Construction of Probabilistic Inferences by Constraint Satisfaction

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Abstract

It has been shown that in decision making evaluations of evidence and attributes are modified. In three studies it was investigated if this finding of coherence shifts generalizes to real-world probabilistic inference decisions which are made from given probabilistic cues. Using a within-subjects design, cue validities were measured before, after (Exp. 1) and during decision making (Exp. 2 & 3). It was found that even in environments with considerable real-world cue knowledge (weather forecasts) and in decisions for which the application of fast-and-frugal heuristics has been claimed (city-size decisions) the validity of cues was systematically modified. These shifts indicate that subjective cue validities are not fixed parameters, but that they are changed to form coherent representations of the decision situation. The findings conflict with the basic assumption of complex decision models and the fast-and-frugal heuristics approach, which claim that probabilistic inferences are made in a unidirectional manner. They corroborate the parallel constraint satisfaction approach to decision making.

Keywords: Decision Making, Connectionism, Parallel Constraint Satisfaction, Fast-and-Frugal Heuristics, Bounded Rationality

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The only thing we know for sure is that the future is uncertain. This is not to say that chaos rules the world. Some things are quite stable over years and sometimes ages. There are good reasons to assume that ten years from now the sun will rise in the east and a bottle of Coke will contain a beverage that actually tastes like Coke. Conversely, it is hard to forecast next autumn’s weather and the quality of a future vintage of German Riesling. In order to cope with uncertainty, organisms can capitalize on probabilistic relations between cues and future events (i.e., between predictors and criteria). In repetitive situations, these relations can be learned by experience. When making a decision, individuals can rely on probabilistic inferences to predict a criterion (e.g., a consequence of a behavior) from the presence of a predictor (e.g., a discriminative stimulus). In line with the Brunswikian approach to probabilistic inferences (Brunswik, 1955), the predictor variables are referred to as cues that differ in validity (i.e., the conditional probability that options with a positive cue value are better on the criterion than options with a negative cue value).

Natural environments provide multiple cues for inference. How do people deal with this complexity? Inspired by Herbert Simon's notion of bounded rationality (Simon, 1955, 1982), judgment and decision researchers commonly assume that individuals employ a couple of heuristics that reduce complex tasks to simpler ones (see Gilovich, Griffin, & Kahneman, 2002; Kahneman, Slovic, & Tversky, 1982, for overviews). A considerable amount of work on probabilistic inferences stems from Gigerenzer and his research group (Gigerenzer, Hoffrage, & Kleinbölting, 1991; Gigerenzer, Todd, & ABC Research Group, 1999); they described a couple of fast-and-frugal heuristics in a very precise manner.

Perhaps the best-known example is the Take the Best (TTB) heuristic. It comprises three steps. First, it selects the cue with the highest validity and looks up the cue values of the alternatives. Second, if one alternative has a higher value than the others, the search for information is stopped (otherwise one goes back to Step 1 and tries the second best cue). The third step contains the decision rule: Predict or choose the winning alternative that is the one with the highest value. Assume for example, that you wish to go to a sunny place to spend your holidays. There are two alternatives on your list, region A and region B. You consult a weather forecast to predict the criterion (weather). News channels, newspapers, federal and commercial agencies provide a huge set of cues to choose from. Nevertheless, employing TTB makes the task very easy. You simply look up the prediction from the source you subjectively consider as most reliable (e.g., the forecast from the biggest news channel in your country). If the cue (the forecast from this channel) discriminates between the two regions, you base your decision on this one piece of information. You choose, for example, to go to region B if the weather forecaster predicts sunshine for B and rain for A.

The TTB heuristic perfectly mirrors the basic assumptions of the fast-and-frugal heuristics approach: One-reason decision making, unidirectional reasoning, and invariance of cue validities. Heuristics cope with complexity by ignoring information. In accordance with the principle take the best ignore the rest (Gigerenzer & Goldstein, 1999, p. 81), it is proposed that
many, if not most, of our decisions and judgments are based on one reason. Interestingly, decisions relying on partial processing of information can yield quite accurate results compared to normative standards as evidenced by computer simulations and experimental studies (e.g., Czerlinski, Gigerenzer, & Goldstein, 1999; Payne, Bettman, & Johnson, 1988).

According to the second assumption, fast-and-frugal heuristics work always in one direction. They start from given cues and infer the criterion. Of course, the hierarchy of cue validities can be changed through learning. Nevertheless, cue validities are conceived as given during a particular task. As such, they provide the hard constraints under which inference processes evolve. The same principle of unidirectional reasoning from the cues to the criterion underlies most complex decision models such as the normative standards provided by Bayesian reasoning, weighted additive rules for cue integration (WADD; Gigerenzer et al., 1999; Payne et al., 1988; cf. Bergert & Nosofsky, 2007) and evidence accumulation models (Busemeyer & Townsend, 1993). According to the WADD rule, individuals compute a weighted sum of cue values and cue validities for all available options and choose the option with the highest weighted sum. Unidirectional reasoning also dovetails with the utility theory's axiom of preference invariance. Specifically, utility theory assumes that the weights of attribute dimensions are fixed during a decision task (von Neumann & Morgenstern, 1944).

The selection of decision strategies in probabilistic inferences has been tested by investigating choices and information search. The findings often conflict with the assumption that heuristics (e.g., TTB, equal weight rules) are the dominantly employed decision strategies even in environments in which the application of these rules would be predicted by the model (e.g., Bröder, 2000; Glöckner, 2006, 2007a; Newell, Weston, & Shanks, 2003; for a different interpretation of the results, see Gigerenzer, 2004). In fact, WADD rules seem to be the default strategy (Bröder, 2003; Glöckner & Betsch, in press). However, in support of the fast-and-frugal heuristics approach Rieskamp and Otto (2006) have shown that individuals could learn to apply simple heuristics from repeated feedback. In the current work we investigate decision strategies on a more general level by measuring the stability of subjective cue validities in the decision process.

The notion of unidirectional reasoning and invariance of cue validities is, to the best of our knowledge, shared by almost all complex decision models as well as all fast-and-frugal heuristics that have been considered in the recent literature on probabilistic inferences (but see, Pennington & Hastie, 1992). This assumption is tackled by findings from research on multi-attribute decision making and legal decision making. Simon, Krawczyk and Holyoak (2004) measured the subjective importance of outcome dimensions before, during and after choices were made. Specifically, they presented participants with job offers differing on four dimensions: commute, office size, vacation and salary. The authors provide convincing evidence for coherence shifts showing that subjective weights of the outcome dimensions change in the course of the decision process (see also Simon et al., in press). Similar changes were found for the evaluations of arguments in decisions about complex criminal cases (Glöckner, 2007b;
Holyoak & Simon, 1999; Simon, Pham et al., 2001; Simon, Snow, & Read, 2004; Simon, 2004). These findings are also in line with other evidence for systematic attribute and cue distortions before and after decisions (Brownstein, Read, & Simon, 2004; Carlson & Russo, 2001; Russo, Meloy, & Medvec, 1998; for an overview see Brownstein, 2003) which operate towards increasing consistency and decreasing dissonance.

The results concerning coherence shifts further validate a constructivist view on decision making (Holyoak & Simon, 1999; Montgomery, 1989; Payne, Bettman, & Johnson, 1992; Pennington & Hastie, 1992; see also Slovic, 1995). Specifically, they support the connectionist approach to judgment and choice (Betsch, 2005; Glöckner, 2006; Glöckner & Betsch, 2008; Holyoak & Simon, 1999; Read, Vanman, & Miller, 1997; Thagard & Millgram, 1995). Accordingly, decision making is conceived as a holistic process characterized by an automatic, parallel consideration of multiple pieces of information which are integrated based on consistency-maximizing processes (see below). The aim of the present research was to test whether coherence shifts obtained in multi-attribute outcome-based decisions and complex legal cases generalize to simple decisions based on probabilistic inferences in different domains.

In particular, we investigated the stability of cue validities during (a) decision tasks on choosing a holiday region upon inspection of well known weather forecasts and (b) on selecting the larger city based on different cues (Gigerenzer & Goldstein, 1996). From a Parallel Constraint Satisfaction (PCS) approach (Glöckner & Betsch, 2008) we derive the assumption that even in simple probabilistic inference tasks, cue validities are changed to form a coherent representation of the decision. We aim to extend the perspective that consistency-maximizing processes are limited to complex decisions in which it is possible to construct stories (Pennington & Hastie, 1992; cf. Simon, 2004). The null hypothesis, derived from the above mentioned complex decision models and the fast-and-frugal heuristics approach, is that cue validities remain stable during the task. First, we provide a sketch of the connectionist PCS rule. Then computer simulations of the model are reported in which the predictions of coherence shifts in probabilistic inference decisions (and further predictions) are mathematically derived and their stability is investigated. Finally, three laboratory experiments are reported that test the hypothesis of coherence shifts.

A PCS Approach to Probabilistic Inferences

The model described below builds on a number of different approaches to decision making. Similar to dominance structuring models (e.g., Montgomery, 1989; Svenson, 1992), it draws on the assumption that (re)structuring of information is an integral part of the decision process. PCS is based on principles of perception (McClelland & Rumelhart, 1981; Read & Miller, 1998), which have been emphasized as an important framework for future research in judgment and decision making (Maule, 2005). Therefore it is a plausible mechanism from an
evolutionary perspective (cf. Gilbert, 1991), which might account for the automatic processes in decision making that have been highlighted by several authors (Dougherty, Gettys, & Ogden, 1999; Holyoak & Simon, 1999; Kahneman & Frederick, 2002; Montgomery, 1989; Simon, Krawczyk, & Holyoak, 2004; Thagard & Millgram, 1995; Weber, Goldstein, & Barlas, 1995).

Our PCS approach postulates that consistency maximizing processes, which can be simulated by connectionist networks, are the core operating processes of decision making (Glöckner & Betsch, 2008; cf. Simon, 2004). We argue that individuals use these Gestalt-like processes to form consistent interpretations of the task. Dependent on the structure of the task, advantages of one or the other interpretation (i.e., option) are automatically accentuated and enter awareness. The resulting (more or less conscious) mental representations are the basis for decisions. The option with the highest activation within this mental representation is chosen. In the case that the mental representation does not reach an aspired level of consistency, deliberate processes set in to support consistency maximizing in the network. In the following, for simplicity, we will focus only on the part of the model which describes the automatic consistency maximizing processes (i.e., the primary network; cf. Glöckner & Betsch, 2008) which is also described in length in Glöckner (2006, 2007).

In order to model simple probabilistic inference tasks, we adopted the general connectionist approach proposed for preferential and complex legal decision making. Thereby, the degrees of freedom in the model were reduced by specifying the structure of the network. As depicted in Figure 1, the first set of units represents options, the second set represents cues, and a special unit represents the general concept of validity of information. The links between options and cues reflect cue information (e.g., that cues speak for or against options). The links between validity of the information and cues reflect the initial subjective value of cue validities. Using a parallel constraint satisfaction algorithm (McClelland & Rumelhart, 1981; Read, Vanman, & Miller, 1997), activations of the nodes are changed to reach a consistent solution (activation pattern) that satisfies the constraints in the system (fixed links and weights). The final activation of the option nodes indicates the valence of the options, whereas the final activation of the cue nodes represents posterior subjective cue validities. The number of itera-

\[ a_i(t + 1) = a_i(t) \times (1 - \text{decay}) + \begin{cases} \text{input}_i < 0 & \text{input}_i \times (a_i(t) - \text{floor}) \\ \text{input}_i \geq 0 & \text{input}_i \times (\text{ceiling} - a_i(t)) \end{cases} \]

\[ \text{input}_i(t) = \sum_{j=1}^{a_{ij}} w_{ij} \times a_j(t) \]

\( a_i(t) \) represents the activation of the node \( i \) at iteration \( t \). The parameters \( \text{floor} \) and \( \text{ceiling} \) stand for the minimum and maximum possible activation (in our model set to a constant value of -1 and +1). \( \text{Input}_i(t) \) is the activation node \( i \) receives at iteration \( t \), which is computed by summing up all products of activations and connection weights \( w_{ij} \) for node \( i \). \( \text{Decay} \) is a constant decay parameter.
tions of the relaxation algorithm to form the coherent pattern of activations can be used as an estimate for the expected decision time for the model.2

Figure 1

Figure 1. The general model for probabilistic inferences is depicted. Boxes represent nodes; lines represent links, which are all bi-directional. Connection weights can range from -1 to +1 and are labeled w. Using the iterative updating algorithm coherence is produced in the network by changing activations a. The special node general validity has a constant activation of +1 and is used to supply the network with energy.

Simulation

Method

Simulations were computed on probabilistic inferences between two options based on three cues which differ in their validity. Specifically, we conducted a simulation of one critical decision task in which the most valid cue makes an opposite prediction to the two lower cues. The simulated decision task appears in the left upper corner of Figure 2a. The simulation was based on the network model presented in Figure 1 and the iterative updating algorithm described in Footnote 1. The initial validity of the most valid cue $w_{v1}$ was manipulated. Choice

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2 The proposed PCS model shares some structural similarities with recent formulation of decision field theory (Busemeyer & Townsend, 1993; Busemeyer & Johnson, 2004) and other evidence accumulation models (Usher & McClelland, 2001, 2004). Due to the assumption of unidirectional connections, however, these theories would not predict changes in subjective cue validities.
predictions (i.e., choices for option 1), changes of the subjective cue validity of cue 2 (i.e., $a_{C_2}$, the activation of cue 2), and the prediction for the decision time (i.e., the number of iterations to find a stable solution) were recorded as dependent variables.

**Fixed parameters.** PCS models have been simulated using different sets of parameters and it has been argued that findings are robust against changes of these (Thagard, 1989). We selected parameters roughly oriented on the initial work by McClelland and Rumelhart (1981).³

The links between cues and options stand for the cue values (i.e., the cues’ information about the options). Positive cue values are represented by excitatory links of $w = 0.01$, negative cue values are represented by inhibitory links of the same strength, $w = -0.01$. Thus, the critical decision task was represented by: $w_{C_1-O_1} = w_{C_2-O_2} = w_{C_3-O_2} = 0.01$ and $w_{C_1-O_2} = w_{C_2-O_1} = w_{C_3-O_1} = -0.01$. The fact that only one option can be chosen was implemented by a strong negative link between the option nodes, $w_{O_1-O_2} = -0.50$. The decay parameter was set to 0.05. The stability criterion used for terminating the process was 10 cycles with no energy changes bigger than $10^{-6}$.

Choices for option 1 are computed by comparing $a_{O_1}$ and $a_{O_2}$.

**Manipulation.** The initial validity of cue 1 was systematically manipulated in 60 steps by changing the initial cue validity parameter of cue 1 $w_{V_1}$ from 0.40 to 1.00. Specific numeric constellations can lead to unsystematic variation in termination of the PCS mechanism. To reduce these effects and to produce more stable results, for each level of cue 1 the initial cue validities for cues 2 and 3 were additionally manipulated for each level of $w_{V_1}$ in a limited interval. Specifically, for each $w_{V_1}$ the weight of the second cue $w_{V_2}$ was varied from 0.30 to 0.34 and the weight of cue 3 $w_{V_3}$ was varied from 0.10 to 0.14. These variations were fully crossed resulting in 25 data points for each level of $w_{V_1}$ which were aggregated into one value. Finally, to smoothen the curve data points were aggregated for each 5 consecutive levels of $w_{V_1}$. Thus, each data point represents the average of 125 simulation results.

**Results and Interpretation**

The results of the simulation for the critical decision task with different initial validities of cue 1 (x-axis) are depicted in Figures 2a to 2c. The graph of Figure 2a shows the proportion of choices for option 1. Option 1 choices are predicted if the final activation of option 1 is higher than the final activation of option 2. The graph shows an inflection point around which choices switch from option 2 to option 1. Conceptually, this can be interpreted as the point at which the validity of the highest cue is about equal to the validity of the concurring lower cues considering the constraints in the system. Parallel to the changes of choice proportion the posterior subjective validity of the second cue (i.e., activation of cue 2) is decreasing (Figure 3) McClelland and Rumelhart (1981) used link weights between 0.005 (weak) and 0.30 (strong). The decay was 0.08.
2b). The inflection point of the choice function falls together with the maximum of decision times (i.e., number of iterations $t$ to find a stable state; Figure 2c).

The reverse pattern – namely a decrease in the posterior subjective validity of the first cue – can be observed using a systematic variation of the initial cue validity of cue 2. More generally, from the PCS rule the specific hypothesis can be derived that cue validities are changed within the decision process according to the success of their predictions in the constraint satisfaction process (i.e., if the cue speaks for or against the winning option, Simon, 2004). This PCS rule thereby predicts that these changes are inherent in the structuring process that drives the decision and are not confined to post-decisional processes (cf. Festinger, 1964). As discussed above, the prediction of changing cue validities stands in clear contrast to the prediction of most complex decision models and fast-and-frugal heuristics.

![Figure 2a](image_url)

**Figure 2a.** Choice predictions from a simulation of the proposed connectionist model are depicted. The simulated decision situation appears in the left upper corner. Each data point represents the average of 125 simulation results. The fixed parameters of the simulation were $w_{c1-o1} = w_{c2-o2} = w_{c3-o2} = +0.01$, $w_{c1-o2} = w_{c2-o1} = w_{c3-o1} = -0.01$ and $w_{o1-o2} = -0.50$. The initial validity parameter of cue 1 $w_v$ was manipulated from 0.40 to 1.00 as denoted on the x-axis. For each level of cue 1 $w_v$ was varied from 0.10 to 0.14 and $w_{o1}$ was varied from 0.30 to 0.34 resulting in 25 crossed constellations. The decay parameter was set to 0.05. The stability criterion used for terminating the process was 10 cycles with no energy changes bigger than $10^{-6}$. Choices for option 1 are computed by comparing $a_{c1}$ and $a_{c2}$. 

**Figure 2b.** The graph shows the result of the simulation in Figure 2a concerning the changes in cue validity of cue 2. On the x-axis the variation of the initial validity of cue 1 is shown, on the y-axis the resulting subjective validity for cue 2 $a_{c2}$ is shown.

**Figure 2c.** The graph shows the result of the simulation in Figure 2a concerning the number of iterations of the PCS algorithm to find a stable solution which can be used as a predictor for decision times. On the x-axis the variation of the initial validity of cue 1 is shown, on the y-axis the number of iteration to find a stable solution is shown.
Tests for Robustness

Manipulation. Although we aimed to select parameters which are commonly used in simulations of parallel constraint satisfaction models, it is important to show that the results also hold for other sets of parameters. Eight further simulations were run to substantiate the results. (To make comparisons easier, the parameters used in the basic simulation reported above are marked by ‘*’.) First, we tested robustness against variations of the decay parameter \( D \) by using levels of 0.01, 0.05* and 0.10. Second, we varied the stability parameter \( S_t \) at three levels \( 10^{-3}, 10^{-6} \) and \( 10^{-8} \). Third, we changed the weight of the negative connection between the option nodes \( O_1 \) and \( O_2 \) \( (O_w) \) to be -0.40, -0.50* and -0.60. Finally, we tested robustness against different levels of weights for links between cues and options by setting all excitatory and inhibitory link weights \( (C_w) \) to \( 0.01/-0.01^*, 0.02/-0.02 \) and \( 0.10/-0.10 \).

Results and Interpretation. In all simulations it was observed that the essential findings concerning choices and changes in posterior cue validities remained robust against all variations of parameters (Figures 3a and 3b). The choice function was invariant against the parameter variations except for the strength of the negative weight between the option nodes \( (O_w) \). Manipulation of \( O_w \) led to a horizontal shift of the choice function and a change in steepness. However, and more importantly, the inflection point of the cue validity function was equally shifted and remained in line with the inflection point of the choice function. The finding that decision times reach a maximum at the joint inflection point of the choice and the cue validity function was less stable and could be replicated with a part of the parameter sets only (Figure 3c). There was, overall, a tendency that the maximum of decision times was shifted to the right with respect to the inflection point of the choice function. Furthermore, there were substantial changes in the shape of the function for the low decay level \( (D = 0.01) \), high cue links \( (C_w = 0.10/-0.10) \), and strongly negative option links \( (O_w = -0.60) \). This might be best explained by over-activation of nodes and might indicate that in order to derive useful decision time predictions, a good balance between activation and decay has to be found. However, decision time predictions of the PCS rule are not tested in this paper, and thus we refrain from discussing further possible reasons for these divergences in details here.
Figure 3a

Figure 3b
Figures 3a-c. The graphs show the result of the simulation in Figures 2a-c and the eight additional simulations with different sets of parameters for choices, subjective cue validity of cue 2 and decision time. The decay parameter \( D \), the weight of the links between cues and options \( C_w \), the weight of the links between the two options \( O_w \) and the stability criterion for terminating the consistency maximizing process \( S_t \) are manipulated.

Finally, inspection of the data on a non-aggregated level showed that the raw data concerning choices and cue validities were quite stable, whereas there was considerable noise in the decision time predictions.

The simulations show that a substantial decrease in the validity of the most valid cue can be expected if the other cues overrule this cue. Note, however, that substantial increases in the validity of the most valid cue would be predicted in the alternative case. Thus, aggregation over participants with heterogeneous choices could produce zero effects. This problem can be circumvented by selecting decision tasks with low differences between the cue validities (cue dispersion) and a high number of cues pointing against the most valid cue. According to the PCS rule, this should lead to more homogeneous decision behavior and, thus, should reduce the likelihood for zero effects. Therefore, in the experiments, decision tasks with two options were chosen, in which one highly valid cue made a prediction for one option, whereas three
other cues pointed towards the other option. Furthermore, only relatively valid cues were chosen to assure low cue dispersion.

In the first experiment, we aimed to test whether changes in cue validities also occur for cues for which participants have a huge number of learning experiences, namely for prominent weather forecasts. Students can hardly avoid recognizing (at least occasionally) weather forecasts in TV channels, newspapers and the internet and they certainly cannot avoid experiencing the weather on their way to university. Hence, for average German students a sufficient number of (at least implicit) learning trials can be expected for some of the prominent German weather forecasts used in the experiment.

**Experiment 1**

Participants had to decide between two places for a holiday based on weather forecasts (cues) from different prominent sources. The subjective validity of the cues was measured before and after the decision. Based on the results of the simulations, we predicted that cue validities are changed to form a consistent representation of the decision task. In detail, we hypothesized that the subjective validity of one highly valid cue conflicting with the majority of other valid cues should decrease and the validity of the other cues should increase. Additionally, by analyzing choice behavior we tested against the null hypothesis that individuals applied a TTB heuristic.

**Method**

**Participants and design.** Participants were 74 students (60 female, 14 male; mean age 21) from different majors at the University of Erfurt, who took part in a one-hour experimental battery and were rewarded with € 6.00. They were randomly assigned to two between-subjects conditions in which cue values were manipulated (cue values). Each participant was presented with four cues (cue) which were measured before and after the decision (time). This resulted in a 2 (cue values) x 4 (cue) x 2 (time) mixed model design with the two latter variables being within-subject factors.

**Procedure.** The experiment was entirely computer-directed and consisted of three parts. The complete instruction can be found in the appendix. In the first part, we measured subjective validities. Participants rated the validity of several well-known sources of weather forecasts on a horizontal scroll bar ranging from -100 (no trust at all) to +100 (trust absolutely). The sources, which will be called cue 1 to cue 4, were two TV stations, one newspaper, and one

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4 TTB was used as comparison standard for pragmatic reasons. Without this assumption no predictions concerning choices could be derived from the fast-and-frugal heuristics approach. Note, however, that all our conclusions concerning coherence shifts do not depend on this assumption.
Internet source. In the second part, lasting about 30 minutes, participants worked on a couple of unrelated tasks. In the third part, participants were to make decisions between two holiday regions A and B based on different 7-day-ahead weather forecasts. After the decision, we had participants rate the subjective validity of the sources of weather forecast using the same measure as in part 1.

The cue values were manipulated between participants to rule out that validity changes might depend on effects of repeated measurement only. In condition 1, cue 2 predicted rain for region A and sun for region B and all other cues predicted the reverse pattern. In condition 2, cue 1 predicted rain for region A and sun for region B and all other cues predicted the reverse pattern. Thus, according to the PCS model, in both conditions most people should choose region B. But whereas in condition 1 the validity of cue 2 should decrease, in condition 2 a decrease in the validity of cue 1 was expected. At the same time, in both conditions, all other cue validities should increase from pre- to post-test. In contrast, TTB predicts that participants should choose region A or B, depending on the prediction of the most valid cue only. As mentioned above, fast-and-frugal heuristics in general as well as complex decision models predict that cue validities should be stable.

Results

Analysis of the choices revealed that almost all of the participants chose region B (condition 1 = 90%, condition 2 = 100%) as predicted by the PCS rule (under the assumption that the three cues taken together are stronger than the one very valid but diverging cue). According to the cue validity rating in the pre-test, for 23 participants the cue predicting against the majority of cues was initially rated the most valid cue. Twenty-one of these participants (91.3%) chose region B although the most valid cue made a prediction for region A. For these participants it can be ruled out that they did apply TTB.

A 2 (cue values) x 4 (cue) x 2 (time) mixed-model analysis of variance (ANOVA) with time and cue as within-subjects factors and the validity ratings as dependent variables revealed significant main effects for cue, $F(2.3, 164.5) = 106.2, p < .001, \eta^2 = .60^6$; and for time, $F(1, 72) = 4.13, p < .05, \eta^2 = .05$. The main effect of cue was driven by considerable differences in the subjective validity between cues. The differences could be attributed to systematically different learning experiences concerning these natural cues. Means of subjective validities (with standard error in parentheses) for cues 1 to 4 were 53.4 (2.1), 73.6 (1.9), 27.4 (2.4), 71.9 (2.5). The main effect for time was caused by the higher general ratings in the post-test (Figure 4).

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5 The cues 1 to 4 were SAT1 (TV channel), ZDF (TV channel), BILD (newspaper) and “www.wetter.de.”

6 A Greenhouse Geisser correction was used because Mauchly’s test of sphericity turned out to be significant. The same correction was applied to all further repeated measurement analyses, if the assumption of sphericity was violated.
More interestingly and in accordance with our hypothesis, the three-way interaction of cue values, time and cue turned out to be highly significant, $F(2.7, 194.6) = 5.0, p < .01, \eta^2 = .07$. Inspection of Figure 4 shows that all changes in cue validity were in the predicted directions: in condition 1, the subjective validity was reduced for cue 2 and increased for all other cues, and in condition 2, a decrease for cue 1 and an increase for all other cues was observed.

**Discussion**

The results corroborate the assumption that cue validities are changed to form a consistent representation of a decision situation even for supposedly well learned real-world cues and in a context in which the construction of stories was impossible. Choice data provide converging evidence by showing that individuals did not use TTB. Choices indicate a compensatory information integration and are thus in line with the predictions of the PCS model. Hence, the findings provide evidence in favor of the connectionist approach to probabilistic inferences.
It might have been the case, however, that cue validities were not changed during the decision but after the choice was made. Dissonance theory predicts that many (but not all) decisions cause cognitive dissonance (Festinger, 1964). To achieve cognitive consistency, individuals tend to change the evaluation of alternatives after a decision which leads to a spreading-apart effect. Accordingly, the chosen alternative is rated more favorably, and the rejected alternatives are rated more negatively.

At the moment, we cannot rule out an alternative interpretation of the results in terms of post-decisional dissonance reduction (however, see Exp. 2). Nevertheless, even such an interpretation has some important theoretical implications. As a common denominator, dissonance theory and the connectionist approach share the assumption that individuals strive for consistency or coherence (see also Egan, Santos, & Bloom, 2007). Not surprisingly, parallel constraint satisfaction models have been used to model dissonance reduction (Shultz & Lepper, 1996). Most notably, the notion of cognitive consistency implies that our mind processes information in a holistic, interactive fashion. Information units mutually influence each other and thus cannot be considered in isolation. This view differs markedly from the fast-and-frugal approach. If individuals apply heuristics, their decisions often should not evoke cognitive dissonance because the simple strategies avoid conflict between pieces of information (which is a requisite for dissonance). Application of TTB, for example, requires only consideration of one cue. If a person considers cue 2 as the cue with highest validity and the forecast predicts fine weather in region A and bad weather in region B, then you can immediately select region A. The chosen option has only positive outcomes (fine weather) and the option not chosen only negative outcomes (bad weather). Under such conditions, dissonance theory predicts the absence of a spreading-apart effect, simply because no dissonance has arisen (Frey, 1981).

**Experiment 2**

To further substantiate the hypothesis that coherence shifts occur during the inference process, we conducted a second experiment. Materials and procedures were the same as in Experiment 1, with the exception that the choice measure was removed. We tested the prediction that people change cue validities upon perceiving and thinking about the decision situation even without committing themselves to a certain option (cf. Simon et al., 2001). Note that dissonance theory predicts the absence of dissonance reduction processes under these conditions: “We must accept the fact that dissonance-reduction processes do not automatically start when a decision is made… The decision must have the effect of committing the person.” (Festinger, 1964, p. 42).
Method

*Participants and design.* Sixty-five students of the University of Erfurt (5 male, 60 female; mean age 20 years) participated in the experiment, which was run as part of a one-hour experimental battery. Students received € 6.00 for their participation. No between-subject manipulation was used. Cue validities of four cues were again measured in a pre- and a post-test resulting in a 4 (cue) x 2 (time) within-subjects design.

*Procedure.* Procedure and materials were the same as in condition 1 of the previous study, with the exception that the actual decision was removed. First participants judged the subjective validity of the cues. After a 30-minute distraction phase they could inspect weather forecasts and were asked to think about a possible solution of the task. However, they were instructed not to make a final decision. Subsequently, participants were asked to judge the subjective validity of the cues again. The exact modifications of the instruction compared to Experiment 1 can again be found in the appendix.

Results and Discussion

A 4 (cue) x 2 (time) repeated measurement ANOVA with time and cue as within-subjects factors and the subjective cue validity as the dependent variable was used to analyze the data. The analysis revealed a significant interaction of cue and time, $F(1.9, 123.6) = 7.5, p = .001, \eta^2 = .11$. The significant main effect for cue was replicated, $F(2.2, 141.5) = 67.7, p < .001, \eta^2 = .51$. Inspection of Figure 5 shows that the results of the present and the previous experiment converge. As expected, the subjective validity of cue 2 was decreased and the validity of the other cues was increased by merely reflecting on the decision task. Thus, the second study shows that shifts in validities can occur even before a decision is made. It rules out alternative interpretations in terms of post-decisional dissonance reduction and provides further support for a connectionist approach to probabilistic inferences.
Figure 5. The results of Experiment 2 are presented using the same difference scores as in Figure 4. Error bars represent the SE for difference scores.

Against these results and in favor of the fast-and-frugal heuristics approach at least three arguments might be raised. First, it might be criticized that the environment used in Experiments 1 and 2 has not been shown to be an environment in which TTB should be applied (however, see Footnote 4). Other heuristics could have been used instead. Note, however, that all fast-and-frugal heuristics as well as the mentioned complex decision models predict ignorance of information and unidirectional reasoning from cues to options. Therefore, none of them can account for our findings. Second, it might be argued that our results have been observed in preference decisions (i.e., which region to select for a holiday) that are based on probabilistic inferences (i.e., likelihood of sunshine, given certain probability cues) instead of pure probabilistic inferences (e.g., city-size tasks; Gigerenzer & Goldstein, 1996). Glöckner (2006) has claimed that structural differences between probabilistic inferences and preference decisions (i.e., high correlation of cues vs. no correlation of attributes) are indeed likely to influence the selection of decision strategies. Thus, our findings should not be prematurely generalized, although it can be argued that many everyday problems (e.g., consumer decisions) have a problem structure similar to the ones used in our experiments.

As a third caveat, one might criticize our measurement method for the dependent variable cue validity. The rating of how much a person would trust information does not exactly correspond with the definition of cue validity used by Gigerenzer et al. (1991). According to Gig-
erenzer et al. (1991), cue validity is defined by the conditional likelihood that one option is better on a criterion given a positive cue value [e.g., $p$ (more sunshine | positive whether forecast)]. From a Brunswikian perspective (Brunswick, 1955), our definition refers to the cue usage by individuals (subjective cue validity), whereas the cue validity definition by Gigerenzer et al. (1991) alludes to the relation between cues and decision criterion in the environment (objective cue validity). Gigerenzer et al. (1999) argue that over time both should converge because individuals learn these relations in interaction with their environment. However, repeatedly reported sampling errors and the mere direction of sampling are likely to produce substantial misrepresentation (Fiedler et al., 2000; Fiedler, 2000). Empirical data indicate that misrepresentations of cue validities are indeed very common and not at all rare events (Glöckner, 2006). To evaluate descriptive models for decision making, it is more appropriate to measure cue utilization than estimations of the objective cue validity (e.g., by using ratings of the objective cue validity). The former aim to measure influences on choices directly, whereas the latter are cognitive interstage products which could affect decisions but have a less direct influence on choices. Nevertheless, according to the finding that coherence shifts also change background knowledge (Simon, 2004), an effect on estimations of objective cue validities can be expected as well.

We aimed to rule out the objections more fundamentally by conducting a third experiment. Thereby, we intended to replicate the results of Experiment 2 in city-size decision tasks which have been the drosophila for research on fast-and-frugal heuristics (Gigerenzer et al., 1999). In the city-size task individuals have to select the larger of two cities based on different cues (e.g., the city is / is not a state capital). It has been argued that individuals should apply TTB in such decisions because the strategy leads to very good decisions (Gigerenzer & Goldstein, 1996). Furthermore, cue validities were measured by subjective ratings of conditional likelihoods instead of ratings of cue usage (i.e., how much a person trusts information in decision tasks).

**Experiment 3**

**Method**

*Participants and design.* Sixty-three students of the University of Erfurt (8 male, 55 female; mean age 20.8 years) participated in the experiment, which was run as part of a 45-minute experimental battery. Students received € 5.00 for their participation. Four cue validities were measured in pre- and post-tests resulting in a 4 (cue) x 2 (time) repeated measurement design.

*Procedure.* In the experiment, individuals had to think about which of two cities is larger, based on the following cues: the city is or is not a state capital; the city has or does not have a university, an international airport and/or a first league soccer team (i.e., team in the 1. Bundesliga). As in the previous experiment, participants did not make any decision to avoid
post-decisional dissonance effects. First, the definition of cue validity based on conditional likelihood was explained to the participants. They were instructed to estimate the validity of the four cues on a scale from 0% to 100% using a horizontal scroll bar. After a filler-task of approximately 15 minutes, the participants were presented with structurally the same decision task as in the previous experiments. The most valid cue (state capital) pointed against all lower cues. Participants should imagine participating in a quiz show where they have to decide which of two cities is larger without exact knowledge about the cities’ populations. They were told that city A is a state capital but has no university, no international airport, and no first league soccer team, whereas city B is no state capital but has a university, an international airport, and a soccer team in the highest league. Participants were instructed not to make a decision yet, because the quizmaster would soon provide additional important information, but that they should try to understand the information set. Then the cue validity was measured using the same method as in the pre-test. Afterwards participants were informed that the quizmaster was not allowed to give the additional information, and that they had to decide without it. Finally, confidence in the decision was measured using a horizontal scroll bar ranging from -100 (very unconfident) to +100 (very confident). The complete instruction is provided in the appendix.

Results and Discussion

Exploratory analyses revealed that three participants repeatedly produced extreme outliers (+/- 3 SD) for the difference between cue validity ratings from pre- to post-test. These three persons were excluded from the analysis. Again, a repeated measurement ANOVA with time and cue as within-subjects factors and subjective cue validity as the dependent variable was computed to analyze the data. It indicated a significant interaction of cue and time, $F(2.7, 158.3) = 3.1, p = .03, \eta^2 = .05$. In line with the predictions of the PCS rule and the findings of experiments 1 and 2, the presentation of the decision task systematically influenced the cue validity ratings (Figure 6). The validity of the highly valid cue state capital was substantially decreased. In contrast to the previously reported studies we did not find a substantial increase in the validities of the other cues.
A significant main effect for cue was found, $F(2.7, 158.6) = 27.3, p < .001, \eta^2 = .32$. The average confidence ratings for the cues international airport, state capital, university, and soccer team (with $SE$ in parentheses) were 74.6 (2.85), 73.7 (2.54), 59.1 (2.59), and 48.5 (3.18). As intended by the selection of the cues, state capital was considered the most valid cue in the pre-test. In the post-test, however, the international airport was considered the most valid cue indicating that consistency maximizing processes do not only account for minor changes of validity ratings but can even lead to alterations in the ordinal cue hierarchy.

Analyses of the choices showed that the large majority of participants selected city B (76.7%) against the prediction of the initially most valid cue (i.e., state capital) and that only a minority selected city A (23.3%). This again provides converging evidence against the application of TTB.

Confidence judgments were analyzed by computing correlations between confidence and a weighted additive difference (WADD-DIFF) score which indicates the difference between the weighted evidence for the options. The WADD-DIFF score was calculated by subtracting the weighted sum of the total evidence for city B from the weighted total evidence for city A and taking the absolute value of the result. High (low) values indicate large (small) advantages of one city over the other. Two separate WADD-DIFF scores were computed based on the cue validity estimations in the pre- and the post-test respectively. For both WADD-DIFF scores
correlation coefficients were computed with the confidence judgment after the decision. According to the PCS rule (and other weighted compensatory models) it is expected that confidence should increase with increasing WADD-DIFF scores. For the pre-test scores, there was no significant correlation ($r = -.11, p > .05$), but for the post-test scores a highly significant correlation was found ($r = .63, p < .001$). In line with the predictions of the PCS rule, confidence decreases with increasing difference between the (weighted) evidence in favor of each city in the post-test scores. Remember that the consistency maximizing processes are assumed to accentuate the advantage of the favored option when perceiving the decision task. Confidence judgments are based on the posterior perception of the decision after coherence shifts occurred (i.e., the posterior cue validities). Thus, it lends additional support for the PCS rule that only the posterior WADD-DIFF score are correlated with confidence.

**General Discussion**

We studied the stability of cue validities during simple decisions based on probabilistic inferences. From the PCS approach we derived the assumption that cue validities are changed during the decision process to form a consistent representation of the decision situation. Findings from the three experiments using a within-subjects design strongly corroborate this hypothesis. The first two experiments show that such changes are observed in environments for which comprehensive learning experiences can be expected. The third experiment indicates that these coherence shifts also occur in environments for which it has been claimed that fast-and-frugal heuristics are applied. All three experiments illustrate that coherence shifts are not limited to contexts in which stories about the situation can be constructed. It was furthermore demonstrated that coherence shifts can be found for cue validity ratings based on direct estimations of cue usage as well as for estimations of conditional likelihoods.

The second and third experiment indicate that coherence shifts are initiated before a decision is made. Thus, shifts in validities cannot be attributed to post-decisional reduction of dissonance (Festinger, 1964; see also Simon & Holyoak, 2002). The findings converge with those obtained in recent studies on multi-attribute choice and legal decision making (e.g., Simon et al., 2004; Simon, Snow, & Read, 2004; Simon, 2004). They add into an accumulating body of evidence supporting the validity of a connectionist approach to judgment and decision making (Glöckner, 2006, 2007a, 2007b; Glöckner & Betsch, in press; Holyoak & Simon, 1999; Simon et al., 2004; Thagard & Millgram, 1995). The evidence is in line with findings from other recent studies on probabilistic inference decisions that show that the PCS rule predicts choices, decision times and confidence ratings generally better than fast-and-frugal heuristics and complex decision models (Glöckner, 2006; Glöckner & Betsch, in press; Glöckner & Betsch, 2008; but see Bröder & Gaissmaier, 2007).

Our results challenge the notion of unidirectional decision making – one of the implicit assumptions of most decision models discussed for probabilistic inferences. Fast-and-frugal
heuristics and most complex decision models involve a unidirectional decision process starting from the information as a given parameter and inferring the criterion from it. Processes of restructuring of cues are not part of both kinds of strategies. Our findings, however, suggest that individuals take the entire set of information into account and actively change predictors to form a consistent representation of the decision situation. As such, probabilistic inferences seem to involve holistic, bidirectional processes.

Against the background of the influential bounded rationality argument (Simon, 1955), the question arises how the mind can perform such complex computations that take even modern computers several seconds to solve. With the PCS model we suggest that people use automatic processes that have been evolutionarily evolved from more basic processes of perception. Not incidentally, the proposed PCS model (like most other current parallel constraint models) was developed on the basic algorithm proposed by McClelland and Rumelhart (1981) to account for word perception. As Maule (2005) pointed out, a central theme in future research on decision making will be the question of how individuals perceive and represent the given information in decision situations. Our research puts forward that the automatic system does a great deal of work in restructuring given information. Hence, the fundamental argument of Herbert Simon (1955, 1982) stating that people do not have the cognitive capacity to perform complex computations has to be qualified by adding “serially and deliberately.” However, reading Simon closely reveals that he already anticipated this possibility:

My first empirical proposition is that there is a complete lack of evidence that, in actual choice situations of any complexity, these [EU] computations can be, or are in fact, performed... but we cannot, of course, rule out the possibility that the unconscious is a better decision-maker than the conscious.

(Herbert Simon, 1955, p.104)
References


Appendix

Instructions Experiment 1 [Originally instructions were in German]

Pre-Test / Post-Test

For the following questions, please try to make as accurate judgments as possible.

Please estimate how much you would trust the pieces of information for predicting the weather in a holiday destination 7 days from now. Please try to provide as accurate ratings as possible.

- Weather forecast of SAT1
- Weather forecast of ZDF
- Weather forecast of the Bild Zeitung
- Weather forecast in the internet at www.wetter.de

Presentation of the decision situation

Please read the description of the situation and make a decision!

Please imagine you are planning a holiday and you consider the places A and B. It is very important for you that the sun is shining during your holiday. Therefore, you extensively collect information of different weather forecasts. The holiday starts in 7 days and today is the last possibility to book a holiday flat. The flats which are offered in places A and B are of equal standard from your perspective. However, the places are far away from each other and the weather could be different in the two places.

The following information is available to you:

The SAT1 weather forecast predicts rain for place A and sunshine for place B.

You see that the ZDF weather forecast predicts sunshine for place A and rain for place B.

You buy a Bild newspaper and find the following predictions: sunshine in place A, rain in place B.

In the internet you find at www.wetter.de the forecast that there will be sun in place A and rain in place B.

Which holiday flat would you book? Place A Place B
Modifications to Instructions Experiment 2

Presentation of the decision situation

Please read the description of the situation thoroughly!

[Middle part equivalent to Experiment 1]

Please take the time to reflect for which place you would decide, based on the available information!

Ready

Instructions Experiment 3

In the following study you should estimate how much information a certain attribute provides for the number of inhabitants of a city. Please try to make accurate and differentiated judgments.

Pre-Test / Post-Test

The predictive power of an attribute for the size of a city is called VALIDITY. The validity of an attribute is defined as the likelihood that a city which has this attribute is larger than a city that does not have the attribute. Such an attribute could be that a city has or does not have a cathedral. A city with a cathedral will tend to be larger than a city without a cathedral. The validity of the attribute (i.e., the likelihood that a city with a cathedral is larger than a city without a cathedral) will be larger than 50%. Please estimate the validity of 4 attributes. [page break]

Please estimate as accurately and differentiated as possible the validity (predictive power) of the following properties for the population of German cities with more than 100,000 inhabitants. Reminder: The validity of an attribute is defined as the likelihood that a city with the attribute is larger than a city without the attribute.

Presentation of the decision situation

Please imagine you have reached the 1-million-dollar question in a quiz show. The question is: Which city has more inhabitants: city A or city B? Unfortunately, you do not know the exact population numbers. You know that A and B are German cities with more than 100,000 inhabitants. Furthermore you remember the following attributes:

City A is a state capital          City B is no state capital
City A has no university          City B has a university
City A has no international airport  City B has an international airport

City A has no first league soccer team  City B has a first league soccer team

Please do not decide yet because the quiz master has indicated that he will provide important additional advice. Try to understand the information set as well as possible.