ is a balanced improving path. Proposition: Then $p(T) \neq p(0)$, i. e. the price path is non-circular.

The proof is basically the same as the one of Theorem 1.

In a similar way as done before in Section 7 we may define a "quasi-utility function" $\tilde{V}(p) = \tilde{U}(p;\hat{q}(\hat{x}(p,\hat{X}(p)),\hat{X}(p))$ which then can be used in the same way as an indicator for the presence of balanced improvement paths between any two price vectors.

Again we may prove a theorem similar to Theorem 2.

Theorem 2a: Assumptions: 1. Preferences are adaptive; 2. Balanced improving paths are non-circular. 3. For any price vector p^0 the upper bound $\overline{A}(p^0)$ is smooth. Proposition: If and only if $\tilde{V}(p^1) > \tilde{V}(p^0)$ we have $p^1 \in A(p^0)$.

The proof is similar to the proof of Theorem 2.

The assumption of adaptiveness of the preference structure in this case is closely linked to imitative preferences. Take a world with just two distinct commodities 1 and 2. Assuming imitative preferences a change in the relative price of the two goods has long run demand effects which are stronger than short run demand effects with given preferences. After good 1 has become cheaper and good 2 has become dearer we expect a prime effect for given preferences of higher demand for good 1 and lower demand for good 2. Now, that the macro-vector of demand has changed imitative preferences imply a further rise in demand for good1 and a further decline in the demand for good 2. Thus imitative preferences have an amplifying effect on the reaction of demand upon a change in relative prices.

Assume now that everybody else is characterised by imitative preferences. Only the person we investigate is anti-imitative: she reduces her preferences for good 1 when others buy more of good 1. We then apply the heuristic of adaptive preferences: basket a and basket b are such that basket b has more of good 2 and less of good 1 than basket a. Switching back to direct utility functions we assume $U(x^a;\hat{q}(x^b),\hat{X}(x^b)) > U(x^b;\hat{q}(x^b);\hat{X}(x^b))$. By the heuristic of adaptive preferences we would then have to find $U(x^a;\hat{q}(x^a),\hat{X}(x^a)) > U(x^b;\hat{q}(x^a);\hat{X}(x^a))$. But this person being anti-imitative, this may not be the case, because $\hat{X}_1(x^a) > \hat{X}_1(x^b)$ and $\hat{X}_2(x^a) < \hat{X}_2(x^b)$ (due to imitative preferences of the other citizens) may induce the anti-imitative person to exhibit the inequality $U(x^a;\hat{q}(x^a),\hat{X}(x^a)) < U(x^b;\hat{q}(x^a);\hat{X}(x^a))$, thereby violating the heuristics of adaptive preferences.

Thus, it is not the case that any interpersonal influences on preferences induce adaptive preferences. But imitative preferences may have this property, because they essentially reinforce the substitution effects of a change in relative prices.

10 The Meaning of the two Theorems and the Concept of Rationality

Homo Oeconomicus has gone into disrepute. Happiness research and experimental economics indicate that people in their actions don't comply with the axioms of the rational maximiser of an exogenously given utility function. Many people speak of "irrational behaviour" which characterises the actual actions of people. I discuss some of the results of this literature more in detail below in section 12.

On the other hand, most people, indeed also most economists, take democracy as the appropriate form of government. Democracy relies on an implicit axiom which I want to call the rationality axiom of democracy: democracy only can work with a certain minimum average degree of rationality of its citizens. Thus by advocating democracy we assume a certain degree of rationality of the average citizen.

We also, as discussed in section 2, are interested in conceptualising civil liberty or freedom. Economics represents liberty in its models by means of the concept of preferences. If a person chooses among different available alternatives her choice as such cannot be criticised as being irrational. This choice is just the outflow of her preferences. This is the tradition of revealed preference. But rationality of the agent may be cast in doubt, if the observer observes more than one choice, say, through time. Rationality may then be tested by means of the criterion of consistency. In revealed preference theory the strong axiom takes this role. The Samuelson-Houthakker theorem tells us that the strong axiom is necessary and sufficient to prove that actions of the agent are consistent with un-contradictory preferences. The strong axiom uses as a criterion a series of improvements in terms of revealed preference.

The revealed preference theory and its test by means of the strong axiom is a non-temporal theory. As rationality test or consistency test the series of revealed preference improvements is not allowed to return to the initial basket. In contrast, the theory of adaptive preferences presented here is a temporal theory. I investigate improvements through time and I admit endogenously determined changes of preferences through time. My theory is a generalisation of the neoclassical theory of revealed preference with the assumption of fixed preferences. Mathematically speaking, none my concepts, assumptions and proofs exclude the possibility that preferences are fixed. The concept of improvement (in this case: through time) is central to my approach. I also use non-circularity of a series of improvements for a kind of "global" rationality test. It is a kind of rationality across changing preferences.

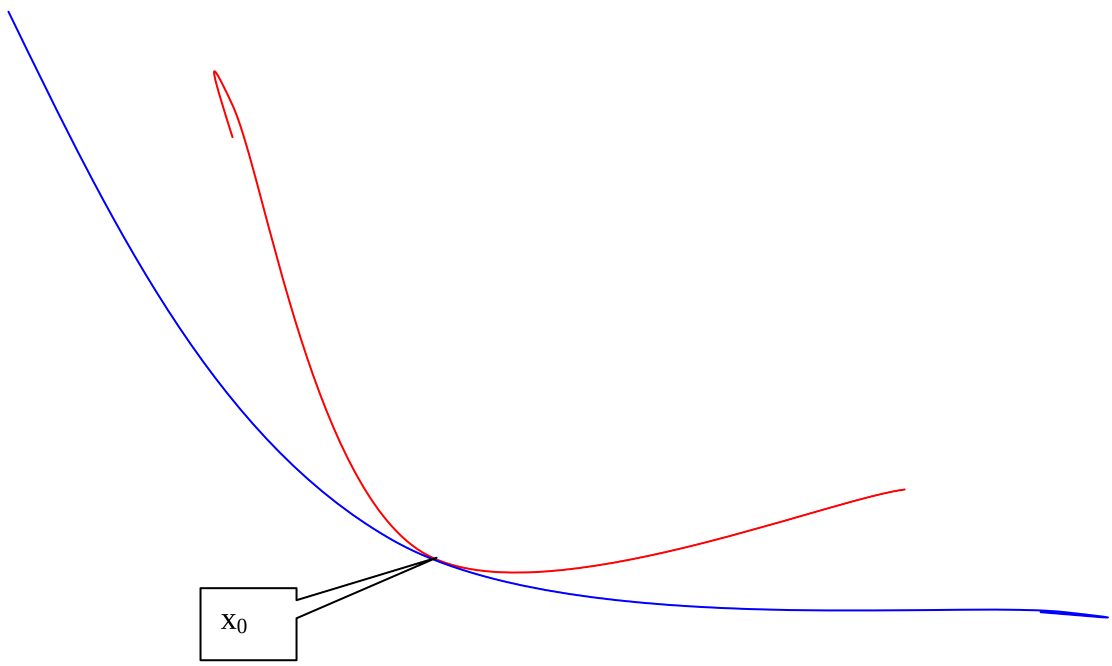
Theorem 1 (and Theorem 1a) shows that laws of motion of preferences which conform to smooth adaptive preferences are sufficient to pass the global rationality test of non-circularity of balanced improving paths.

Theorem 2 (and Theorem 2a) shows that laws of motion of preferences which are adaptive and which conform to the rationality test of non-circularity of balanced improving paths lead to a characterisation of the commodity space in terms of balanced improvement paths which looks like a fixed preference structure. Moreover my earlier theorem of 2005 shows that under certain conditions like absence of "path dependence" of long run demand functions non-circularity implies adaptive preferences. Thus, whenever the laws of motion of preferences do not conform to adaptive preferences there is the high probability that the global rationality test of non-circularity of balanced improvement paths will not be passed.

Thus, rationality or consistency even under endogenously changing preferences is intimately linked with an exogenous structure on the commodity space which looks like a fixed preference structure. The question arises: is the neoclassical model perhaps valid, if we give it the right interpretation? Is it the appropriate static projection of the laws of motion of adaptive preferences? Can we then use it as a heuristic device for a more complex real world?

I take an analogy from physics. In the first half of the 17th century Galileo posited a very simple law of falling bodies. His critics at the time, following the authority of Aristotle, pointed out that he must be wrong, because one could observe that different bodies took different times to reach the floor. Well, Galileo was right, if you interpreted his law appropriately: as then could be fully understood in the 19th century. Galileo was literally correct for bodies falling without the interference of friction. But even though friction was better understood only much later, physics in the 17th century almost immediately made use of Galileo's ingenious discovery. It obviously helped Newton to find the law of gravitation. We then see that Galileo's discovery was much more than the discovery of a completely unrealistic and thus uninteresting limiting case.

We may then surmise that also the neoclassical model of rational action under exogenously given preferences is more than an unrealistic and thus uninteresting limiting case. We may re-interpret it as the static photo- image of the "preference film" of moving adaptive preferences. This photo-image has relevance, because it can be a picture of long run reactions to, say changes in prices, and thus it may also have normative relevance. Adaptive and imitative preferences seem to operate as an amplifier of reactions under given preferences upon changes in, say, relative prices or other constraints. The graph depicting indifference curves of preferences corresponding to x_0 and of quasi-preferences indicating the lower bound of $A(x^0)$ also indicates the difference between the short run elasticity of substitution (for given preferences) and the long run elasticity of substitution including the amplifying effect of induced preference changes.



Graph 4 (like Graph 3): Amplifying effect of induced preference changes as a characteristic of adaptive preferences.

These considerations also lead to an answer to the question: are there reasons of evolutionary advantage for the law of preference motion which I call adaptive preferences? I believe the answer is: yes. What I have shown is the close interconnection between adaptive preferences and non-circularity of balanced improving paths. Imagine now that preferences are "counter-adaptive". We would then have to consider it highly likely that there are many cyclical balanced improving paths. But then, by continuity considerations, we also could find balanced improving paths which end up "below" their starting point. If such a preference structure prevails it would be highly dangerous for survival to deviate from a stationary state by means of an improving path. In their struggle for survival thus "counter-adaptive" preferences have a fundamental disadvantage against adaptive preferences, because the latter, in their attempt to improve the status quo are not misled into downward circles.

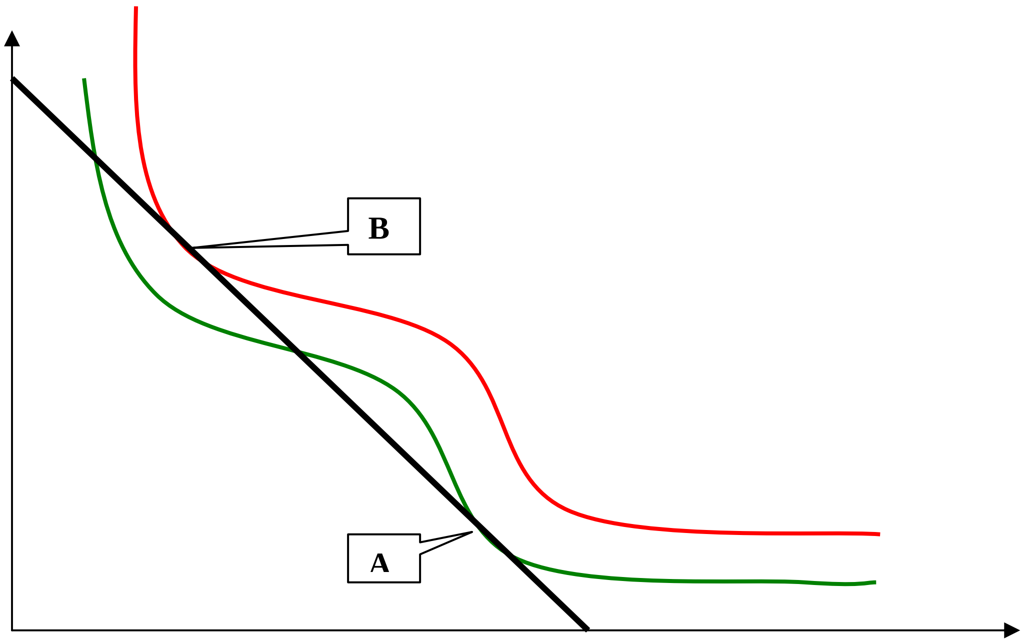
Moreover we should note that fixed preferences are the "borderline case" between adaptive preferences and counter- adaptive preferences: Adaptive preferences are characterised by the inequality $U(x;\hat{q}(x)) \geq U(x;q)$ for any q. Fixed preferences are the case that $U(x;q)$ is the same for all q, so that we have $U(x;\hat{q}(x)) = U(x;q)$ for any q. But then we are "close" to cases in which we can find q so that $U(x;\hat{q}(x)) < U(x;q)$ which then easily leads to the possibility of circular balanced improving paths. Thus, small "random" changes in the law of motion of preferences starting from fixed preferences may easily lead into the realm of "counter- adaptive" preferences. Adaptive preferences then may have a survival value which is superior to fixed preferences.

I now return to the concept of rationality and to the rationality axiom of democracy. The traditional utility maximising model with fixed preferences, in particular in the forms used, say, by the Chicago School, essentially is a completely static model of rationality. People know what they want. Learning is only about the means by which to achieve given goals. The goals themselves are given. The model of adaptive preferences allows for a development of rationality, for an "unfolding" of rationality, also in terms of the goals one wants to achieve. Thus education, schooling, character building (strengthening of will power for example, fairness, etc) do not fit well into a model of fixed preferences, but I believe are compatible with a model of adaptive preferences.

The model of adaptive preferences then is a model of potential rationality, rather than necessarily actual rationality.

11 The Impossibility of Global Optimisation

It is easily possible that the "quasi-utility-function" $V(x)$ does not exhibit nice convexity properties. Thus $A(x^0)$ may not be a convex set. As an example take the case of two goods and of two "quasi-indifference curves" like the red and the green one in the graph below.



Graph 5: Two stable equilibria, B reachable from A by a balanced improving path

The black straight line may be seen as a budget constraint. Given this budget constraint and given adaptive preferences the consumer converges either to basket A or to basket B, depending on his initial preferences. With preferences eventually corresponding to A the consumer maximises his utility by staying at A. With preferences eventually corresponding to B the consumer stays at B. But by Theorem 2 there exists an improving path starting with preferences corresponding to A and ending at B or even slightly below B. The problem is that this path initially runs above the budget line, because by Theorem 2 it stays above the green quasi-indifference curve and thus early on it must be above the budget line.

A is a stable point, yet within the budget constraint there are other points, like B, which are "better" than A in the sense that they could be reached by a balanced improving path. Note that this, in a sense, suboptimal stable solution is not a matter of lacking credit for borrowing to go beyond the budget temporarily. It is a problem of a lacking will to leave point A with preferences corresponding to A. The person we look at does not really think very much about baskets like B, which are far away from A. She has no reason to do problem solving for problems of which she is not aware.

What about paternalism? We imagine that an authority with superior knowledge concerning the structure of $V(x)$ of the person might induce the person to borrow money in order to move in small improving steps from A to a point sufficiently below B so that in the long run the loan can be repaid. We might think of a doctor advising the patient to change her life to become healthier. There are many other examples of this kind. But there obviously is no guarantee that such advice will be sought, and, if sought, will be observed. Moreover, if the true world is more than two-dimensional and if there are many sub-optimal equilibrium baskets it may also be quite difficult to know about appropriate movements away from the equilibrium to another one.

But the main reason I believe that global optimisation is impossible is social welfare. Consider the quasi indifference curves to be derived from some kind of quasi welfare function for a society. If it is democratically governed it is difficult to see how people could themselves enact changes so that society could move from A to B on an improvement path. Preferences of voters correspond then to the point A. Thus there will be strong resistance against a move away from A, if this involves borrowing for the purpose of achieving a goal quite far away from the status quo and at actual preferences not at all favoured over the status quo.

12 "Preferences" of Organisations

Methodological individualism prescribes that preferences are characteristics of individuals, not of groups of individuals. But it may be appropriate to ask questions about the "behaviour" of organisations like for example firms or bureaus of a larger bureaucracy, knowing that such "behaviour" may in the end have to be explained by the behaviour of the individuals involved in this organisation. I see a fruitful analogy with my approach of adaptive preferences. My thesis is: organisations, in particular firms, "behave" as if they had a preference structure which corresponds to adaptive preferences. Indeed, it seems to me that such an approach also sheds some light on individual behaviour. Behavioural economics these days tries to model the human being (or his/her brain) as if it were a multi-person entity. Thus, for example, Thaler and Sunstein[7] in their book "Nudge" split the human actor in two persons, the "doer" and the "planner" – and so do many more behavioural economists[8]. Nevertheless, from the perspective of normative economics we need to attach preferences to individuals, not only to "persons" who in a behavioural model represent only part of an individual. We may then, by analogy, learn something about individual behaviour and preferences, if we attach "preferences" to an organisation and relate these "preferences" to the preferences of individuals, who are members of the organisation.

Take a bureau, part of a bureaucracy. It is financed by the taxpayer and it is not faced with acute survival problems. It performs certain functions 1, 2, 3 and a large part of its personnel is specialised in one of these functions. Let x_1, x_2, x_2 be the number of people dedicated to function 1, 2, and 3 respectively. The opportunity now arises to change the weights of the three functions and the taxpayer even offers additional jobs of Δx, if the bureau does more work for function 1 and reduces the amount of work for functions 2 and 3. This would imply a positive Δx_1 and negative Δx_2 as well as Δx_3. We have, of course $\Delta x = \Delta x_1 + \Delta x_2 + \Delta x_3 > 0$. Let $\Delta z = |\Delta x_1| + |\Delta x_2| + |\Delta x_3|$. Thus Δz represents the required restructuring in quantitative terms, whereas Δx represents the "gain" in jobs, due to the restructuring. We expect the "preference" of the bureau for or against restructuring to depend on the ratio $\frac{\Delta x}{\Delta z}$. If Δz is sufficiently large and this ratio $\frac{\Delta x}{\Delta z}$ is sufficiently small the bureau will oppose the restructuring, because the internal resistance among employees who have to fear to be reallocated within the bureau is likely to be quite strong. We may express this in terms of

[7] Richard Thaler and Cass Sunstein, Nudge, New Haven and London, 2008, Part I, Humans and Econs, p.15 – 100.
[8] Werner Güth,…

indifference curves in the three dimensional space representing x_1, x_2, x_2. The indifference curve going through the status quo point $x_1^0, x_2^0, x^0{}_2$ can be expected to be quite "curved" showing only a moderate elasticity of substitution. Thus the "preferences" of the organisation exhibit exactly what we mean by adaptive preferences: the actual distribution of jobs across different functions is preferred more strongly to other hypothetical distributions than it would be preferred if it were not the actual distribution. Organisations exhibit a strong preference for the status quo. One example of such organisations is a university with a high degree of decision autonomy. It is very difficult to internally agree on a change in the distribution of professorships among different departments even if such change would go with a rise in the number of available professorships.

Economics has developed the concept of x-inefficiency which describes in general terms what I here have described as strongly adaptive "preferences" of organisations.

The same idea can be applied to political decisions. In the realm of politics special interest groups play an important role. Following Olson, and perhaps refining his approach, we might show that special interest groups can be formed best for special interests which consist of the defence of existing, status quo based rents. Thus the community, the nation, as a political entity exhibit strongly adaptive "preferences".

The traditional liberal (in the European meaning of "liberal") answer to the strong status quo orientation, i.e. the strongly adaptive "preferences" of organisations and political entities has been "competition". Competitive pressure on any organisation, like a firm, a bureaucracy and perhaps also like a state tends to reduce status quo based rents of its members, because without appropriate changes the organisation may not survive in the competitive struggle and thus the rents would be gone anyway. Under competition among railway companies the stoker on the electric locomotive will be gone.

I believe that this function of competition to eliminate x-inefficiency is quantitatively more important than the function to eliminate allocative inefficiency. Yet, while economics has a reasonably good theory of allocative efficiency (albeit, under the assumption of fixed preferences) it does not have a good theory of x-inefficiency and its relation to competition. It is my suggestion that my approach of adaptive preferences might help to develop such a theory.

One point of theory here is perhaps important. We can ask the question: why can't an organisation, say, like a self-governing university, improve performance by trading the different status quo based rents and thereby obtain Pareto- improvements, i.e. mutual advantages? The answer is that it is not possible within organisations to agree on the monetary equivalent of such rents. One reason is that the positions, from which such rents derive, are themselves no tradable property rights. Any attempt at such monetisation would make cooperation of the members of the organisation impossible. - But we are far away from an axiomatic theory which can appropriately deal with x-inefficiency.

13 Behavioural Economics and Adaptive Preferences

Modern behavioural economics[9] has discussed quite a few "anomalies" of human behaviour, which seem to be inconsistent with rational maximisation of a fixed utility function[10]. Also

[9] A forerunner of modern behavioural economics was Günter Schmölders (1903-1991), Professor of Public Finance in the University of Cologne. He was particularly interested in "Steuerpsychologie", i.e. Tax Payer Psychology and made important contributions there. See for example Günter Schmölders, Das Irrationale in der öffentlichen Finanzwirtschaft, Probleme der Finanzpsychologie, Hamburg 1960. His work has been completely disregarded by the economics profession outside of the German speaking part of the world. His insights cover many of the "discoveries" of modern behavioural economics.

"happiness research" is partly at odds with the traditional neoclassical model. In this section I do not discuss all these empirical findings. Rather I want to indicate why I believe that most of the stable patterns of "non-standard" behaviour are compatible with the idea of adaptive preferences.

Take the "endowment" effect[11]. In a random allocation students who are given a coffee mug value it (on average) twice as high as students who did not get a coffee mug. This obviously is inconsistent with standard utility maximising under fixed preferences. If transaction costs are negligible, valuation of a coffee mug should be about the same whether you own or whether you do not own the mug at this moment. For, you can always transform yourself at negligible transaction cost from an owner of a mug to a non-owner or vice versa.

But the "endowment effect" which seems to be observable quite generally (not only with coffee mugs) is of course consistent with adaptive preferences. You value the basket which you consume higher relative to other baskets than you would value it if you consumed a different basket.

The difference between Graph 1 above and the endowment effect with the coffee mug is that in Graph 1 we stipulated perfectly divisible goods, whereas the coffee mug allocated at random among students is not divisible. In this experiment ownership of the mug only could obtain the value zero (no ownership) or the value unity (ownership). Also purchasing and selling the mugs involved only integer numbers of mugs. At the moment my point is a different one: it appears that most insights from behavioural economics concerning non-standard behaviour are compatible with the formula also used in my set-up of adaptive preferences: "resistance to change" or "status quo bias[12]".

Thus, for example, the recent experiments by Selten et al on "Anspruchsanpassungstheorie" concerning goal setting in the tradition of Herbert Simon's satisficing model show that the present actual point achieved has a strong influence on goals. Goals adapt to the current status quo.

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[10] One (incomplete) survey is in Richard Thaler and Cass Sunstein, Nudge, New Haven and London, 2008, Part I, Humans and Econs, p.15 – 100.

[11] Thaler and Sunstein, op. cit p.33.

[12] Samuelson, William and Richard Zeckhauser, Status Quo Bias in Decision Making, Journal of Risk and Uncertainty, 1988, p. 7-59.

The robust empirical result of loss aversion also conforms to the general idea of "resistance to change".

The strong influence on choice exerted by the "default option" again is consistent with "status quo bias". Here the status quo is the default option. In a broader sense, which so far has not been worked out with any precision, we can expect the default option, unless manipulated by another person, is a kind of "natural" continuation of a status quo. For, in the case of collective decisions by more than one person, a decision, say, by a committee or a legislative body, always has to be an explicit decision. The default option is "no decision". The status quo can be defined to be the state of affairs in case of "no decision". Thus the default option, so to speak by definition, is the status quo. In the case of individual decisions a "decision" need not be explicit. Thus here we cannot simply identify the default option with the status quo of "no decision". Nevertheless we can expect on average the default option to be closest to the "no decision" alternative.

Thaler and Sunstein[13] enumerate psychological effects in terms of rules of thumb: anchoring (p.23), availability (p.24), representativeness (p.25). They all can be seen as simplifying strategies to come to grips with cognitive difficulties of arriving at the "right" decision. Obviously these strategies always have the result that the starting point of the cognitive exercise has a strong influence on the outcome. To the extent that this starting point can be identified with the status quo they then are in accordance with the "status quo bias". "Mindless choosing" has a similar effect.

Very strong is the tendency to "follow the herd" in one's decisions[14]. Imitative behaviour corresponds to my concept of adaptive preferences. This was discussed in section 9 above. A particular form of "following the herd" is "following advertisements". Advertising is a commercial reality; and it is undisputed among experts that advertising has a strong impact on purchasing decisions. Some of this impact may be explained by "informative advertising". But we may also explain part of the impact of advertising by the following the herd effect: people (not fully explicitly) infer from advertisements that other people buy the advertised product; and indeed many products are advertised as being the product with high popularity among customers.

[13] Thaler and Sunstein, op.cit.
[14] Thaler and Sunstein, op. cit. Chapter 3, Following the Herd.

Happiness research has shown that material well-being is not the most important influence on a cardinal measure of happiness[15]. On the other hand this research also shows that the relative income position within a reference group is quite important for a cardinal measure of happiness[16]. I suggest that these results are consistent with the hypothesis of adaptive preferences. Adaptive preferences are a preference structure with respect to goods consumed. Thus, the actual consequence of happiness for consumption of goods is what we need to investigate. If the relative income position is important we can infer that consumers are also guided by others in their consumption patterns, in particular by others, who have at least as high an income as they have themselves. Obviously the importance of relative income positions within a reference group is closely connected with a well known human treat: envy. But if people envy others for the goods which those others can afford to buy then we expect that they will be strongly influenced in their consumption patterns by those others. Hence we observe imitative preferences.

Much more could be said about this topic of the relation between "non-standard" behaviour and adaptive preferences.

Here I do not deal in detail with the policy recommendations of the Thaler- Sunstein book or with similar policy recommendations by others. The title of the Thaler- Sunstein book is "Nudge", by which they mean influencing people's choice in a very soft form, to "nudge" them; and this to their own advantage. They call this philosophy, as others also do, "libertarian paternalism". Up to a point I can agree with this philosophy; but this is not the topic of this paper.

Given the indivisibility involved in observations of the endowment effect or other non-standard effects I suggest the distinction between a <u>virtual choice space</u> of perfectly divisible goods, in which my formal theory of adaptive preferences applies, and its "representation" in the real world in the form of an <u>actual choice space</u> where goods are not perfectly divisible. Whereas Theorem 2 requires the virtual choice space of perfect divisibility, Theorem 1 and also Theorem 3 of section 14 below do not.

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[15] Easterlin, Richard, Income and Happiness: Towards a Unified Theory, Economic Journal 2001, 111, p. 465-484. Bruno Frey and Alois Stutzer, What Can Economists Learn from Happiness Research?, Journal of Economic Literature, 2002, XL, p. 402-435

[16] Ada Ferrer-i-Carbonell, Income and well-being: An empirical analysis of the comparison- income effect, Journal of Public Economics, 2005, 89, p 997-1019, and numerous other publications.

<u>14 Cost Benefit Analysis and Adaptive Preferences</u>

The main reason why I have developed the concept of adaptive preferences is that I wanted to find a way how to do welfare economics when preferences are influenced by the results of economic policy decisions. In this last section I then apply the apparatus to the most applied field of welfare economics, which is cost benefit analysis.

For simplicity of presentation I do not present the most general case of my theory, as far as yet developed. For this section I assume that the preference space has the same dimension $N = n$ as the goods space. I further assume that the matrix α has the same positive number on the main diagonal. In that case the equation (1) reads

$$\frac{dq}{dt} = \alpha(x - q)$$

where α can be understood to be a positive real number. Then we have that preferences corresponding to x are represented by x itself. i.e. $\hat{q}(x) = x$.

I now introduce the relation "more similar than… to.." . We write $q(x)\overline{q}$ for "q is more similar to x than is $\overline{q}$ and this relation between q and $\overline{q}$ is fulfilled if we have the following inequalities

$$|x_i - q_i| \le |x_i - \overline{q}_i| \text{ for } i = 1,2,.....n$$

$$\text{and } (x_i - q_i)(x_i - \overline{q}_i) \ge 0 \text{ for } i = 1,2,.....n$$

In words: q is more similar to x than is $\overline{q}$, if the distance between q and x is smaller in each vector component than the distance between $\overline{q}$ and x and if in addition q and $\overline{q}$ are on the same side of x in each vector component. To put it differently: q is said to be more similar to x than $\overline{q}$ if any reasonable distance measure would yield the result that the distance between x and q is smaller than the distance between x and $\overline{q}$. Note that this definition of "similarity" to a third vector does not make each pair of vectors comparable in terms of similarity.

Now I introduce a variant of the assumption of smooth adaptive preferences: If preferences q are more similar to corresponding preferences $\hat{q}(x) = x$ than are preferences $\overline{q}$ then $U(x;q) \ge U(x;\overline{q})$. Given the differential equation (1) in its form for this section it is clear that as long as x remains the same q becomes more similar to x through time. In other words, the two forms in which we have defined smooth adaptive preferences earlier and in this section are equivalent as long as the present form of equation (1) holds.

Cost-benefit analysis is used all the time in policy making, but also in private decisions by individuals or firms or associations. It is a method, which intellectually isolates certain parts of the world from the rest of the world and then concentrates on these parts, which appear to be relevant for the issue at hand. Parliament has to decide whether to change a certain law. A firm has to decide whether to make a certain investment in order to enlarge its production capacity. An individual has to decide whether to accept a certain job offer or not. The "rest of the world" generally is represented by the money involved in the particular decision. It is "money" and market prices which make sure that the wider context of the particular decision is taken account of[17]. To the extent that this kind of representation of the interdependence of everything with everything is appropriate, the "money form" of this representation makes decision taking vastly simpler than it would otherwise be. This vast simplification is the prerequisite for a world in which a very large number of decisions can take place simultaneously. Without such simplification the number of feasible simultaneous decisions would have to be very much lower. Society could not have obtained its present degree of complexity and could not draw on its present high degree of the division of labour[18]. Without the money form of representation of the wider world the status quo bias in the form of "non-decision" would be absolutely dominant.

Economists have investigated the conditions under which it is appropriate to do this partial equilibrium exercise which is involved in any cost-benefit analysis. The general presumption here is the all-round existence of reasonably competitive markets. Without going into the details of these analyses it is so far clear that they all rely on the assumption that members of the economy are people who maximise an ordinal utility function which is exogenously given.

In the following I show that there is hope that we can carry over such analyses to a world of adaptive preferences.

[17] Hayek, F.A. von, The Use of Knowledge in Society, AER 1945
[18] "The greatest improvement in the productive powers of labour, and the greater part of the skill, dexterity, and judgement with which it is anywhere directed, or applied, seem to have been the effects of the division of labour." Adam Smith, Wealth of Nations, Book 1, Chapter 1, first sentence.

Cost-benefit analysis is about policy choices between different alternatives. In the simplest case we answer a single "yes or no" question. Should a certain bridge be built? Should a certain legislative proposal be adopted or not? Does a certain innovation enhance welfare or not? I look at this simplest case. Thus, we may have to decide between two paths of consumption vectors x(t) and z(t). With them preferences change by means of the differential equations discussed before. Let $q(t)$ be the preferences corresponding to path $x(t)$ and let $r(t)$ be the preferences corresponding to path $z(t)$. Thus we have the differential equations

$$\dot{q} = \alpha(x - q) \text{ and } \dot{r} = \alpha(z - r)$$

We can assume that the two alternative paths have identical starting points so that $z(0) = x(0)$ and $r(0) = q(0)$.

In the following I treat the paths $x(t)$ and $z(t)$ asymmetrically. $x(t)$ is the "default option", i.e. the state of the world, in the case of "non-decision": the law is not changed, the bridge is not built, or the innovation is not introduced. It does not mean that $x(t)$ is a stationary path. It could be a growing or shrinking economy. Any kind and number of decisions may be taken in the economy at large which have an expected influence on $x(t)$. Following the partial equilibrium philosophy of cost-benefit analysis we take these other developments as given and only look at the particular decision at hand: whether to change the world by implementing $z(t)$ or not, the latter implying that the world is $x(t)$.

Now, the decision between $x(t)$ and $z(t)$ is taken so to speak with "preferences corresponding to $x(t)$". I will not make this precise here. It is precise in the case that $x(t)$ is a stationary path and that preferences already correspond to this stationary path. If, as realistically is the case, $x(t)$ is not stationary, and also not expected to be stationary by the decision makers, we may have a large array of possibilities concerning the preferences expected to be prevailing along the path. But, I adhere to the assumption that the movement of the preferences through time corresponds to the specified differential equations. Yet, we have some freedom in the choice of initial preferences $q(0) = r(0)$. They need not correspond to the initial value $x(0)$ of $x(t)$.

Let us now observe that changes due to certain measures like building a bridge or changing the law or product innovations evolve through time. The impact of these measures rises through time. Thus it is of interest to consider divergences between $z(t)$ and x(t) which become larger through time. Such a pair of paths I call: a pair of paths with "monotonic divergence". The precise definition is the following:

Definition: There is monotonic divergence between $x(t)$ and $z(t)$ on the time interval $[0,T]$ with $z(0)=x(0)$ if the following holds: 1. for each component i, $i=1,2,....n$. either $z_i(t)-x_i(t)>0$ for all $t\in(0;T]$ or $z_i(t)-x_i(t)<0$ for all $t\in(0;T]$ or $z_i(t)=x_i(t)$ for $t\in[0;T]$. 2. If $z_i(t)-x_i(t)>0$ then $z_i(t)-x_i(t)\geq z_i(\tau)-x_i(\tau)$ for $t>\tau$; if $z_i(t)-x_i(t)<0$ then $z_i(t)-x_i(t)\leq z_i(\tau)-x_i(\tau)$ for $t>\tau$.

A further distinction is important: the project implementing $z(t)$ rather "the default" $x(t)$ either can be in the general trend of goings- on or it can be the opposite, i.e. against the trend. The theorem I prove below only applies trend-projects. This concept refers to the relation of actual consumption to actual preferences. If there is a prevailing general trend which deviations from the default take we expect preferences to lag behind the actual developments, due to adaptive preferences. As an example take food. We have observed a trend of substitution from food which requires a lot of work of the cook in the household towards ready made food products which require much less work in the household. The tastes of people lagged behind this development and thus also retarded it, because people who were used to self-cooked food first had aversions against the "new" labour saving mode of nourishment.

Definition: A project which implements $z(t)$ rather than the "default" $x(t)$ with monotonic divergence is called a trend project, if on the time interval $[0,T]$ for each $i=1,2....n$ the following inequality holds $(z_i(t)-x_i(t))(x_i(t)-q_i(t))\geq 0$.

Thus, for a monotonically diverging trend project we have: if $z_i(t)-x_i(t)>0$ then $x_i(t)-q_i(t)\geq 0$. If $z_i(t)-x_i(t)<0$ then $x_i(t)-q_i(t)\leq 0$. For such a project the "direction" of the change, as indicated by the "i´s" with inequalities $z_i(t)-x_i(t)>0$ and the "i´s" with inequalities $z_i(t)-x_i(t)<0$ is in line with the recent direction of the economy at large as traced by the lag between of preferences relative to actual consumption, i.e. as traced by the inequalities $x_i(t)-q_i(t)\geq 0$ and $x_i(t)-q_i(t)\leq 0$.

For trend projects we then can prove the following:

Theorem 3 (or Theorem on Cost-Benefit Analysis with Adaptive Preferences): Assume smoothly adaptive preferences as defined in this section. Assume that the pair of paths $x(t)$ and $z(t)$ is characterised by monotonic divergence on a time interval $[0,T]$ and that $z(0) = x(0)$. Assume further that the project involved is a trend project. Then, for any $t \in [0,T]$ we have

$U(z(t); r(t)) \geq U(z(t); q(t))$.

Proof: Solving the differential equations for $q(t)$ and $r(t)$ yields the n equations

$$r_i(t) - q_i(t) = \alpha e^{-\alpha t} \int_0^t e^{\alpha \tau} (z_i(\tau) - x_i(\tau)) d\tau \quad \text{for } i = 1,2...n$$

If $z_i(t) - x_i t) \geq 0$ then by monotonic divergence we have

$$z_i(t) - x_i(t) \geq z_i(\tau) - x_i(\tau) \quad \text{for } t \geq \tau$$

And thus

$$r_i(t) - q_i(t) \geq 0 \text{, but}$$

$$r_i(t) - q_i(t) \leq \alpha e^{-\alpha t} \int_0^t e^{\alpha \tau} (z_i(t) - x_i(t)) d\tau \leq z_i(t) - x_i(t)$$

Then, by the characteristic of a trend project we have

$$r_i(t) - z_i(t) \leq q_i(t) - x_i(t) \leq 0$$

which, together with $r_i(t) - q_i(t) \geq 0$, implies

$$q_i(t) \leq r_i(t) \leq z_i(t)$$

Similarly we show for $z_i(t) - x_i t) \leq 0$ that

$$r_i(t) - q_i(t) \leq 0 \text{, but}$$

$$r_i(t) - q_i(t) \geq \alpha e^{-\alpha t} \int_0^t e^{\alpha \tau} (z_i(t) - x_i(t)) d\tau \geq z_i(t) - x_i(t)$$

and therefore

$$q_i(t) \geq r_i(t) \geq z_i(t)$$

Thus we have shown that $r(t)(z(t)q(t)$ (in words: $r(t)$ is more similar to $z(t)$ than is $q(t)$, which by the assumption of smooth adaptive preferences implies $U(z(t); r(t)) \geq U(z(t); q(t))$.

Q.E.D.

What is the economic meaning of this theorem? Assume a society is confronted with the decision to either go path $x(t)$ or to go path $z(t)$. As discussed before $x(t)$ is the "default option" and thus preferences which influence the decision are those which evolve along the path $x(t)$. If, with these anticipated preferences $q(t)$ society decides to choose path $z(t)$ then – the theorem says – having chosen path $z(t)$ and thus now being associated with the unanticipated preferences $r(t)$, society will not regret to have preferred $z(t)$ over $x(t)$, provided the project was a trend project with monotone divergence.

Take a few examples. Government decides to build the bridge. Then the ensuing impact of the bridge on preferences a fortiori justifies ex post that it was right to have built the bridge. Or: Parliament decides to change a law, which encourages the movement of consumption in the already established trend. Then the ensuing impact of the new law on preferences a fortiori justifies the change in the law. Or: we are in a market economy: an innovative firm introduces a new product in the market. It calculates this to be profitable given the pre-existing preferences of consumers. Then a fortiori the innovation will be profitable with the preference change induced by the new product. Innovations tend to be trend projects in the sense defined above.

We should note that project characteristics of monotone divergence and being in trend are sufficient conditions for the ex-post justification of a project undertaken with ex-ante preferences. I have little doubt that further research will provide other classes of projects which also have this property. But there will of course be exceptions.

The general message then is: it remains possible to do cost-benefit analysis even with endogenously changing preferences as long as "law of motion" of the preferences conforms to smoothly adaptive preferences. Thus the advantages of decentralisation (say, by markets) of social decisions which economists implicitly or explicitly rely on by explicitly or implicitly assuming fixed preferences can remain valid also if preferences are not fixed, but smoothly adaptive.

But we also should note the following: the converse is not true. If with preferences $r(t)$ the choice of $z(t)$ over $x(t)$ is justified and thus this choice is not regretted afterwards, then there is no guarantee that $z(t)$ will be chosen, given that the decision is taken with (expected) preferences $q(t)$. In other words: for the class of changes investigated a decision for change taken with present preferences is a sufficient condition for the decision to be right in view of the feedback of the change on preferences; but it is not a necessary condition. Thus, with adaptive preferences there is less change in terms of legislation, investment in infrastructure and innovations than could be justified. In this sense adaptive preferences generate a conservative bias.

As discussed in section 10 above I believe that there are good evolutionary reasons for the hypothesis that human preferences are adaptive in the sense of the word used in this paper. But, beyond that speculation about the evolution of human nature, adaptive preferences (with the special case of fixed preferences included) also seem to be an anthropological requisite of the astounding success of world society to generate wealth so much above the level of subsistence and with so high a world population. As said in this section before, the high complexity (high degree of the division of labour) of a modern wealth generating society as we observe it in the OECD countries requires decision making in the partial equilibrium mode, i.e. in the cost-benefit mode described above in this section.

Assume now the opposite of adaptive preferences. Thus, with the choice of $z(t)$ rather than the default option $x(t)$, the attractiveness of $z(t)$ declines in comparison with the preferences of $x(t)$, under which the decision to choose $z(t)$ had been made. In that case, from the welfare economic point of view it would be much more difficult to justify the partial-equilibrium i.e. the decentralised decision mode. Also psychologically, disappointment about decisions would abound and thus people would become much prone to avoid decisions altogether. In a loose sense we then can consider the success of the Western "wealth machine" itself to be a proof of the hypothesis of adaptive preferences.

Economic evolution led to a highly complex and highly successful economic system in the western world. By the very concept of evolution in the Darwinian tradition this development came about by highly decentralised decision making. In so many cases small parts of society did not take the default option of the status quo. The institutional set-up happened to be one in which decisions for change on average tended to be decisions for the better rather than for the worse. There was an "efficiency filter" for change built in the institutions which provided that the changes were efficient in most cases. But all these concepts of efficiency relied on a measuring rod of fixed preferences. My theory is that we can extend this traditional theory to a more evolutionary measuring rod, i.e. adaptive preferences: changes which are efficient given ex ante preferences remain efficient for ex-post preferences and thus are not regretted afterwards.