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Max Planck Institute for Research on Collective Goods, Kurt-Schumacher-Str. 10, D-53113 Bonn
http://www.coll.mpg.de

Competitive Equilibrium in Markets for Votes[1]

Alessandra Casella[2] **Aniol Llorente-Saguer**[3]
Thomas R. Palfrey[4]

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[2]Columbia University, NBER and CEPR, ac186@columbia.edu
[3]Max Planck Institute for Research on Collective Goods, llorente@coll.mpg.de
[4]Caltech, trp@hss.caltech.edu

Abstract

We develop a competitive equilibrium theory of a market for votes. Before voting on a binary issue, individuals may buy and sell their votes with each other. We define the concept of *ex ante vote-trading equilibrium*, and show by construction that an equilibrium exists. The equilibrium we characterize *always* results in dictatorship if there is any trade, and the market for votes generates welfare losses, relative to simple majority voting, if the committee is large enough or the distribution of values not very skewed. We test the theoretical implications by implementing a competitive vote market in the laboratory using a continuous open-book multi-unit double auction.

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1 Introduction

When confronted with the choice between two alternatives, groups, committees, and legislatures typically rely on majority rule. They do so for good reasons: as shown by May (1952), in binary choices, majority rule is the unique anonymous, neutral and monotonic rule. In addition, majority rule creates incentives for sincere voting: in environments with private values, it does so regardless of the information that voters have about others' preferences or voting strategies. A long tradition in political theory analyzes conditions under which majority voting over binary choices yields optimal public decisions or has other desirable properties.[1]

It has long been realized, however, that majority rule has an obvious weakness: it fails to reflect intensity of preferences, and an almost indifferent majority will always prevail over an intense minority. Political scientists and economists have conjectured that a solution could come from letting votes be freely traded, as if they were commodities. Just as markets allocate goods in a way that reflects preferences, vote markets may allow voters who care more about the decision to buy more votes (and hence more influence), compensating other voters with money transfers (see, e.g., Buchanan and Tullock, 1962, Coleman, 1966, Haefele, 1971, Mueller, 1973, Philipson and Snyder, 1996, Parisi, 2003). Whether this could lead to preferable outcomes remains debated. Even ignoring other critiques on distributional and philosophical grounds, scholars have recognized that in the absence of full Coasian bargains vote trading imposes externalities on third parties. Riker and Brams (1973), for instance, present examples in which exchanges of votes across issues are profitable to the pair of voters involved, and yet the committee obtains a Pareto inferior outcome. McKelvey and Ordeshook (1980) test the hypothesis in experimental data and conclude that the examples are not just theoretical curiosities, but can actually be observed in the laboratory.

To this date, there is no theoretical work that clearly identifies when we should expect vote trading inefficiencies to arise in general, and when instead more positive results might emerge. There is no general model of decentralized trade in vote markets, and central questions about equilibrium allocations when voters can exchange votes with each other remain unanswered.

This article seeks to answer, in the context of a relatively simple environment, two fun-

[1] Condorcet (1787) remains the classic reference. Among modern formal approaches, see for example Austen-Smith and Banks (1996), Ledyard and Palfrey (2002) or Dasgupta and Maskin (2008).

damental questions about vote trading in committees operating under majority rule. First, from a positive standpoint, what allocations and outcomes will arise in a competitive equilibrium where votes can be freely exchanged for a numeraire commodity? Second, what are the welfare implications of these equilibrium outcomes, compared to a purely democratic majority rule institution where buying and selling of votes is not possible?[2]

To answer these questions, we develop a competitive equilibrium model of vote markets where members of a committee buy and sell votes among themselves in exchange for money. The committee decides on a binary issue in two stages. In the first stage, members participate in a perfectly competitive vote market; in the second stage, all members cast their vote(s) for their favorite alternative, and the committee decision is taken by majority rule.

A market for votes has several characteristics that distinguish it from neoclassical environments. First, the commodities being traded (votes) are indivisible. Second, these commodities have no intrinsic value. Third, because the votes held by one voter can affect the payoffs to other voters, vote markets bear some similarity to markets for commodities with externalities: demands are interdependent, and an agent's own demand is a function of not only the price, but also the demands of other voters. Fourth, payoffs are discontinuous at the points in which majority changes, and at this point many voters may be pivotal simultaneously.

These distinctive properties create a major theoretical obstacle to understanding vote trading. In the standard competitive model of exchange, equilibrium, as well as other standard concepts such as the core, typically fails to exist (Park, 1967; Kadane, 1972; Bernholtz, 1973, 1974; Ferejohn, 1974; Schwartz 1977, 1981; Shubik and Van der Heyden, 1978; Weiss, 1988, Philipson and Snyder, 1996, Piketty, 1994). The following simple example illustrates both the potential inefficiency of majority rule and the problem of nonexistence of equilibrium. Suppose the two alternatives are M and E, and there are three voters, Maud and Mary who prefer M over E, and Eve who prefers E over M. The three voters have different intensities of preferences, which we represent in terms of valuations or willingness to pay. Maud's valuations for M and E are 10 and 0, respectively; Mary's valuations for M and E

[2] These issues apply more broadly than just to committee decision making and the political process. For example, corporations make key decisions by shareholder votes. These votes can be exchanged in open competitive asset markets. The connections between shareholder voting and the trading of voting shares in competitive asset markets is emerging as an important issue in the the study of corporate governance and corporate control. See for example, Demichelis and Ritzberger (2007) and Dhillon and Rossetto (2011) and the references they cite.

are 12 and 0, respectively; Eve's valuations for M and E are 0 and 30, respectively. Thus majority rule leads to decision M, but from a utilitarian point of view E is the efficient decision.

If it were possible to exchange votes for money, then vote trading could in principle lead to a Pareto improvement. For example, Eve could buy both of the other votes for a price of 13 for each vote. However, as pointed out at least as early as Ferejohn (1974) (or, more recently, Philipson and Snyder, 1996), nonexistence of a market clearing equilibrium price is a serious and robust problem. At any positive price, Eve demands at most one vote: any positive price supporting a vote allocation where either side has more than two votes cannot be an equilibrium; one vote is redundant and so at any positive price there is excess supply. Eve could buy Maud's vote at a price of 11. But again the market will not clear: Mary's vote is worth nothing and Mary would be willing to sell it for less than 11. In fact, any positive price supporting Eve's purchase of one vote cannot be an equilibrium: the losing vote is worthless and would be put up for sale at any positive price. But a price of zero cannot be an equilibrium either: at zero price, Eve always demands a vote, and there is excess demand. Finally, any positive price supporting no trade cannot be an equilibrium: if the price is at least as high as Eve's high valuation, both Maud and Mary prefer to sell, and again there is excess supply; if the price is lower than Eve's valuation, Eve prefers to buy and there is excess demand.[3]

Other researchers have conjectured, plausibly, that nonexistence arises in this example because the direction of preferences is known, and hence losing votes are easily identified and worthless (Piketty, 1994). According to this view, the problem should not occur if voters are uncertain about others voters' preferences. But in fact nonexistence is still a problem. In our example, suppose that Eve, Maud, and Mary each know their own preferences but do not know the preferences of the other two: they know only that the other two are equally likely to prefer either alternative. A positive price supporting an allocation of votes such that all votes are concentrated in the hands of one of them still cannot be an equilibrium: as in the discussion above, one vote is redundant, and the voter would prefer to sell it. But any positive price supporting an allocation where one individual holds two votes cannot be an equilibrium either: that individual holds the majority of votes and thus dictates the outcome;

[3] Philipson and Snyder (1996) and Koford (1982) circumvent the problem of nonexistence by formulating models with *centralized* markets and a market-maker. Both papers argue that vote markets are generally beneficial.

the remaining vote is worthless and would be put up for sale. Finally, a price supporting an allocation where Eve, Maud, and Mary each hold one vote cannot be an equilibrium. By buying an extra vote, each of them can increase the probability of obtaining the desired alternative from 3/4 (the probability that at least one of the other two agrees with her) to 1; by selling their vote, each decreases such a probability from 3/4 to 1/2 (the probability that the 2-vote individual agrees). Recall that Eve's preferences are the most intense, and Maud's the weakest. If the price is lower than 1/4 Eve's high valuation, Eve prefers to buy, and if the price is higher than 1/4 Maud's low valuation, Maud prefers to sell. The result must be either excess demand, or excess supply, or both, in all cases an imbalance. These examples are robust: nonexistence is a serious problem for the standard competitive equilibrium approach to vote markets.

In this paper, we respond to these challenges by modifying the standard competitive model in a natural way. Specifically, we propose a notion of equilibrium that we call *Ex Ante Competitive Equilibrium*. As in the competitive equilibrium of an exchange economy with externalities, we require voters to form demands taking as fixed the equilibrium price and other voters' demands. To solve the non-convexity problem, we allow for mixed –i.e., probabilistic– demands.[4] This introduces the possibility that markets will not clear exactly. Thus, instead of requiring that supply equals demand in equilibrium with probability one, in the ex ante competitive equilibrium we require market clearing *in expectation*. Ex post the market is cleared through a rationing rule. The central point is that deviations from market clearing, although possible ex post, cannot be systematic: they have zero mean and are unpredictable. It is a minimum requirement that still ties the price and the allocations to the powerful discipline of market equilibrium, while allowing the probabilistic demands that overcome the existence problem.[5]

Returning to the Maud, Mary, and Eve example with privately known direction of preference illustrates how our approach works. In that example, there is a unique ex ante

[4]Making votes continously divisible does not solve the non-convexity problem because there always remains a discontinuity at the point where one trader holds half the votes.

[5]Kultti and Salonen (2005) also propose overcoming equilibrium existence problems in a market for votes by allowing for mixed demands, but without imposing any notion of market balance. In their model, as in ours, the number of voters is finite, there is aggregate uncertainty, and the market does not clear ex post. Other general equilibrium models have used mixed demands, for example Prescott and Townsend (1982). However, in Prescott and Townsend markets clear exactly, even with mixed demands, because there is a continuum of agents and no aggregate uncertainty. Equilibria with mixed strategies and rationing are common in the literature on Bertrand competition with capacity constraints (for example, Gertner, 1985, Maskin, 1986).

competitive equilibrium with trade: Maud supplies her one vote, Eve demands one vote, and Mary mixes 50/50 between supplying one vote and demanding one vote. The equilibrium price is 3.[6] Expected supply and expected demand are both equal to 3/2. Realized demand however is stochastic. With probability 1/2, Maud and Mary both supply one vote and Eve demands one vote, and with probability 1/2 Mary and Eve both demand one vote, while Maud supplies one vote. With an anonymous rationing rule either Maud or Mary sells a vote to Eve in the first case, and either Mary or Eve buys a vote from Maud in the second case. It is straightforward to verify that with anonymous rationing all voters are optimizing.

In our model individuals have privately known preferences, and we prove, by construction, the existence of an ex ante competitive equilibrium. The equilibrium we characterize has a striking property: as in the example, whenever there is trade, there is a dictator: i.e., when the market closes, one voter owns $(n+1)/2$ votes. For any committee size, the dictator must be one of the two individuals with most intense preferences. When the number of voters is small, and the discrepancy between the highest valuations and all others is large, the market for votes may increase expected welfare. But if the discrepancy is not large, expected welfare may fall. One can see this clearly in the example: the equilibrium does not change if we lower Eve's valuation from 30 to, say 15, at which point the market outcome is inefficient. The condition for efficiency gains becomes increasingly restrictive as the number of voters rises. Indeed, if valuations are independent of the direction of preferences, we prove that the market must be *less* efficient than simple majority voting without a vote market (i.e., no-trade), if the number of voters is sufficiently large.

We test our theoretical results in an experiment by implementing the market in a laboratory with a continuous open-book multi-unit double auction. For several decades there have been hundreds of experiments on competitive markets and many experiments on voting in committees. The experiment we conduct is the first study we are aware of that brings together these disparate strands of the experimental literature. Because vote markets are so different from standard markets, this required some innovations to the standard computerized market trading environment.

The transaction prices we observe are higher than the Ex Ante Competitive Equilibrium prices, but fall with experience and converge to values that for most treatments are consistent

[6] For this example, the equilibrium strategies are the same if direction of preference is common knowledge, but the equilibrium price is 6 insteaed of 3. Generally both the equilibrium price and the equilibrium strategies will be more complicated when the direction of preference is common knowledge. See Casella et al. (2012) for further discussion and examples.

with our theoretical predictions in the presence of some risk-aversion. The overpricing is more noticeable in larger markets, possibly mirroring the more complex environment subjects face, and the difficulty in acquiring a large enough number of votes to hold a majority stake in a relatively short period of trading time. The frequency of dictatorship is lower than predicted, but increases significantly with experience, reaching 80 percent in small markets and late trials. Remarkably, after the long years of disagreement in the literature, the empirical efficiency of our laboratory markets tracks the theory very closely.

Two other strands of literature are not directly related to the present article, but should be mentioned. First, there is the important but different literature on vote markets where candidates or lobbies buy voters' or legislators' votes: for example, Myerson (1993), Groseclose and Snyder (1996), Dal Bò (2007), Dekel, Jackson and Wolinsky (2008) and (2009). These papers differ from the problem we study because in our case vote trading happens *within* the committee (or the electorate). The individuals buying votes are members, not external traders, groups or parties. Second, vote markets are not the only remedy advocated for majority rule's failure to recognize intensity of preferences in binary decisions. The mechanism design literature has proposed mechanisms with side payments, building on Groves-Clarke taxes (e.g., d'Apremont and Gerard-Varet 1979). However, these mechanisms have problems with bankruptcy, individual rationality, and/or budget balance (Green and Laffont 1980, Mailath and Postlewaite 1990). A more recent literature has suggested alternative voting rules without transfers. Casella (2005), Jackson and Sonnenschein (2007) and Hortala-Vallve (forthcoming) propose mechanisms whereby agents can effectively reflect their relative intensities and improve over majority rule, by linking decisions across issues. Casella, Gelman and Palfrey (2007), Casella, Palfrey and Riezman (2008), Engelmann and Grimm (forthcoming), and Hortala-Vallve and Llorente-Saguer (2010) test the performance of these mechanisms experimentally and find that efficiency levels are very close to theoretical equilibrium predictions, even in the presence of some deviations from theoretical equilibrium strategies.

The rest of the paper is organized as follows. Section 2.1 defines the basic setup of our model. Section 2.2 introduces the notion of Ex Ante Competitive Equilibrium and presents the rationing rule. Section 2.3 shows existence by characterizing an equilibrium. Section 2.4 compares welfare obtained in equilibrium to simple majority rule without trade. We then turn to the experimental part. Section 3 describes the design of the experiment, and section 4 describes the experimental results. Section 5 concludes, and the Appendices contain detailed proofs and the experimental instructions.

2 The Model

2.1 Setup

Because we define a new equilibrium concept, we present it in a general setting. Some of the parameters will be specialized in our analysis and in the experiment. Consider a committee of N voters, $N = \{3, 5, ..., n\}$, n odd, deciding on a single binary issue through a two-stage procedure. Each voter i is endowed with an amount m_i of the numeraire, and with $w_i \in \mathbb{Z}$ indivisible votes, where $\mathbb{Z}$ is the set of integers. Both $m = (m_1, ..., m_n)$ and $w = (w_1, ..., w_n)$ are common knowledge. In the first stage, voters can buy votes from each other using the numeraire; in the second stage, voters cast their vote(s), if any, for one of the two alternatives, and a committee decision, C, is taken according to the majority of votes cast. Ties are resolved by a coin flip. The model of exchange focuses on the first stage and we simply assume that in the second stage voters vote for their favorite alternative.[7]

The two alternatives are denoted by $A = \{\alpha, \beta\}$, and voter i's favorite alternative, $a_i \in A$, is privately known. For each i the probability that $a_i = \alpha$ is equal to η_i and $\eta = (\eta_1, ..., \eta_n)$ is common knowledge. Let $S_i = \{s \in \mathbb{Z} \geq -w_i\}$ be the set of possible demands of each agent.[8] That is, agent i can offer to sell some or all of his votes, do nothing, or demand any positive amount of votes. The set of actions of voter i is the set of probability measures on S_i, denoted Σ_i. We write $S = S_1 \times ... \times S_n$ and let $\Sigma = \Sigma_1 \times ... \times \Sigma_n$. Elements of Σ are of the form $q_\sigma : S \to \mathbb{R}$ where $\sum_{s \in S} q_\sigma(s) = 1$ and $q_\sigma(s) \geq 0$ for all $s \in S$.

We allow for an equilibrium in mixed strategies where, ex post, the aggregate amounts of votes demanded and of votes offered need not coincide. A *rationing rule* R maps the profile of voters' demands to a feasible allocation of votes. We denote the set of feasible vote allocations by $X = \left\{ x \in \mathbb{Z}_+^n \middle| \sum x_i = \sum w_i \right\}$. Formally, a rationing rule R is a function from realized demand profiles to the set of probability distributions over vote allocations: $R : S \to \Delta X$ where $x_i \in [\min(w_i, w_i + s_i), \max(w_i, w_i + s_i)]\ \forall i$ and $R(s) = w + s$ if $\sum s_i = 0$. Hence, a rationing rule must fulfill several conditions: a) R cannot assign less (more) votes than the initial endowment if the demand is positive (negative), b) R cannot assign more

[7] Equivalently, we could model a two stage game and focus on weakly undominated strategies.

[8] Negative demands correspond to supply.

(less) votes than the initial endowment plus the demand if the demand is positive (negative) and c) if aggregate demand and aggregate supply of votes coincide, then all agents' demands are satisfied.

The particular (mixed) action profile, $\sigma \in \Sigma$, and the rationing rule, R, jointly imply a probability distribution over the set of final vote allocations that we denote as $r_{\sigma,R}(x)$. In addition, for every possible allocation we define the probability that the committee decision coincides with voter i's favorite alternative, a probability we denote by $\varphi_{x,a_i,\eta} := \Pr(C = a_i | x, \eta)$ - where $x \in X$ is the vote allocation and $a \in A$.

Finally, we define voters' preferences. The preferences of voter i are represented by a von Neumann Morgenstern utility function u_i, a concave function of the argument $v_i 1_{C=a_i} + m_i - (x_i - w_i) p$, where $v_i \in [\underline{v}, \overline{v}]$ ($\underline{v} \geq 0$, $\overline{v}$ finite) is a privately known valuation earned if the committee decision C coincides with the voter's preferred alternative a_i, 1_x is the indicator function, m_i is i's endowment of the numeraire, $(x_i - w_i)$ is i's net demand for votes, and p is the transaction price per vote.

We can now define $U_i(\sigma, R, p)$, the ex ante utility of voter i given some action profile, the rationing rule, and a vote price p:

$$U_i(\sigma, R, p) = \sum_{x \in X} r_{\sigma,\Re}(x) \begin{bmatrix} \varphi_{x,a_i,\eta} \cdot u_i(v_i + m_i - (x_i - w_i) p) \\ +(1 - \varphi_{x,a_i,\eta}) \cdot u_i(m_i - (x_i - w_i) p) \end{bmatrix}$$

One can see in the formula that the uncertainty about the final outcome depends on three factors: a) the action profile, b) the rationing rule, and c) the preferences of other voters.

2.2 Ex Ante Competitive Equilibrium

Definition 1 *The set of actions σ^* and the price p^* constitute an* ***Ex Ante Competitive Equilibrium*** *relative to rationing rule R if the following conditions are satisfied:*

1. *Utility maximization: For each agent i, σ_i^* satisfies*

$$\sigma_i^* \in \underset{\sigma_i \in \Sigma_i}{\arg Max} U_i\left(\sigma_i, \sigma_{-i}^*, R, p^*\right)$$

2. Expected market clearing: In expectation, the market clears, i.e.,

$$\sum_{s \in S} q_{\sigma^*}(s) \sum_{i=1}^{n} s_i = 0$$

The definition of the equilibrium shares some features of competitive equilibrium with externalities (e.g., Arrow and Hahn, 1971, pp. 132-6). Optimal demands are interrelated, and thus equilibrium requires voters to best reply to the demands of other voters. In contrast, the standard notion of competitive equilibrium for good markets requires agents to best-respond only to the price. The difference between the Ex Ante Competitive Equilibrium and the competitive equilibrium with externalities is that the former notion requires market clearing only in expected terms. Thus ex post market imbalances are possible in equilibrium, but Ex Ante Competitive Equilibrium requires that such imbalances not be systematic and predictable. This is the important qualification. In the spirit of rational expectations equilibria, deviations from market clearing–"errors"–are realized ex post with positive probability but their size and direction cannot be predicted. It is a minimal requirement that still preserves the notion of market equilibrium as market clearing.

The fact that demand and supply do not necessarily balance is the reason for the rationing rule. In general, the specification of the rationing rule can affect the existence (or not) of the equilibrium, and if an equilibrium exists, its properties.

One interpretation of the rationing rule is as the representation of the ex post market-clearing process in the competitive equilibrium. In the spirit of Green (1980)'s important contribution to the theory of effective demand, trade is highly decentralized, search incomplete, and rationing stochastic. Unsatisfied demand and supply may coexist, but zero out in expectation.[9] In its full specification, the rationing rule would then be derived endogenously, a function of the random matching of the individuals in the market. A simpler take is to specify a rationing rule, as we do here, interpreting it either as a reduced form of such decentralized process, or as an exogenous institution. For most of this paper, we focus on one specific anonymous rule, rationing-by-voter ($R1$), according to which each voter either fulfills his demand (supply) completely or is excluded from trade. After voters submit their orders, demanders and suppliers of votes are randomly ranked in a list, with all rankings having the same probability. Then demands are satisfied in turn: the demand of the first voter on the

[9] The literature mentioned earlier on oligopolistic markets with capacity constraints interprets rationing similarly .

list is satisfied with the first supplier(s) on the list; then the demand of the second voter on the list is satisfied with the first supplier(s) on the list with offers still outstanding, and so on. In case someone's demand cannot be satisfied, the voter is left with his initial endowment, and the process goes on with the next of the list. $R1$ is reminiscent of All-or-Nothing (AON) orders used in securities trading: the order is executed at the specified price only if it can be executed in full. AON orders are used when the value of the order depends on it being executed in full: traders want to ensure that they will not be saddled with partially filled orders of little value. It is this feature that makes $R1$ particularly well-suited to a market for votes.[10]

2.3 Equilibrium

For the rest of the paper we assume that each voter prefers either alternative with probability 1/2 ($\eta_i = 0.5$) and is initially endowed with one vote ($w_i = 1$). In addition, we assume for now that all individuals are risk-neutral, and normalize the initial endowment of money to zero ($m_i = 0$ for all i), allowing for negative consumption of the numeraire. The value of m_i plays no role with risk-neutrality, and thus the restriction here is with no loss of generality. We will return later to the assumption of risk-neutrality.

In this section we prove by construction the existence of an Ex Ante Competitive Equilibrium when the rationing rule is $R1$.

Theorem 1 *Suppose $\eta_i = \frac{1}{2}$, $w_i = 1$, and $m_i = 0$ $\forall i$, agents are risk neutral, and $R1$ is the rationing rule. Voters are ordered according to increasing valuation: $v_1 < v_2 < ... < v_n$. Then for all n and $\{v_1, ..., v_n\}$ there exists an Ex Ante Competitive Equilibrium with positive trade. In equilibrium, voters 1 to $n-2$ offer to sell their vote with probability 1; voters $n-1$ and n demand $\frac{n-1}{2}$ votes with probabilities γ_{n-1} and γ_n respectively, and offer their vote otherwise. The probabilities γ_{n-1} and γ_n and the equilibrium price p depend on n and the realization of $\{v_1, ..., v_n\}$, but for all n and $\{v_1, ..., v_n\}$, p is always such that voter $n-1$ is just indifferent between selling his vote and demanding a majority of votes.*

Proof. Follows from Lemma 1, below. ■

[10] See for example the description of AON orders by the New York Stock Exchange http://www.nyse.com/futuresoptions/nysearcaoptions/. As a rationing rule, $R1$ fulfills the conditions given in section 2.1 but by construction makes it possible for both sides of the market to be rationed.

The theorem establishes two results that are at the center of our contribution. The first is the existence of an Ex Ante Competitive Equilibrium with positive trade. Relaxing market balance by allowing it to hold in expectation reestablishes the existence of an equilibrium, even in the presence of all the anomalies that characterize a market for votes. Our equilibrium notion thus has some bite: it suggests precise, testable predictions that we will be able to confront to the experimental data.

Equally important, the Ex Ante Competitive Equilibrium has properties that are economically intuitive. Because votes are only valuable when they make their owner pivotal, the market can be loosely thought of as analogous to auctioning off the right to be dictator on the issue. Thus it is not surprising that the equilibrium price is precisely the price at which the second highest valuation voter is indifferent between "winning" the dictatorship or selling his vote and letting someone else be dictator. Allowing for expected market balance not only reestablishes the existence of an equilibrium with trade, but leads to equilibrium strategies and a price that reflect the special nature of a market for votes.

While Theorem 1 establishes the existence of a nontrivial equilibrium for any n and any profile of distinct valuations, two qualifications are in order. First, the theorem applies relative to rationing rule $R1$. As in other equilibrium existence results (for example, Simon and Zame 2000), the fine details of the rationing rule can be important. While $R1$ seems natural and intuitive, we do not have an existence result or a characterization for arbitrary rationing rules. We do know, however, that Theorem 1 has some degree of robustness to the rationing rule. We have investigated an alternative anonymous rule, $R2$ or rationing-by-vote, where each vote supplied is allocated with equal probability to each voter with outstanding unsatisfied demand. We show in Appendix II that under reasonable conditions (satisfied in our laboratory markets), if the number of voters is not too large then an Ex Ante Competitive Equilibrium exists with $R2$. Moreover, the equilibrium we characterize is very similar under $R2$ and $R1$: the price is such that the second highest value voter is indifferent between demanding a majority of votes or selling his vote; all voters with lower values offer their vote for sale, and the highest value voter demands a majority of votes. Thus, the equilibrium competition for dictatorship does not depend on $R1$.

Second, trivial equilibria exist in which there is no trade: all voters demand zero, and all are indifferent over all trading strategies, given the zero-trade strategies of the other voters. A more interesting possibility is that there may be other equilibria with positive trade. Although we have not found a counter-example, we do not have a general result about uniqueness of positive-trade equilibria. We know, and prove in Appendix III, that the equilibrium characterized in Theorem 1 is unique for any profile of valuations when $n = 3$.

To discuss Theorem 1 further, we need to characterize it more precisely.

Lemma 1 *For all n, there exists a finite threshold $\mu_n \geq 1$ such that if $v_n > \mu_n v_{n-1}$, then $\gamma_{n-1} = (n-1)/(n+1)$, $\gamma_n = 1$, and $p = v_{n-1}/(n+1)$. If $v_n \leq \mu_n v_{n-1}$, then γ_{n-1}, γ_n, and p are the solutions to the system:*

$$\gamma_n = \frac{2n}{n+1} - \gamma_{n-1} \tag{1}$$

$$p = \frac{2 - 4\phi - (1-4\phi)\,\gamma_{n-1}}{2\,(n-1) - (n-3)\,\gamma_{n-1}} v_n \tag{2}$$

$$p = \frac{2 - 4\phi - (1-4\phi)\,\gamma_n}{2\,(n-1) - (n-3)\,\gamma_n} v_{n-1} \tag{3}$$

$$\gamma_{n-1} \in \left[\frac{n-1}{n+1}, \frac{n}{n+1}\right], \quad \gamma_n \in \left[\frac{n}{n+1}, 1\right] \tag{4}$$

where $\phi = \binom{n-1}{\frac{n-1}{2}} 2^{-n}$.

Proof. In Appendix I. ■

We do not reproduce here the explicit equations for γ_{n-1}, γ_n, and p because they are not particularly transparent. They are in the Appendix, together with the derivation of the threshold $\mu_n = (n-1)(n+5)\left[(n+1)\left(n+3-\binom{n-1}{\frac{n-1}{2}}2^{-(n-3)}\right)\right]^{-1}$. But the logic of the equilibrium is quite clear. Positive demand for votes comes only from the two voters at the upper end of the valuations distribution. The equilibrium price, together with the two voters' randomization probabilities, depends exclusively on n and on the realization of v_n and v_{n-1}. If v_n is sufficiently large, relative to v_{n-1}, the price that leaves voter $n-1$ indifferent between offering his vote for sale and demanding a majority of the votes is low enough to guarantee that voter n strictly prefers demanding a majority of the votes himself, rather than selling. In such a case, voter $n-1$ is the only one randomizing: voter n demands a majority of votes with probability 1 ($\gamma_n = 1$), and all others offer their vote for sale. The price is simply $p = v_{n-1}/(n+1)$. If instead v_n and v_{n-1} are close enough, then it is possible for both voter n and voter $n-1$ to randomize between selling their vote and demanding a majority. The randomization probabilities and the price must satisfy expected market balance (1) and the indifference conditions for both voters, (2) and (3). For v_n and v_{n-1} in this range, both the price p and γ_n are increasing in v_n/v_{n-1}, while γ_{n-1} is decreasing.[11]

The threshold μ_n depends on n but the relationship is not obvious: the size of the electorate influences the equilibrium price, the number of votes that must be purchased to achieve a majority, the expected market balance condition, and the probability of obtaining one's favorite outcome in the absence of trade. All four factors affect μ_n. The threshold μ_n is non-monotonic and always close to 1: $\mu_n = 1$ at $n = 3$, it increases to a maximum of 1.03 at $n = 7$, and declines thereafter, rapidly converging to 1 as n gets large.

Notice, for clarity, that individual voters are not assumed to know others' values. It is only in equilibrium that the identity of the two highest value voters is revealed by their best response strategies, given the price and others' strategies.

Three observations follow from Lemma 1. First of all, the only actions selected with positive probability are either selling or demanding enough votes to hold a majority. Because of the importance of pivotality, the demand for votes is only positive when it concerns the full package of votes necessary to acquire the right to dictatorship.

The second observation then follows immediately: in the equilibrium described in the lemma, any vote market outcome where trade occurs *always results in dictatorship*. The only instance in which the equilibrium does not induce dictatorship is when: (1) $v_n < \mu_n v_{n-1}$, and both voter n and voter $n-1$ randomize between selling and demanding a majority of votes, and (2) the realized action for both is to sell. In such a case, every voter on the market offers to sell; there is no trade, and the decision is taken through simple majority. But such an outcome is both infrequent–its probability is bounded above by $1/(n+1)^2$–and not very interesting–it arises from the failure to have any trade. In all other cases, after rationing, either voter n or voter $n-1$ will have a majority of votes. A market for votes does not distribute votes somewhat equally among high valuation individuals: if any trade occurs, our ex ante competitive equilibrium concentrates *all* decision-power in the hands of a single voter.

Third, rationing occurs with probability 1: there is always either excess demand or excess supply of votes. The larger the committee size, the closer to 1 the probability that both voter n and voter $n-1$ demand votes, and thus the higher the probability of positive excess demand. In the limit, for electorates whose size is unbounded, demand exceeds supply by

[11] Note that at higher p, the indifference of v_{n-1} requires higher γ_n: the probability of being rationed when demanding must be higher.

one vote with probability approaching 1. Relative to the amounts traded, the imbalance is of order $O(1/n)$, and thus negligible in volume, recalling analyses of competitive equilibria with non-convexities (in particular the notion of Approximate Equilibrium, where in large economies allocations approach demands.[12]) But, contrary to private goods markets, in our market the imbalance is never negligible in its impact on welfare: it always triggers rationing and shuts $(n-1)/2$ voters out of the market.

2.4 Welfare

Whenever there is trade, in the equilibrium characterized by Theorem 1 decision-power is concentrated in the hands of a single voter. And because the probability of trade itself is bounded below by $1 - 1/(n+1)^2$, the probability of dictatorship is always close to 1 and converges to 1 at large n. It is true that the dictator will be one of the two highest value voters, but concentrating all decision power in the hands of a single agent does not bode well for the ex ante welfare properties of the institution. In this section we compare the welfare obtained in the Ex Ante Competitive Equilibrium to a situation without market, where voters simply cast their votes for their favorite alternative. Our main result is that if n is sufficiently large, the vote market must be inefficient relative to the majority rule outcome with no trade.

Call W_{MR} ex ante expected utility in the absence of trade (under majority rule), and W_{VM} ex ante expected utility with the vote market, both evaluated before individual valuations are realized. Denote by $F_n(\mathbf{v})$ with $\mathbf{v} = (v_1, ..., v_n)$ the joint probability distribution of the vector of valuations. We assume that the density function $f_n(\mathbf{v})$ exists, is continuously differentiable and everywhere strictly positive on $[\underline{v}, \overline{v}]^n$, with $\underline{v} > 0$. We can write:

$$\begin{aligned} W_{MR} &= \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{v_n} .. \int_{\underline{v}}^{v_2} \left(\frac{v_1 + .. + v_n}{n} \right) \sum_{i=(n-1)/2}^{n-1} \binom{n-1}{i} \left(\frac{1}{2} \right)^{(n-1)} f(v_1, .., v_n) dv_1..dv_n \quad (5) \\ &= \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{v_n} .. \int_{\underline{v}}^{v_2} \left(\tfrac{v_1 + .. + v_n}{n} \right) \left[\frac{1}{2} + \phi \right] f(v_1, .., v_n) dv_1..dv_n \end{aligned}$$

[12] See for example Starr (1969), and Arrow and Hahn (1971).

$$
\begin{aligned}
W_{VM} \;=\; & \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{v_n} .. \int_{\underline{v}}^{v_2} [\gamma_n(v_n, v_{n-1}) \left(\frac{2-\gamma_{n-1}(v_n, v_{n-1})}{2}\right) \left(\frac{v_n}{2n} + \frac{v_1 + .. + v_n}{2n}\right) + \\
& + \gamma_{n-1}(v_n, v_{n-1}) \left(\frac{2-\gamma_n(v_n, v_{n-1})}{2}\right) \left(\frac{v_{n-1}}{2n} + \frac{v_1 + .. + v_n}{2n}\right) + \qquad (6) \\
& + [1-\gamma_n(v_n, v_{n-1})][1-\gamma_{n-1}(v_n, v_{n-1})] \left(\frac{v_1 + .. + v_n}{n}\right) \left[\frac{1}{2} + \phi\right]] f(v_1, .., v_n) dv_1..dv_n
\end{aligned}
$$

where as before $\phi = \binom{n-1}{\frac{n-1}{2}} 2^{-n}$, and the notation makes explicit the dependence of probabilities γ_n and γ_{n-1} on v_n and v_{n-1}. Equation (5) is transparent: if majority rule is implemented, ex ante expected utility is defined by the average valuation and by the probability of being pivotal, always strictly higher than 1/2. These are, respectively, the first and second expression inside the integrals. Ex ante expected utility with the vote market is defined in equation (6). The three terms inside the integrals correspond to ex ante per capita utility in the three possible scenarios: dictatorship by voter n, dictatorship by voter $n-1$, and lack of trade, in each case weighted by the corresponding probability. In the first two cases, expected utility equals half the expected average valuation of non-dictators plus the full valuation of the dictator. In the third case, the outcome is dictated by majority rule. Because the probabilities γ_n and γ_{n-1} depend on v_n and v_{n-1}, the expression cannot be simplified. But the complication is a matter of notation only. We can establish:

Proposition 1 *Consider a sequence of vote markets, indexed by the size of the electorate n. For any sequence of distribution functions $\{F_n\}$, there exists a finite $\overline{n}$ such that if $n > \overline{n}$ $W_{n,MR} > W_{n,VM}$.*

Proof. To simplify notation, we now write $\gamma_n(v_n, v_{n-1})$ as γ_n, and similarly for γ_{n-1}. From $v_n \geq v_{n-1}$:

$$
\begin{aligned}
W_{n,VM} \leq & \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{v_n} .. \int_{\underline{v}}^{v_2} [(1-(1-\gamma_n)(1-\gamma_{n-1})) \left(\frac{v_n}{2n} + \frac{v_1 + .. + v_n}{2n}\right) + \\
& + (1-\gamma_n)(1-\gamma_{n-1}) \left(\frac{v_1 + .. + v_n}{n}\right) \left[\frac{1}{2} + \phi\right]] f_n(v_1, ..v_n) dv_1..dv_n
\end{aligned}
$$

Consider any realization of $(v_1, ..v_n)$. Note that:

$$\left(\frac{v_n}{2n} + \frac{v_1 + .. + v_n}{2n}\right) \quad < \quad \frac{v_1 + .. + v_n}{n}\left[\frac{1}{2} + \phi\right] \tag{7}$$
$$\Longleftrightarrow \quad \frac{v_n}{(v_1 + .. + v_n)/n} < 2n(2^{-n})\binom{n-1}{\frac{n-1}{2}}$$

But $\lim_{n \longrightarrow \infty} 2n(2^{-n})\binom{n-1}{\frac{n-1}{2}} = \infty$,[13] while for any F_n such that every random variable v_i has bound positive support, $v_n/(v_1 + .. + v_n)/n$ is always finite[14]. Hence:

$$\lim_{n \longrightarrow \infty}\left[\begin{array}{c}(1-(1-\gamma_n)(1-\gamma_{n-1}))\left(\frac{v_n}{2n} + \frac{v_1+..+v_n}{2n}\right) \\ +(1-\gamma_n)(1-\gamma_{n-1})\left(\frac{v_1+..+v_n}{n}\right)\left[\frac{1}{2} + 2^{-n}\binom{n-1}{\frac{n-1}{2}}\right]\end{array}\right] \tag{8}$$
$$< \lim_{n \longrightarrow \infty}\left(\frac{v_1 + .. + v_n}{n}\right)\left[\frac{1}{2} + 2^{-n}\binom{n-1}{\frac{n-1}{2}}\right]$$

if $[1-(1-\gamma_n)(1-\gamma_{n-1})] > 0$ for all n. But, from Lemma 1, $[1-(1-\gamma_n)(1-\gamma_{n-1})] \geq [1 - 1/(n+1)^2]$ for all n. Hence (8) holds for any realization of values, and thus must hold in expectation:

$$\begin{aligned}\lim_{n->\infty} W_{n,VM} &\leq \lim_{n->\infty}\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{v_n} .. \int_{\underline{v}}^{v_2}[(1-(1-\gamma_n)(1-\gamma_{n-1}))\left(\frac{v_n}{2n} + \frac{v_1 + .. + v_n}{2n}\right) + \\ &\quad +(1-\gamma_n)(1-\gamma_{n-1})\left(\frac{v_1 + .. + v_n}{n}\right)\left[\frac{1}{2} + 2^{-n}\binom{n-1}{\frac{n-1}{2}}\right]]f_n(v_1, ..v_n)dv_1..dv_n \\ &< \lim_{n->\infty}[\int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{v_n} .. \int_{\underline{v}}^{v_2}\left(\frac{v_1 + .. + v_n}{n}\right)\left[\frac{1}{2} + 2^{-n}\binom{n-1}{\frac{n-1}{2}}\right] f_n(v_1, ..v_n)dv_1..dv_n] \\ &= \lim_{n->\infty} W_{n,MR}\end{aligned}$$

The functions $W_{n,VM}$ and $W_{n,MR}$ are continuous in n, and thus there must exist a number $\overline{n}$ such that if $n > \overline{n}$ $W_{n,VM} < W_{n,MR}$. ■

Why does the vote market fare worse, in ex ante welfare terms, than simple majority? The

[13] Stirling's approximation for factorial terms states that at large n, $n! = n^n e^{-n}\sqrt{2\pi n}(1 + O(1/n))$. Thus at large n, $\left(2n(2^{-n})\binom{n-1}{\frac{n-1}{2}} - 2n/\sqrt{2\pi(n-1)}\right) \longrightarrow 0$. But $\lim_{n->\infty} 2n/\sqrt{2\pi(n-1)} = \infty$, and thus $\lim_{n->\infty} 2n(2^{-n})\binom{n-1}{\frac{n-1}{2}} = \infty$.

[14] We are using here the assumption $\underline{v} > 0$. Weaker conditions would be sufficient too (for example, $\underline{v} > 0$ for a positive but arbitrarily small fraction of voters). No requirement that $\underline{v}$ be strictly positive is necessary at all if we constrain the valuation draws to be iid.

intuition is simple and clarifies the mechanisms of the model. With a market for votes, when trade occurs, a voter's equilibrium ex ante probability of obtaining his preferred alternative is 1 if he is dictator, and 1/2 if he is not (the probability that the dictator will agree with him). Because ex ante the probability of being dictator is of order $1/n$, the total probability is 1/2 plus a term of order $1/n$. Without trade, the corresponding probability is 1/2 plus the probability of being pivotal, which in this model is of order $1/\sqrt{n}$ in large electorates. Although in both cases the probability of obtaining one's preferred alternative decreases with n and tends to 1/2 aymptotically, the speed of convergence is slower with simple majority voting: as the proposition states, there is always an electorate size large enough that simple majority voting with no trade leads to higher ex ante utility than the market for votes.[15]

At a large enough size of the electorate, as the proposition states, the conclusion holds for any distribution F. In small electorates, the welfare properties of the market for votes must depend on the correlation of valuations across voters and on the shape of F: the larger the expected disparity between the highest expected valuation (or the two highest) and all others, the less costly is the concentration of votes brought by the market. Notice however that for the market to bring welfare gains such disparity must be pronounced enough. Suppose for example that valuations were independent draws from a Uniform distribution on $[0, 1]$. Then the expected highest draw v_n is twice the expected average of all other draws, and $2n/(n-1)$ times the average of valuations v_1 to v_{n-2}. And yet the market leads to a decline in expected welfare:

Proposition 2 *Suppose* $(v_1, .., v_n)$ *are independent draws from a Uniform distribution with support* $[0, 1]$. *Then, for all* n, $W_{MR} > W_{VM}$.

Proof. In Appendix I. ■

The larger the number of voters, the larger is the expected cost of dictatorship, and the more skewed the distribution of valuations must be for the market to be efficiency enhancing. The result is illustrated in Figure 1. The figure plots the area in which the vote market dominates majority rule (dark grey), and the area in which the reverse is true (light grey).

[15] We thank Laurent Bouton for this intuition. In our model the probability of favoring either alternative is independent of a voter's valuation. If asymmetries between the supporters of the two alternatives were introduced, the result may change, but the equilibrium of the vote market would need to be rederived. In Casella, Palfrey and Turban (2012), we discuss one example where the groups supporting either alternative have known and different sizes. The superiority of majority voting is confirmed, both theoretically and experimentally.

The vertical axis is the ratio of the second to the highest valuation, $\frac{v_{n-1}}{v_n}$, and the horizontal axis is the ratio of the average of valuations v_1 to v_{n-2} to the highest valuation, a ratio we call $\frac{\overline{v}_{n-2}}{v_n}$. The figure has four panels, corresponding to $n = 5$, 9, 51, and 501. The two cases $n = 5$ and $n = 9$ are the committee sizes we study in the experiment, and the symbols in the figures correspond to the experimental valuations' profiles we discuss in the next section. At $n = 501$, the vote market dominates majority rule only if the highest valuation is at a minimum about twenty times as high as the average of valuations v_1 to v_{n-2}.F

The welfare analysis, logically straightforward given Theorem 1, contradicts common intuitions about vote markets. In the absence of budget inequalities and common values, a market for votes is often believed to dominate simple majority rule because it is expected to redistribute voting power from low intensity voters to high intensity voters.[16] Although such a redistribution is confirmed in the Ex Ante Competitive Equilibrium we characterize, it occurs in extreme fashion: the efficiency conjecture generally fails because *all* decision power is concentrated in the hands of a single individual.

3 Experimental Design

A model of vote markets is difficult to test with existing data: actual vote trading is generally not available in the public record, and in many cases is prohibited by law. We must turn to the economics laboratory. Exactly how to do this, however, is not obvious. Like most competitive equilibrium theories, our modeling approach abstracts from the exact details of a trading mechanism. Rather than specifying an exact game form, the model is premised on the less precise assumption that under sufficiently competitive forces the equilibrium price will emerge following the law of supply and demand. But because of the nature of votes, a vote market differs substantially from traditional competitive markets, and our equilibrium concept is non-standard. Does this new competitive equilibrium concept applied to the different voting environment have any predictive value? Will a laboratory experiment organized in a similar way to standard laboratory markets (Smith, 1965) lead to prices, allocations and comparative statics in accord with ex ante competitive equilibrium? These are the main questions we address in this and the next section.

Experiments were conducted at the Social Sciences and Economics Laboratory at Caltech during June 2009, with Caltech undergraduate students from different disciplines. Eight

[16] See for example, Piketty (1994).

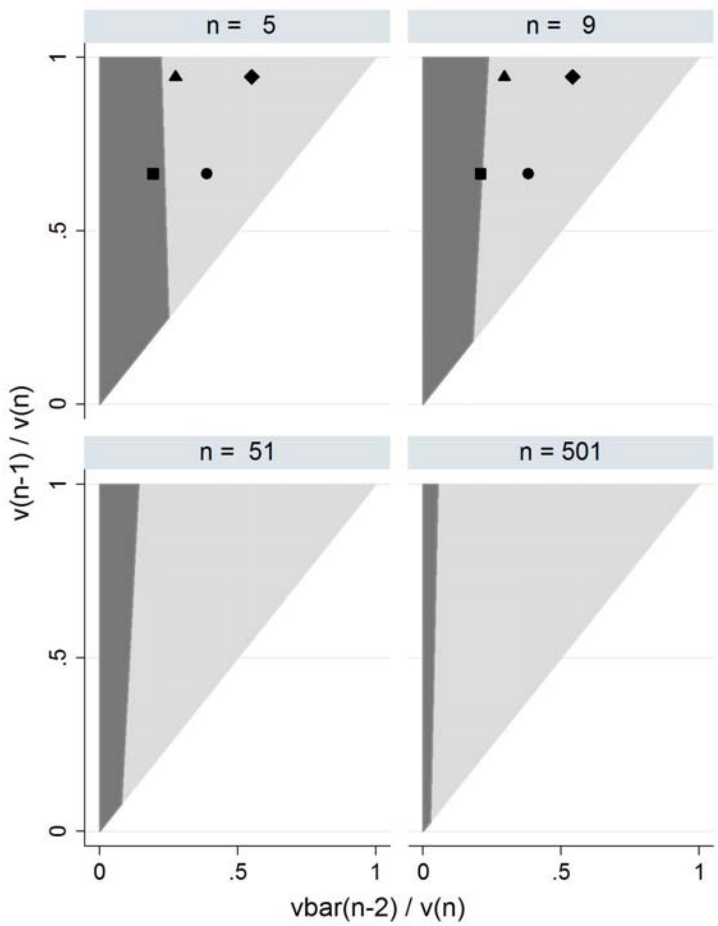

Figure 1: Welfare graphs. The graphs show the area in which majority rule dominates vote markets (light grey), and the area in which vote markets dominate majority rule (dark grey). The symbols corresponds to different experimental treatments. Triangle: HB, Diamond: HT, Square: LB and Circle: LT.

sessions were run in total, four of them with five subjects and four with nine. No subject participated in more than one session. All interactions among subjects were computerized, using an extension of the open source software package Multistage Games.[17]

The voters in an experimental session constituted a committee whose charge was to decide, through voting, on a binary outcome, X or Y. Each voter was randomly assigned to be either in favor of X or in favor of Y with equal probability and was given a valuation that s/he would earn if the voter's preferred outcome was the committee decision. Voters knew that others would also prefer either X or Y with equal probability and that they were assigned valuations, different for each voter, belonging to the range [1,1000], but did not know either others' preferred outcome or the realizations of valuations, nor were they given any information on the distribution of valuations.

All voters were endowed with one vote. After being told their own private valuation and their own preferred outcome, but before voting, there was a 2 minute trading stage, during which voters had the opportunity to buy or sell votes. After the trading stage, the process moved to the voting stage, where the decision was made by majority rule. At this stage, voters simply cast all their votes which were automatically counted in favor of their preferred outcome. Once all voters had voted, the results were reported back to everyone in the committee, and the information was displayed in a history table on their computer screens, viewable throughout the experiment.

We designed the trading mechanism as a continuous double auction, following closely the experimental studies of competitive markets for private goods and assets (see for example Smith, 1982, Forsythe, Palfrey and Plott 1982, Gray and Plott, 1990, and Davis and Holt, 1992). At any time during the trading period, any committee member could post a bid or an offer for one or multiple votes. Bid and offer prices (per vote) could be any integer in the range from 1 to 1000. New bids or offers did not cancel any outstanding ones, if there were any. All active bids or offers could be accepted, and this information was immediately updated on the computer screens of all voters. As with the model's rationing rule $R1$, a bid or offer for more than one unit was not transacted until the entire order had been filled. However, active bids or offers that had not been fully transacted could be cancelled at any time by the voter who placed the order. The number of votes that different voters of the committee held was displayed in real time on each voter's computer screen and updated

[17] Documentation and instructions for downloading the software can be found at http://multistage.ssel.caltech.edu.

with every transaction. There were two additional trading rules. At the beginning of the experiment, voters were loaned an initial amount of cash of 10,000 points, and their cash holdings were updated after each transaction and at the end of the voting stage. If their cash holdings ever became 0 or negative, they could not place any bid nor accept any offer until their balance became positive again.[18] Second, voters could not sell votes if they did not have any or if all the votes they owned were committed.

Once the voting stage was concluded, the procedure was repeated with the direction of preference shuffled: voters were again endowed with a single vote, valuation assignments remained unchanged but the direction of preferences was reassigned randomly and independently, and a new 2-minute trading stage started, followed by voting. We call each repetition, for a given assignment of valuations, a *round*. After 5 rounds were completed, a different set of valuations was assigned, and the game was again repeated for 5 rounds. We call each set of 5 rounds with fixed valuations a *match*. Each experimental session consisted of 4 matches, that is, in each session voters were assigned 4 different sets of valuations. Thus in total a session consisted of 20 rounds.

The sets of valuations were designed according to two criteria. First, we wanted to compare market behavior and pricing with valuations that were on average low (L), or on average high (H); second, we wanted to compare results with valuations concentrated at the bottom of the distribution (B), and with valuations concentrated at the top (T). This second feature was designed to test the theoretical welfare prediction: when valuations are concentrated at the bottom, the wedge between the top valuations and all others is larger, and thus the vote market should perform best, relative to majority voting. The B treatments correspond to the triangle (HB) and square (LB) symbols in Figure **??**

For either $n = 5$ or $n = 9$, each of the 4 combinations, LB, LT, HB and HT, thus corresponds to a specific set of valuations. We call each a *market*. The exact values are reproduced in Table 1 and plotted in Figure 2.[19]

Sessions with the same number of voters differed in the order of the different markets, as described in Table 2. Because, for given n, the equilibrium price depends only on the second

[18] The liquidity constraint was rarely binding, and bankruptcy never occurred. By the end of the last market, all subjects had positive cash holdings after loan repayment.

[19] Thus the experiment has 8 markets (4 markets for each of $n = 5$ and $n = 9$). We obtained the exact valuation numbers by choosing high values with no focal properties ($v_n = 957$ and $v_{n-1} = 903$ for H, and $v_n = 753$ and $v_{n-1} = 501$ for L) and deriving the remaining valuations through the rule: $v_i = v_{n-1}\left(\frac{i}{n-1}\right)^r$ (with some rounding) with $r = 0.75$ for T and $r = 2$ for B. .

	Valuation Number								
Market	1	**2**	3	**4**	5	**6**	7	**8**	**9**
HB	14	**56**	127	**226**	353	**508**	691	**903**	**957**
HT	190	**319**	433	**537**	635	**728**	784	**903**	**957**
LB	8	**31**	70	**125**	196	**282**	384	**501**	**753**
LT	105	**177**	240	**298**	352	**404**	434	**501**	**753**

Table 1: Valuations in the different markets. In the case of n=5, only valuations in bold were used.

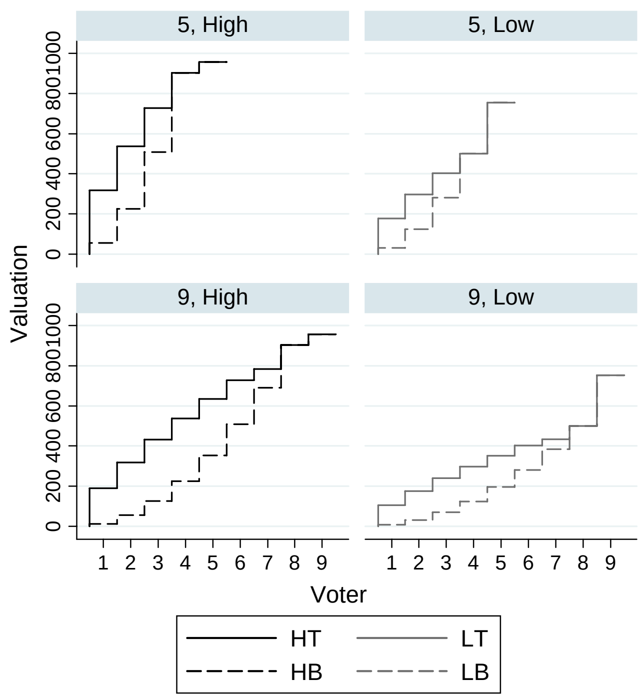

Figure 2: Experimental valuations. Graphs on the top/bottom correspond to treatments with committee size 5/9. Black/Gray graph correspond to treatments with high/low valuations (H/L). Solid/Dashed lines correspond to treatments with valuations concentrated on top/bottom (T/B).

highest valuation, HT and HB markets (and LT and LB markets) have the same equilibrium price. Thus in each session we alternated H and L markets. In addition, because we conjectured that behavior in the experiment could be sensitive to the dispersion in valuations, we alternated B and T markets. With these constraints, four experimental sessions for each number of voters were sufficient to implement all possible orders of markets.

	Match			
Session	1	2	3	4
1	HT	LB	HB	LT
2	LT	HB	LB	HT
3	HB	LT	HT	LB
4	LB	HT	LT	HB

Table 2: Orders of different markets.

At the beginning of each session, instructions were read by the experimenter standing on a stage in the front of the trading room.[20] After the instructions were finished, the experiment began. Subjects were paid the sum of their earnings over all 20 rounds multiplied by a pre-determined exchange rate and a show-up fee of $10, in cash, in private, immediately following the session. Sessions lasted on average one hour and fifteen minutes, and subjects' average final earnings were $29.

4 Experimental Results

We organize our discussion of the experimental results by focusing, in turn, on prices, final vote allocations, and efficiency.

4.1 Prices

Figure 3 shows, as an example, the realized transacted prices in one of our markets: LT with 9 voters. The horizontal line is the number of seconds passed since the opening of the market when the transaction takes place; the vertical dashed lines indicate the end of a round (recall that valuations are maintained across rounds, but the direction of preferences is reassigned randomly). The different colors correspond to different sessions and more importantly to different times within a session when the specific market was played: blue dots are price

[20]The instructions are reproduced in Appendix III.

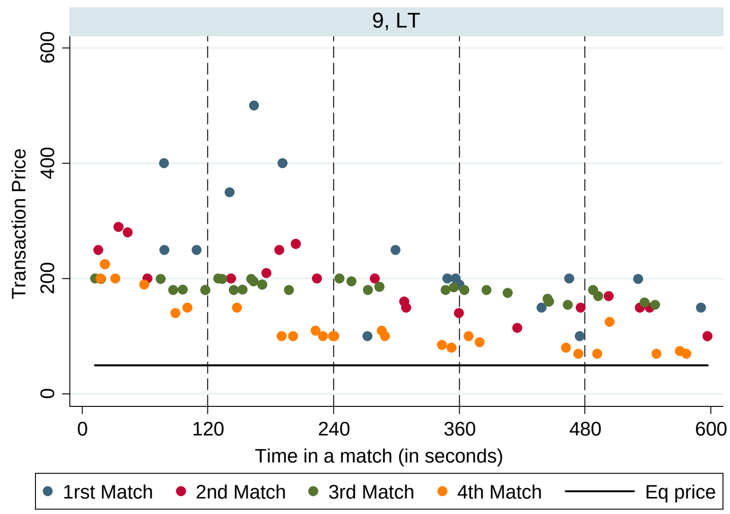

Figure 3: Prices of traded votes in the LT market with 9 subjets. The horizontal line corresponds to the equilibrium price with risk-neutrality. Vertical dashed lines indicate different rounds in a match.

realizations in the session where LT was played first, and where therefore subjects had no experience with the vote market game at all; red dots correspond to a session where the LT market was played second, after five rounds of experience with a different market and a different equilibrium price; green dots to a session where the LT market was played third, and finally yellow dots to a session where it was played last, and where therefore subjects had accumulated most experience, although with different valuations. The black horizontal line is the equilibrium price, which for an LT market with 9 players corresponds to $p = 50$. As described above, in this treatment the voters' valuations ranged from 105 to 753, with the median valuation at 352. The figure makes two points quite clearly. First, over time realized prices fall towards the equilibrium price, and this occurs both for successive rounds in a given session and across sessions, as the LT market occurs later in the order of treatments. Second, even though prices fall with experience, realized prices are above the equilibrium price. These two features are common to the price data in all of our experimental markets, and we organize our discussion around them.

4.1.1 Risk Aversion

Overpricing is a common finding in market and auction experiments, attributed at least in part to the presence of risk-aversion.[21] The equilibrium price plotted in Figure 3 is the equilibrium price with risk-neutrality. It seems intuitive that the equilibrium price would be higher with risk aversion, because buying a majority of votes eliminates all the risk. However, verifying such an intuition requires characterizing the Ex Ante Competitive Equilibrium of the vote market in the presence of risk aversion, and the task is complicated by the fact that the rationing rule itself creates some risk. The following proposition establishes the result and characterizes equilibrium prices for the case of constant absolute risk aversion.

Proposition 3 *Suppose* $u(\cdot) = -e^{-\rho(\cdot)}$ *with* $\rho > 0$*; R1 is the rationing rule. Then for all our experimental treatments the set of strategies in Theorem 1 together with the price* $p = \frac{2}{\rho(n+1)} \ln\left(\frac{1}{2} + \frac{1}{2}e^{\rho v_{n-1}}\right)$ *constitute an ex ante competitive Equilibrium.*

Proof. In Appendix I. ■

If all individuals have CARA utility functions with the same risk aversion parameter, the results of Theorem 1 generalize with little change. As in the case of risk neutrality, at the equilibrium price voter $n-1$ must be indifferent between selling and demanding a majority of votes; hence again the price is a function of v_{n-1}. As discussed in the Appendix, in all our experimental treatments the ratio v_n/v_{n-1} is sufficiently large to support an Ex Ante Competitive Equilibrium where the highest valuation voter, voter n, demands $(n-1)/2$ votes with probability 1. Thus in our experimental parameterization, equilibrium strategies with CARA utility are fully described by the first part of Lemma 1.

As expected, the equilibrium price increases with the risk-aversion parameter ρ. The intuition is not difficult to see. The price p must be such that voter $n-1$ is indifferent between demanding -1 or $\frac{n-1}{2}$ votes. In equilibrium, the probability of being rationed is equal in both cases, but demanding -1 is a riskier lottery because even if non-rationed the outcome depends on the preferences of voter n, who will be dictator. Hence, fixing the price, an increase in risk aversion makes the riskier lottery of selling less attractive relatively to demanding a majority of votes: in order to make voter $n-1$ indifferent, the price must be higher. In fact this straightforward reasoning can tell us more. The upper bound on the price, $\overline{p}$, must correspond to an infinitely risk-averse individual, one who puts full weight

[21] See for example Cox et al. (1988), Goeree et al. (2002), and Goeree et al. (2003).

on the worse realization of the selling lottery, where voter n has opposite preferences and the election is lost. Hence $u(\overline{p}) = u(v_{n-1} - \overline{p}\frac{n-1}{2})$, or $\overline{p} = 2\frac{v_{n-1}}{n+1}$. The upper bound on the price must be exactly twice the risk-neutral price.[22] We can emphasize this observation in a Corollary:

Corollary 1. *For any value of* $\rho > 0$, $p \in (\frac{v_{n-1}}{n+1}, 2\frac{v_{n-1}}{n+1})$.

4.1.2 Convergence towards the equilibrium price.

Figure 4 reports plots of realized prices in all experimental markets for both $n = 5$ and $n = 9$, by round and by order in a session. The two horizontal lines plot the upper and lower boundary on the equilibrium price. The last panel reproduces Figure 3.

The figure shows several regularities. First, experience matters even across markets with different valuations: yellow dots are consistently more likely to belong to the equilibrium price range than other colors, and blue dots consistently less likely to do so. Second, both within and across rounds there is more dispersion in realized prices in $n = 5$ markets: the smaller number of transactions reduces the extent of learning and results in higher variability. Third, convergence towards the equilibrium price is always from above, and in $n = 9$ markets it is towards the upper boundary of the price range.[23] Because the upper boundary is visually indistinguishable in the figure from the correct equilibrium price for even small degrees of risk aversion, the finding is consistent with our model.

The challenge is how to formalize rigorously and test the dynamic price adjustment that the figure suggests. Following Noussair, Plott and Riezman (1995), we estimate $\log(p_{Mmt}) = \alpha_M + \beta_{Mm}(1/t) + \varepsilon_{Mmt}$, where t is the unit of time in the experiment (seconds since the beginning of the match for each transacted price, in our design), M is the index for each market, and m is the index for each match (the order in which the market is played in the experimental session).[24] Thus the parameter α_M captures the asymptotic tendency of price $\log(p_M)$. The full set of parameters' estimates is reproduced in the Appendix; the estimates

[22] Indeed, $\lim_{\rho\to\infty} \frac{2}{\rho(n+1)} \ln\left(\frac{1+e^{\rho v_{n-1}}}{2}\right) = 2\frac{v_{n-1}}{n+1}$. But the result holds not only with CARA, but for all concave $u(\cdot)$.

[23] Convergence from above has been observed in many other market experiments. For example, the early experiments on posted price mechanisms by Plott and Smith (1978) exhibit convergence from above. Experiments where buyer surplus is greater than seller surplus can also lead to this direction of convergence (Smith and Williams 1982). However, neither of these features is present in the markets we study.

[24] Given the dispersion in realized prices shown in the plots, a logarithmic transformation of the prices improves the properties of the regression residuals.

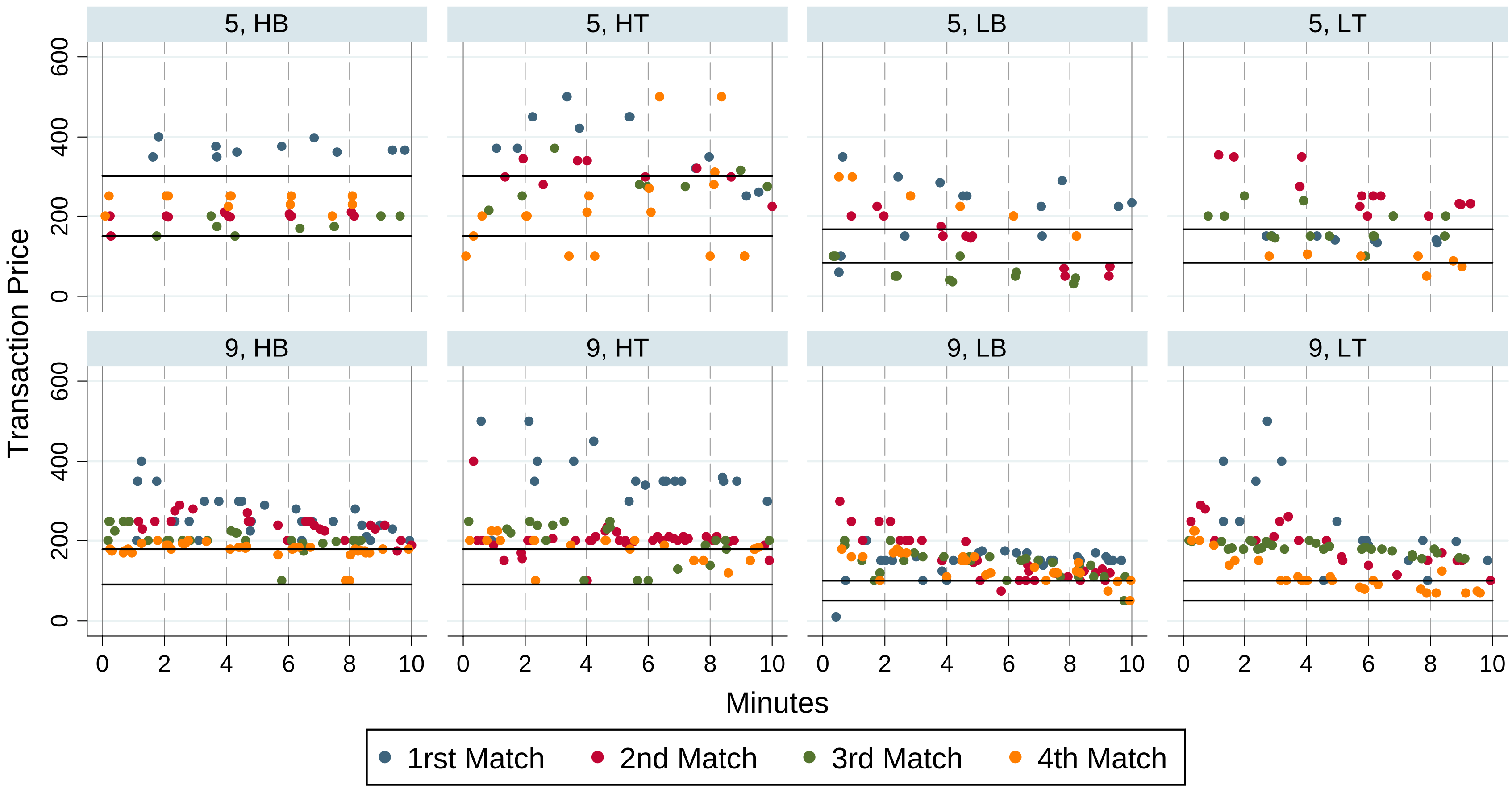

Figure 4. Evolution of transacted prices in each market. Different colors correspond to different matches. The horizontal axis is time (in minutes) since the beginning of the match. The solid horizontal lines are boundaries on equilibrium prices, for different degrees of risk aversion.

		$\widehat{p}^*$	95% Conf. Interval	$[p^{rn}, \overline{p}]$	
$n = 5$	HT	242	[172, 340]	[151, 302]	$\overline{R}^2 = 0.42$
	HB	233	[187, 293]	[151, 302]	$N = 178$
	LT	162	[101, 260]	[84, 168]	
	LB	117	[54, 255]	[84, 168]	
$n = 9$	HT	200	[151, 268]	[90, 180]	$\overline{R}^2 = 0.45$
	HB	196	[172, 224]	[90, 180]	$N = 435$
	LT	143	[100, 202]	[50, 100]	
	LB	136	[107, 174]	[50, 100]	

Table 3: Linear regression of the log of the transacted price on a market dummy and the inverse of the time, in seconds, since the beginning of each match. The time coefficients are allowed to vary across both markets and matches and are reported in the Appendix. For each market, the data are clustered by session and match. Because of the small number of clusters, the standard erros are estimated through a cluster-robust bootstrap estimator, with 2000 repetitions (Cameron, Gelbach and Miller, 2006). The regression does not include four price realizations just above zero that were observed after a dictator had emerged.

of the long-term prices, here converted from logs into levels for ease of reading and denoted $\widehat{p}^*$, are reported in Table 3. The third column reports the range of equilibrium prices, from the risk neutral price p^{rn}, to the upper boundary $\overline{p}$.

The table provides a compact summary of Figure 4. In all $n = 5$ markets the 95% confidence interval for the estimate of p^* overlaps the equilibrium range; in the $n = 9$ markets, the same conclusion holds for the two H treatments and is marginally rejected in the two L treatments.[25] Thus in six of our eight treatments we cannot reject competitive equilibrium pricing with risk aversion.

The theory yields precise comparative statics predictions: for given n, the prices should be higher in H markets than in L markets; and for given H or L valuations, prices should be higher in markets with 5 individuals than in markets with 9. The large standard errors prevent any of the differences from being statistically significant at conventional levels, but in seven out of eight comparisons the point estimates are in line with the theoretical predictions.[26] In addition, for given n, the price should be equal in T and B markets, a strong prediction given that all valuations are different, with the exception of the two highest. The hypothesis cannot be rejected, and the close proximity of the point estimates in three out of

[25] One of the two rejections is for the market depicted in Figure 3.

[26] The exception is $p_{LB5} < p_{LB9}$.

the four cases is noteworthy.

Table 3 confirms what Figure 4 suggested: relative to equilibrium predictions, prices tend to remain higher in $n = 9$ markets. Although differential risk aversion could explain some of the difference, an alternative explanation seems to us more likely: in the experimental design, trading was open for two minutes in both $n = 5$ and $n = 9$ markets, but it seems quite possible that price discovery requires a longer trading period in the larger market. We cannot exclude the hypothesis that prices remain higher in $n = 9$ markets simply because the market has not converged yet.

There is a third hypothesis we can instead reject. Our equilibrium has the somewhat unusual feature that, for any n, equilibrium demands are positive for at most two voters. One might be skeptical about applying competitive equilibrium price-taking in such a model, and conjecture that what we see is the result of low competitive forces. However, that intuition turns out to be wrong, and there is an easy way to see this. Suppose one views this market as a duopsony instead of a competitive market. In a duopsony, buyers have market power so prices should be *lower* than the competitive ones. But this is not what we observe at all. We observe prices somewhat above risk-neutral competitive equilibrium prices, so our data clearly reject the hypothesis that the high value voters (i.e., buyers) are able to exercise market power. The reason in fact is straightforward. When the vote market opens everyone is a (potential) buyer and everyone is a (potential) seller: all voters are competing with each other on both sides of the market. In a monopsony or duopsony, on the contrary, there are one or two designated buyers, and all other market participants are designated as sellers a priori. Because voters can take either side of the market, the vote market is similar to an asset market, where competitive forces or arbitrage will prevent disequilibrium pricing.

4.2 Transactions and Allocations

4.2.1 Transactions

Table 4 summarizes the observed trades. We distinguish between a *transaction* –a realized trade between two voters–and an *order*–an offer to sell or a bid to buy votes that may or may not be realized. Note that both orders and transactions in principle may concern multiple votes: on the purchasing side, it is clearly feasible to demand and buy multiple units; on the selling side, although voters enter the market endowed with a single vote, they could resell in bulk votes they have purchased. For each committee size, the first row is the average number

		LB	LT	HB	HT	Average
$n = 5$	**No. Transactions**	2.3	2.1	2.3	2.3	2.3
	% Unitary	83	95	96	96	92
	% From Offers	77	58	66	72	68
	No. Orders	5.8	5.8	4.4	6.8	5.7
	% Unitary	94	94	92	92	93
	% Offers	90	74	65	52	70
$n = 9$	**No. Transactions**	5.8	5.0	5.9	5.0	5.4
	% Unitary	100	85	97	100	96
	% From Offers	77	59	83	77	74
	No. Orders	16.0	15.1	16.6	14.6	15.6
	% Unitary	98	93	95	93	95
	% Offers	83	67	62	62	69

Table 4: Transactions' Summary

of transactions per two-minute trading round, in the different markets. The transactions can be read as net trades because the percentage of reselling was in all cases lower than 5%.[27] As the table shows, the number of transactions is quite constant across markets, and slightly higher than the theoretical prediction of 2 for $n = 5$, and 4 for $n = 9$. Most transactions concerned individual votes (row 2) and indeed so did most orders in general, not only those that were accepted (row 5).[28] Finally, most transactions also originated from accepted offers (row 3), and again most orders, whether accepted or not, were offers to sell, as opposed to bids to buy (row 6).

4.2.2 Final Allocation of Votes

How close to the theory were the final vote allocations? Figure 5 shows the average number of votes held by voters at the end of each round, compared to the equilibrium prediction. voters are ordered on the horizontal axis, from lowest to highest valuation, but recall that in the experiment voters had no information about others' valuations, or about the ranking of

[27] We define *speculation* as the total number of votes that were both bought and sold by the same player. Averaging across markets, it is 1 percent when $n = 5$ and 3 percent when $n = 9$ (there is no systematic effect across markets). The observed lack of speculation or other attempts to manipulate prices is in line with our modeling of the market as competitive, even with the small number of traders in the market.

[28] By allowing multiunit orders, but requiring them to be filled before any transaction could occur, our trading mechanism builds in a salient feature of the R1 rationing rule. It's interesting that traders rarely exploited this feature. To the extent that our experimental results are close to the theory, this suggests some robustness to the rationing rule.

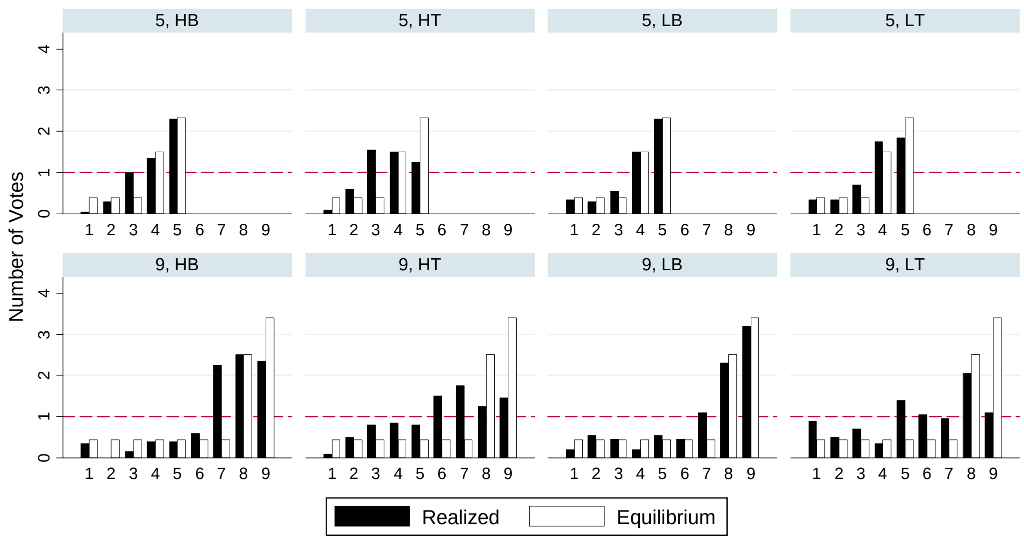

Figure 5: Average amount of votes held by subjects at the end of the trading stage, and equilibrium (expected) allocation, ordered by valuation. The dotted line indicates no trade.

their own valution. Markets with valuations concentrated at the bottom of the distributions (B) appear to conform to the theory quite well: the highest valuation voters end the round with a large fraction of the votes. In particular, if the market is LB, the distribution of votes across voters does increase sharply for the highest valuation voters, exactly as the theory suggests, and this is true for both $n = 5$ and $n = 9$. In markets with valuations concentrated at the top of the distribution (T), the results deviate from the theory, most clearly in $n = 9$ markets: the highest valuation voters demand fewer votes on average than their equilibrium demand, and the number of votes held increases smoothly as the valuations increase.

Not all deviations from equilibrium allocations have welfare consequences: the important question is the concentration of votes the theory predicts at the top of the distribution of values. Is dictatorship observed in the experiment? Table 5 shows the frequency with which either the highest or the second highest value voter concluded a trading round owing a majority of the votes, for different markets and different committee sizes. For given n, the first row in the table reports the frequency of dictatorship over the full data set (20 rounds for each market); the second row reports the frequency over the last two rounds of each match (8 rounds in all), and the last row over the last match (5 rounds in all).[29]

[29] There are some instances where the third highest value trader emerged as dictator: over the full data

n		Market HB	HT	LB	LT	Average
5	All data	50	40	100	60	62.5
	Last 2 rounds	62.5	37.5	100	75	68.75
	Last match	100	100	100	20	80
9	All data	25	0	15	0	10
	Last 2 rounds	25	0	25	0	12.5
	Last match	80	0	0	0	20

Table 5: Observed precentage of rounds in which one of the two highest value subejcts is dictator.

Over the full data set, in $n = 5$ committees, dictatorship emerged just above 62 percent of the times. In the LB market, the data do match the theory perfectly in this regard: out of 20 rounds, in four different experimental sessions, all 20 result in dictatorship. In $n = 9$ committees, where the purchase of four, as opposed to two, votes is required for dictatorship, the results are much weaker, with an average frequency of dictatorship of 10 percent. In T markets, in particular, where valuations are concentrated at the top of the distribution, competition among the higher-value voters clearly works against the concentration of five votes in the hand of the same voter; in B markets, dictatorship does in fact occur, on average 20 percent of the times.

We have seen earlier that realized prices tend to be high, an observation that rationalizes the reluctance to buy the number of votes required to exert full control. But as prices converge towards equilibrium values, we should see an increase in the frequency of dictatorship. For each n, lines 2 and 3 in the Table indeed suggest this conclusion, although the samples become very small. More formal support is provided by a logit regression of the probability of dictatorship by one of the two highest value voters, as function of market characteristics (H and T dummies), time dummies for round and match, and committee size. The regression shows that the probability is significantly higher in the last round of each match, and is significantly lower in T markets, and in committees of 9 voters. The results are reported in Table 6.

The coefficients of the logit regression allow us to assign values to the influence of experience on the probability of dictatorship, as reported in Table 7. The benchmark market is the

set and summing over all markets, this occurred in 15 rounds when $n = 5$ (as opposed to 250 rounds for the two highest valuation traders), and in 10 rounds when $n = 9$ (as opposed to 40 for the two highest valuation traders).

Variable	Coef	95 % Conf. Interval	
Match	0.50	$[-0.23, 1.23]$	$N = 160$
Round	0.29^{**}	$[0.07, 0.50]$	$Pseudo R^2 = 0.384$
Dummies: H	-1.15	$[-2.66, 0.35]$	
T	-1.70^{**}	$[-3.06, -0.35]$	
9	-3.42^{**}	$[-4.95, -1.89]$	
cons	0.77	$[-0.84, 2.38]$	

Table 6: Logit regression of the probability of a dictator as a function of match number, round number, H, T, and n=9 dummies. Data are clustered by n, session, and market, and standard errors are robust. ** significant at 5%.

Market	HT				HB				LT				LB			
Match	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
$n = 5$	4.3	5.6	6.7	7.0	7.0	6.3	5.1	3.8	6.9	7.0	6.4	5.3	4.7	3.5	2.6	1.6
$n = 9$	0.2	0.4	0.6	0.9	1.1	1.7	2.6	3.8	0.7	1.1	1.7	2.5	2.9	4.2	5.5	6.5

Table 7: Point estimates of the average percentage change per round in the probability of dictatorship, for each match.

LB market with $n = 5$, and the constant term, applied to the logistic function, translates into a default probability of dictatorship of 68 percent. Keeping in mind that the standard errors are large, in the benchmark market the regression explains an increase in the probability of dictatorship per round of 4.7 percentage points on average in the first match, an increase per round that persists, although at a declining rate, in later matches. The regression assigns a still larger impact to experience in the other $n = 5$ markets, with a trend across matches that depends on market characteristics. In particular, in the HT market the probability of dictatorship not only increases over successive rounds, as expected, but increases more in later matches, possibly capturing the higher complexity of the equilibrium discovery in a market with more high value voters. This feature is also common to all $n = 9$ markets, again possibly reflecting their complexity.

4.3 Welfare

Large part of the motivation for this paper is the unresolved debate in the literature over the welfare properties of a market for votes relative to simple majority. How high were the subjects' payoffs in the experiment, relative to what the subjects would have earned in the absence of vote trading? And how well does our model predict the realize welfare rankings?

According to the theory, the equilibrium strategy is invariant to the direction of a voter's preferences. In the experiment, voters participated in the market and submitted their orders without information about others' realized preferred alternative— all they knew was that any voter was assigned either alternative as preferred with probability 1/2. Thus a voter's trading behavior should be independent of both its own and other voters' realized direction of preferences. When evaluating the welfare implications of a specific vote allocation, the exact realization of the direction of preferences matters, but the interesting welfare measure is the *average* welfare associated with such an allocation, for all possible realizations of the directions of preferences. It is such a measure that we calculate on the basis of the experimental data.[30]

For each profile of valuations and for the realized allocation of votes at the end of each round, we compute the average aggregate payoff for all possible profiles of preferences' directions, weighted by the probability of their realization. The number we obtain, $W_{VM,r}$, is our measure of experimental payoffs for each round. For given profile of valuations, the standard deviation across rounds gives us a measure of payoffs variability due to the variability in votes allocations generated by trading. We then average $W_{VM,r}$ over all rounds, for given profile of valuations, and obtain W_{VM}, average experimental payoffs per market. We compute the equivalent measure with majority voting, W_{MR} — that is, for each profile of valuations, for all possible realizations of preferences' directions, we resolve the disagreement in favor of the more numerous side; taking into account the probability of each realization, we then compute the average aggregate payoff. To ease the comparison of payoffs across the two institutions, the different markets, and the different committee sizes, we express both W_{VM} and W_{MR} as normalized scores, relative to a ceiling $\overline{W}$, and a floor $\underline{W}$. For each market, $\overline{W}$ is the average maximum payoff, calculated by selecting the alternative favored by the side with higher aggregate valuation, and $\underline{W}$ is the average payoff with random decision-making, where either alternative is selected with probability 1/2. The welfare score then is: $(W_{VM} - \underline{W})/(\overline{W} - \underline{W})$ for the experimental data, and correspondingly for majority voting.[31]

[30] In the experiment, the observed difference in outcomes between the vote market and majority rule depends on the random realization of the subjects' direction of preferences. It could happen for example that all committee members agree, and the outcome would then be trivially identical. But this is a small sample problem. Calculating expected welfare with the realized allocation of votes takes into account all possible realizations of preferences, weighted by their probability, and allows us to derive maximal information from the limited data set.

[31] Note that the only source of variability across rounds comes from the different votes allocations with a market for votes. W_{MR}, $\overline{W}$, and $\underline{W}$ are all constant, for given profile of valuations.

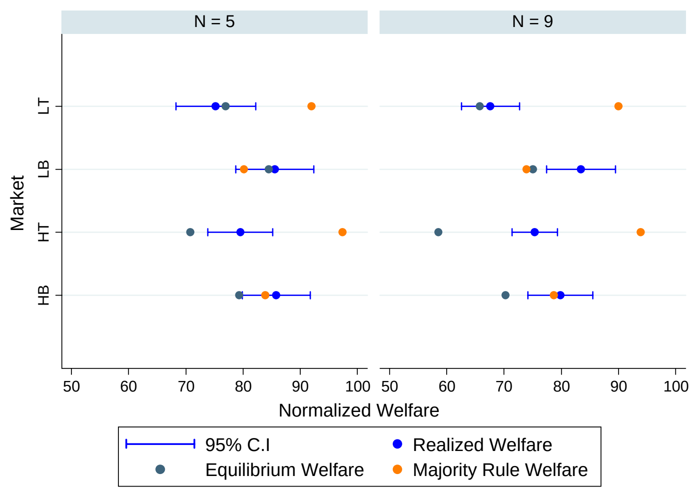

Figure 6: Normalized welfare score per market.

Figure 6 plots, for each market, the normalized welfare score in the experimental data, together with its 95% confidence interval, and for the same profile of valuations the welfare score that corresponds to majority voting (in yellow), and to the theoretical equilibrium (in dark blue).

The figure shows several interesting regularities. First, realized welfare mimics equilibrium welfare more closely in markets with low valuations, indeed very closely in three out of four cases. In markets with high valuations, realized welfare is consistently higher than predicted welfare, reflecting the lower frequency of dictatorship in the data. Second, the theoretical analysis predicts that vote markets should perform better, relative to simple majority with no trade, when valuations are concentrated at the bottom of the distributions. In particular, as shown in Figure 1 vote markets should dominate majority in market LB, be slightly worse in market HB, and substantially worse in markets LT and HT, for both committee sizes. In the data, the market for votes is significantly better than majority with no trade in only one case, the LB market with $n = 9$. It is marginally but insignificantly better in the other B markets, and significantly worse in all T markets, where valuations are concentrated at the top of the distribution. Remarkably, this remains true even though, as we saw, voters deviated from equilibrium towards a more equalitarian allocation of votes, and thus were closer to majority rule than theory predicts.

5 Conclusions

This paper proposes a new competitive equilibrium approach that can be applied to vote markets. Vote markets have a number of unusual properties that require a non-standard approach. Votes are lumpy (as is the public decision), and have no intrinsic value; in fact their value depends on the number of votes held by others, a characteristic that creates both externalities and discontinuities in payoffs. We define the concept of *Ex Ante Competitive Equilibrium*, a concept that combines the price-taking assumption of competitive equilibrium in a market for goods with other less standard assumptions: traders best respond to the demands of other traders, mixed demands are allowed, and market clearing holds only in expectation. Thus deviations from market clearing cannot be systematic and expected, but rationing is typically required ex post, and the equilibrium is defined relative to an anonymous rationing rule.

Using a constructive proof, we establish the existence of an Ex Ante Competitive Equilibrium in a vote market where voters have incomplete information about other members' preferences and a single binary decision has to be made, and we fully characterize the properties of this equilibrium.

A striking feature of the vote allocation in the equilibrium we characterize is that whenever there is trade there is a dictator: with probability one, a single voter acquires a majority position. This feature pins down the welfare properties of the vote market. Because the market results in a dictator, and because the dictator must be one of the two highest valuation (or intensity) voters, the welfare effects depend on the distribution of valuations and on the size of the electorate or committee. In small committees, trade may increase ex ante welfare if there is a sufficient wedge between the two highest valuations and the average of the others, but the required wedge is quite large, and increasing in the size of the committee. When the number of voters is large, the market *always* creates inefficiencies relative to simple majority voting. Intuitively, as n gets larger, the probability of being dictator decreases faster than the probability of being pivotal in the absence of trade.

The theoretical findings are examined using data from a laboratory market experiment with five and nine person committees, where the vote market was conducted as a continuous multiple-unit open-book double auction. By varying the distribution of valuations and the size of the committee, we are able to manipulate the predicted outcomes across the experimental treatments.

The data show overpricing relative to risk neutrality, but are mostly consistent with equilibrium predictions with risk aversion. Estimates of asymptotic price convergence fail to reject the competitive pricing model in six of our eight treatments and reject it marginally in the remaining two treatments. As predicted by the theory, prices did not vary significantly with valuations, if the top two valuations were fixed. In addition, again in line with the theory, prices were higher in smaller committees and in treatments where the top two valuations were higher, although neither of these comparative statics results is statistically significant.

Observed vote allocations were less skewed than predicted when valuations were concentrated at the top of the distribution. They were instead close to equilibrium allocations when the two top valuations were significantly higher than the remainder. In smaller committees, such treatments resulted in dictatorship seventy-five percent of the time, over the full data set, and between eighty and 100 percent of the time when traders were experienced. In larger committees, where the purchase of four votes is required for dictatorship, the frequency of dictatorship was much lower, but dictatorship still arose on average between one fourth and one fifth of the times when the discrepancy between the top two valuations and the others was large. In all cases, the probability of dictatorship increased significantly with experience, suggesting that subjects may require time to discover the equilibrium in such complex markets.

Finally, the welfare results in the experiment were superior to the theoretical prediction in half of our treatments (and indistinguishable in the remainders), reflecting the lower frequency of dictatorship, especially in committees of nine voters. But the predicted welfare ranking, relative to majority voting with no trade, was realized in *all* treatments: average experimental payoffs were significantly lower than majority voting payoffs would have been, with the set of experimental valuations, when the discrepancy between the top valuations and all others was small, and were higher, but insignificantly so, otherwise.

There are still many open questions about vote markets, and in our experiment we explore only the simplest environment. In a companion paper, we study the robustness of the results derived here to different information assumptions and to asymmetries between the supporters of each alternative.[32] A key direction of future research is to extend the model to multiple issues. Here we consider only a one-issue vote market, but in principle the same approach can be applied to the more general case of committees that vote on multiple issues, such as

[32] Casella, Palfrey, and Turban (2012).

legislatures, boards, and standing committees. With multiple issues, the model will then be able to address questions of log-rolling where the market effectively becomes a means for a voter to accumulate votes on issues he cares most about in exchange for his vote on issues he cares less about. The welfare properties of multi-issue vote markets are likely to be more complicated to analyze. Extrapolating from our findings suggests significant inefficiencies, as found in other models of vote exchange that focus on simple bartering examples (Riker and Brams, 1973), rather than taking a general equilibrium approach as we do here. On the other hand, inefficiencies might be mitigated in the presence of multiple issues by the more numerous possibilities for gains from trade, as suggested by abstract models of mechanisms that link decisions across multiple dimensions (Jackson and Sonnenschein, 2007, Casella, 2005, Hortala-Vallve, forthcoming).

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Appendix I

Proofs

Lemma 1. *For all n, there exists a finite threshold $\mu_n \geq 1$ such that if $v_n > \mu_n v_{n-1}$, then $\gamma_{n-1} = (n-1)/(n+1)$, $\gamma_n = 1$, and $p = v_{n-1}/(n+1)$. If $v_n \leq \mu_n v_{n-1}$, then γ_{n-1}, γ_n, and p are the solutions to the system:*

$$\gamma_n = \frac{2n}{n+1} - \gamma_{n-1} \tag{9}$$

$$p = \frac{2 - 4\phi - (1-4\phi)\,\gamma_{n-1}}{2\,(n-1) - (n-3)\,\gamma_{n-1}} v_n \tag{10}$$

$$p = \frac{2 - 4\phi - (1-4\phi)\,\gamma_{n}}{2\,(n-1) - (n-3)\,\gamma_{n}} v_{n-1} \tag{11}$$

$$\gamma_{n-1} \in \left[\frac{n-1}{n+1}, \frac{n}{n+1}\right], \quad \gamma_n \in \left[\frac{n}{n+1}, 1\right] \tag{12}$$

where $\phi = \binom{n-1}{\frac{n-1}{2}} 2^{-n}$.

Proof. We prove the Lemma in two parts. First, we show that there exists a finite threshold $\mu_n \geq 1$ such that if $v_n > \mu_n v_{n-1}$, then $\gamma_{n-1} = \frac{n-1}{n+1}$, $\gamma_n = 1$, and $p = \frac{v_{n-1}}{n+1}$. Second, we show that if $v_n \leq \mu_n v_{n-1}$, then γ_{n-1}, γ_n, and p are the solutions to the system formed by equations(9), (10), (11) and (12).

1. There exists a finite threshold $\boldsymbol{\mu}_n \geq$ 1 such that if $\mathbf{v}_n > \boldsymbol{\mu}_n \mathbf{v}_{n-1}$, then $\boldsymbol{\gamma}_{n-1} = \frac{\mathbf{n-1}}{\mathbf{n+1}}$, $\boldsymbol{\gamma}_n$= 1, and p $= \frac{\mathbf{v}_{n-1}}{n+1}$.

1a. Voter $\mathbf{n-1}$ has no profitable deviation. If voter $n-1$ offers to sell, the total supply of votes is $n-1$ votes. Voter n demands $\frac{n-1}{2}$ votes. Thus total demand equals $\frac{n-1}{2}$, and voter $n-1$'s probability of being rationed is equal to $\frac{1}{2}$. Therefore, $U^{n-1}(-1) = \frac{1}{2}v_{n-1} + \frac{1}{2}p$. If voter $n-1$ demands $\frac{n-1}{2}$ votes, he is again rationed with probability $\frac{1}{2}$, and his expected utility is $U^{n-1}(\frac{n-1}{2}) = \frac{3}{4}v_{n-1} - \frac{n-1}{4}p$. The price that makes him indifferent is exactly $p = \frac{v_{n-1}}{n+1}$. Demanding other quantities is strictly dominated: a smaller quantity can be accommodated with no rationing, but voter $n-1$ would pay the units demanded, and voter n, with a majority, would always decide; a larger quantity is equivalent if $n-1$ is rationed, but is costly and redundant if $n-1$ is not rationed.

1b. Voters $\mathbf{1, 2, ..., n-2}$ have no profitable deviation. Buying votes can only be advantageous if it can prevent both voter $n-1$ and voter n from becoming dictator, but a demand of more than $\frac{n-1}{2}$ is always dominated. Thus the only positive demands to consider are $\frac{n-1}{2}-1$ or $\frac{n-1}{2}$ votes. In the proposed equilibrium, for any $i \in \{1, ..., n-2\}$

$$U^i(-1) = \frac{1}{2}v_i + \frac{n^2-3}{2(n-2)(n+1)}p \tag{13}$$

$$U^i(\frac{n-1}{2}) = \begin{array}{ll} \frac{4n+5}{6(n+1)}v_i - \frac{n^2+n-2}{6(n+1)}p & \text{if } n>3 \\ \frac{3}{4}v_1 - \frac{1}{4}p & \text{if } n=3 \end{array} \tag{14}$$

$$U^i(\frac{n-3}{2}) = \begin{array}{ll} \frac{n+2+(n-1)(1-2^{-(n+1)/2})}{3(n+1)}v_i - \frac{(n-3)(n+5)}{6(n+1)}p & \text{if } n>3 \\ \frac{5}{8}v_1 & \text{if } n=3 \end{array} \tag{15}$$

It is easy to see that selling dominates in the case of $n=3$. In the case of $n>3$, 13 is larger than 14 whenever $\frac{v_{n-1}}{v_i} \geq \frac{n^2-4}{n^2+n-5}$, which holds for any positive n, and 13 is larger than 15 whenever $\frac{v_{n-1}}{v_i} \geq \frac{(n^2-1)(n-2)\left(1-2^{-(n-1)/2}\right)}{n^3+3n^2-19n+21}$, which also holds for any positive n.

1c. Voter n has no profitable deviation if $\mathbf{v}_n \geq \boldsymbol{\mu}_n \mathbf{v}_{n-1}$. In the proposed equilibrium, the expected utilities to voter n from demanding a majority of votes or from deviating and ordering g votes less are given by:

$$U^n(\tfrac{n-1}{2}) = \frac{3n+5}{4(n+1)}v_n - \frac{n^2+2n-3}{4(n+1)}p \tag{16}$$

$$U^n(\tfrac{n-1}{2}-g) = \frac{n-1+4\phi^n(g)}{2(n+1)}v_n - \left(\tfrac{n-1}{2}-g\right)p \tag{17}$$

where $\phi^n(g) = \sum_{i \geq g}^{\frac{n-1}{2}+g} \binom{\frac{n-1}{2}+g}{i} \cdot \left(\frac{1}{2}\right)^{\frac{n-1}{2}+g}$. (16) is larger than (17) whenever $g + 2 \cdot \phi^n(g) \leq \frac{n^2+3n+4}{2(n+1)}$ holds. The left hand side is increasing in g, and therefore reaches its maximum at $g = \frac{n-1}{2}$, that is, when voter n does not buy a vote at all. We need to show that $\phi^n\left(\frac{n-1}{2}\right) \leq \frac{1}{4}\frac{3n+5}{n+1}$. The inequality holds for any n since $\phi^n\left(\frac{n-1}{2}\right) \leq \frac{3}{4}$.

Finally, the expected utility to voter n from offering to sell his vote is given by:

$$U^n(-1) = \frac{n-1}{n+1}\frac{1}{2}\left[v_n + p\right] + \frac{2}{n+1}\left[\frac{1}{2} + \binom{n-1}{\frac{n-1}{2}}2^{-n}\right].$$

Thus $U^n(-1) \leq U^n(\frac{n-1}{2})$ whenever

$$\frac{v_n}{v_{n-1}} \geq (n-1)(n+5)\left[(n+1)\left(n+3-\binom{n-1}{\frac{n-1}{2}}2^{-(n-3)}\right)\right]^{-1} \equiv \mu_n$$

The condition defines the threshold μ_n, and establishes the first part of the Lemma: voter n has no incentive to deviate as long as $v_n \geq \mu_n v_{n-1}$. Note that $\mu_n = 1$ if $n = 3$. Hence for $n = 3$, the equilibrium is fully characterized.

2. If $\mathbf{v}_n \leq \boldsymbol{\mu}_n \mathbf{v}_{n-1}$, then $\boldsymbol{\gamma}_{n-1}$, $\boldsymbol{\gamma}_n$, and p are the solutions to the system formed by equations (9), (10), (11) and (12).

Given that voters $n-1$ and n randomize between demanding a majority and offering their vote, they must be indifferent between these two options. For voter $n-1$, $U^{n-1}(-1) = \frac{1}{2}v_{n-1} + (1-\gamma_n)\phi v_{n-1} + \gamma_n \frac{1}{2}p$ and $U^{n-1}\left(\frac{n-1}{2}\right) = \left(1 - \frac{1}{2}\gamma_n\right)\left(v_{n-1} - \frac{n-1}{2}p\right) + \frac{1}{4}\gamma_n v_{n-1}$; exchanging the indexes, the corresponding equations hold for voter n. The two conditions $U^{n-1}(-1) = U^{n-1}\left(\frac{n-1}{2}\right)$ and $U^{n}(-1) = U^{n}\left(\frac{n-1}{2}\right)$ yield (10) and (11).[33] Equation (9) is the market clearing condition.

Given equations (9), (10) and (11), it must be that γ_{n-1} and γ_n are determined by the following system of equations:

$$\begin{cases} \frac{2-4\phi-(1-4\phi)\gamma_n}{2(n-1)-(n-3)\gamma_n} = \frac{2-4\phi-(1-4\phi)\gamma_{n-1}}{2(n-1)-(n-3)\gamma_{n-1}}\varphi \\ \gamma_{n-1} = \frac{2n}{n+1} - \gamma_n \end{cases} \tag{18}$$

where, as earlier, $\phi \equiv 2^{-n}\binom{n-1}{\frac{n-1}{2}}$ and we define $\varphi = \frac{v_n}{v_{n-1}}$. If $\varphi = 1$, it must be $\gamma_{n-1} = \gamma_n = \frac{n}{n+1}$ and therefore $p = \frac{n+2-4\phi}{n^2+3n-2}v_{n-1}$. If $\varphi > 1$, γ_n is the solution of a quadratic equation in n and φ: $a(n,\varphi)\gamma_n^2 + b(n,\varphi)\gamma_n - c(n,\varphi) = 0$, where

$$\begin{aligned} a(n,\varphi) &= (\varphi-1)(1-4\phi)(n-3)(n+1) \\ b(n,\varphi) &= 2\left[(1-\varphi+2\phi(3\varphi-1))n^2 + (8\phi(2-\varphi)-5+\varphi)n - 2(1-\phi)(\varphi+1)\right] \\ c(n,\varphi) &= 4\left[1+\varphi(n-1)-3n+2\phi(\varphi(n-1)^2+3n-1)\right]. \end{aligned}$$

Given that $a(n,\varphi) > 0$ and $c(n,\varphi) > 0$[34]: 1) the discriminant is positive and the solution well-defined; 2) the requirement $\gamma_n > 0$ selects the solution $\frac{-b(n,\varphi)+\sqrt{b(n,\varphi)^2+4a(n,\varphi)c(n,\varphi)}}{2a(n,\varphi)} > 0$; 3) $\frac{-b(n,\varphi)+\sqrt{b(n,\varphi)^2+4a(n,\varphi)c(n,\varphi)}}{2a(n,\varphi)} \leq 1$ whenever $\varphi \leq \mu_n$. Finally, it is easy to see from (18) that $\gamma_n \geq \gamma_{n-1}$ and therefore $\gamma_{n-1} \in \left[\frac{n-1}{n+1}, \frac{n}{n+1}\right]$ and $\gamma_n \in \left[\frac{n}{n+1}, 1\right]$.

[33] Note that $0 < \varepsilon \leq \frac{1}{4}$ and therefore the numerator is positive and so is the denominator.

[34] To show $c(n,\varphi)$ is positive we need to show tha $\varphi \geq \frac{(3n-1)(1-2\phi)}{(n-1)(2\phi(n-1)+1)}$ holds. Given that $\varphi > 1$ it is sufficient to show that $1 \geq \frac{(3n-1)(1-2\phi)}{(n-1)(2\phi(n-1)+1)}$. This inequality can be rewritten as $\phi \geq \frac{1}{n+1}$ which can be easily show to hold.

2a. Voters $\mathbf{n-1}$ and $\mathbf{n}$ have no profitable deviations. We want to show that $U_i\left(\frac{n-1}{2}-g\right) \leq U_i\left(\frac{n-1}{2}-(g+1)\right)$ for both $i \in \{n-1, n\}$, where $g \in \left\{1, 2, ..., \frac{n-1}{2}\right\}$. The utility of demanding $\frac{n-1}{2}-g$ votes is given by:

$$U\left(\tfrac{n-1}{2}-g\right) = \gamma_j \tfrac{1}{2} v_i + \left(1-\gamma_j\right) \phi^n\left(g\right) v_i - \left(\tfrac{n-1}{2}-g\right) p$$

where $\phi^n\left(g\right) = \sum_{i \geq g}^{\frac{n-1}{2}+g} \binom{\frac{n-1}{2}+g}{i} \cdot \left(\frac{1}{2}\right)^{\frac{n-1}{2}+g}$. is the probability that at least g other players left with a vote agree with i's will; $j = n$ if $i = n-1$, and $j = n-1$ if $i = n$. By demanding fewer than $(n-1)/2$ votes, individual i is never rationed, but if individual j demands a majority, j's will is implemented with probability one. Proving $U_i\left(\frac{n-1}{2}-g\right) \leq U_i\left(\frac{n-1}{2}-(g+1)\right)$ is equivalent to proving $p \geq \left(1-\gamma_j\right) v_i \left(\phi^n\left(g\right) - \phi^n\left(g+1\right)\right)$. Given that this inequality is more restrictive for voter n, it is sufficient to show that $p \geq \left(1-\gamma_{n-1}\right) v_n \left(\phi^n\left(g\right) - \phi^n\left(g+1\right)\right)$ holds. And given that $p \geq \frac{n+2-4\phi}{n^2+3n-2} v_{n-1}$, $1-\gamma_{n-1} \leq \frac{2}{n+1}$ and $v_n \leq \frac{(n-1)(n+5)}{(n+1)(n+3-8\phi)} v_{n-1} \left(= \mu_n v_{n-1}\right)$, it is sufficient to show[35]

$$\frac{(n+1)^2(n+2-4\phi)(n+3-8\phi)}{2(n-1)(n+5)(n^2+3n-2)} \geq \phi^n\left(g\right) - \phi^n\left(g+1\right).$$

But $\phi^n\left(g\right) - \phi^n\left(g+1\right)$ is maximized at $g = \frac{n-1}{2} - 1$, and $\phi^n\left(\frac{n-1}{2}-1\right) - \phi^n\left(\frac{n-1}{2}\right) = \phi$. In addition, $\phi \leq \frac{1}{4}$. Hence the inequality holds whenever $\frac{(n+1)^4}{2(n-1)(n+5)(n^2+3n-2)} \geq \frac{1}{4}$ holds. And this last inequality holds for any n (it can be reduced to $n^4 + n^3 + 7n^2 + 31n - 8 \geq 0$).

Finally, it is easy to see that not offering nor demanding is dominated by selling. Hence, there exist no profitable deviations.

2b. Voters 1 to n-2 have no profitable deviations.

[35] The lower bound on p can be obtained as follows. First, see that $f\left(x\right) = \frac{a-bx}{c-dx}$ is increasing if and only if $ad > bc$. Therefore, we can see from equation **??** that p increases with γ_n iff $\phi \geq \frac{1}{n+1}$ or, alternatively, $\left(n+1\right)\binom{n-1}{\frac{n-1}{2}} 2^{-n} \geq 1$. It is easy to show that this holds by induction. First see that it holds with equality when $n = 3$. Second, it is easy to show that if $\left(n+1\right)\binom{n-1}{\frac{n-1}{2}} 2^{-n}$ is larger that 1 for n, then it must be larger that 1 for $n+2$. Hence, the upper bound on the price can be obtained by setting $\gamma_n = 1$. and the lower bound by setting $\gamma_n = \frac{n}{n+1}$. This gives $p \in \left[\frac{n+2-4\phi}{n^2+3n-2} v_{n-1}, \frac{v_{n-1}}{n+1}\right]$.

The utilities of these voters are given by the following expressions.

$$\begin{aligned}
U(-1) &= \tfrac{1}{2}v + A\phi v + B\tfrac{1}{2}p + C\tfrac{n-1}{2(n-2)}p \\
U\left(\tfrac{n-1}{2}\right) &= A\left(v - \tfrac{n-1}{2}p\right) + B\left(\tfrac{3}{4}v - \tfrac{n-1}{4}p\right) + C\left(\tfrac{2}{3}v - \tfrac{n-1}{6}p\right) \\
U\left(\tfrac{n-3}{2}\right) &= A\left(\left(1-2^{-(n+1)/2}\right)v - \tfrac{n-3}{2}p\right) + B\left(\tfrac{1}{2}v - \tfrac{n-3}{2}p\right) + C\left(\tfrac{2-2^{-(n+1)/2}}{3}v - \tfrac{n-3}{6}p\right) \\
U\left(\tfrac{n-1}{2} - g\right) &= A\phi^n(g)v + (B+C)\tfrac{1}{2}v - \left(\tfrac{n-1}{2} - g\right)p
\end{aligned}$$

where $A = (1-\gamma_n)(1-\gamma_{n-1})$, $B = \left[\gamma_n(1-\gamma_{n-1}) + (1-\gamma_n)\gamma_{n-1}\right]$, $C = \gamma_n\gamma_{n-1}$ and $g \in \left\{2, ..., \frac{n-1}{2}\right\}$.

In ruling out deviations by voters $1, ..n-2$, we begin by showing that demanding any number of votes different from $\frac{n-1}{2}$ or $\frac{n-3}{2}$ is strictly dominated by selling. We then show that selling dominates these two remaining options too.

2b-I. $\mathbf{U}(-1) > \mathbf{U}\left(\frac{n-1}{2} - g\right)$ $\forall \mathbf{g} \in \left\{2, ..., \frac{n-1}{2}\right\}$.

First we show that demanding $\frac{n-1}{2} - (g-1)$ votes is dominated by demanding $\frac{n-1}{2} - g$. $U\left(\frac{n-1}{2} - g\right) \geq U\left(\frac{n-1}{2} - (g-1)\right)$ can be rewritten as

$$p \geq (1-\gamma_n)\left(\gamma_n - \tfrac{n-1}{n+1}\right)\left(\phi^n(g-1) - \phi^n(g)\right)v_i$$

Given that $p \geq \frac{n+2-4\phi}{n^2+3n-2}v_{n-1}$, $(1-\gamma_n)\left(\gamma_n - \frac{n-1}{n+1}\right) \leq \frac{1}{(n+1)^2}$ and $v_i \leq v_{n-1}$ $(\forall i \leq n-2)$, the inequality can be reduced to $\frac{(n+1)^2(n+2-4\phi)}{n^2+3n-2} \geq \phi^n(g-1) - \phi^n(g)$, which holds trivially since $\frac{(n+1)^2(n+2-4\phi)}{n^2+3n-2} > 1$ and $\phi^n(g-1) - \phi^n(g) < 1$. But $U(-1) > U(0)$, and the result follows.

2b-II. $\mathbf{U}_i(-1) > \mathbf{U}_i\left(\frac{n-1}{2}\right)$ **where** $\mathbf{i} \leq \mathbf{n-2}$. The condition corresponds to:

$$\begin{aligned}
&\left[\begin{array}{c} -\frac{1}{2} + (1-\gamma_n)(1-\gamma_{n-1})(1-\phi) + \\ \frac{3}{4}\left[\gamma_n(1-\gamma_{n-1}) + (1-\gamma_n)\gamma_{n-1}\right] + \frac{2}{3}\gamma_n\gamma_{n-1} \end{array}\right] v_i \\
\leq &\left[\begin{array}{c} (1-\gamma_n)(1-\gamma_{n-1})\frac{n-1}{2} + \\ \left[\gamma_n(1-\gamma_{n-1}) + (1-\gamma_n)\gamma_{n-1}\right]\frac{n+1}{4} + \frac{2}{3}\gamma_n\gamma_{n-1}\frac{n^2-1}{6(n-2)} \end{array}\right] p
\end{aligned}$$

Given $v_i \leq v_{n-1}$ and $(1-\phi)v_{n-1} > \frac{n-1}{2}p$, $\frac{3}{4}v_{n-1} > \frac{n+1}{4}p$ and $\frac{2}{3}v_{n-1} > \frac{n^2-1}{6(n-2)}p$, the inequality must hold if it holds when we substitute $(1-\gamma_n)(1-\gamma_{n-1})$, $\left[\gamma_n(1-\gamma_{n-1}) + (1-\gamma_n)\gamma_{n-1}\right]$ and $\gamma_n\gamma_{n-1}$ by their upper bounds $\frac{1}{(n+1)^2}$, $\frac{2}{n+1}$ and $\left(\frac{n}{n+1}\right)^2$ respectively. After simplification, the condition reduces to $\frac{(n-2)\left(n^2+3n+12-6\phi\right)}{n(n^3+3n+2n-18)} \leq \frac{p}{v_{n-1}}$. But using the lower bound on the price,

$p \geq \frac{n+2-4\phi}{n^2+3n-2} v_{n-1}$, a sufficient condition is then: $\phi \leq \frac{n^4+n^3-6n^2+48n-48}{4n^4+6n^3+2n^2-24n-24}$, which holds for any n.

2b-III. $\mathbf{U}_i(-1) > \mathbf{U}_i\left(\frac{n-3}{2}\right)$ **where** $\mathbf{i} \leq \mathbf{n-2}$. The result is trivial in the case of $n = 3$ (and recall that for $n = 3$ the equilibrium described here collapses to the equilibrium described in the first part of Lemma 1). In what follows we will prove the result for $n \geq 5$. For ease of presentation, define $\theta := 1 - \left(\frac{1}{2}\right)^{\frac{n+1}{2}}$. $U_i(-1) > U_i\left(\frac{n-3}{2}\right)$ holds whenever

$$\begin{aligned} &\left[-\tfrac{1}{2} + (1-\gamma_n)(1-\gamma_{n-1})(\theta-\phi) + \tfrac{1}{2}(\gamma_n(1-\gamma_{n-1}) + \gamma_{n-1}(1-\gamma_n)) + \tfrac{\theta+1}{3}\gamma_n\gamma_{n-1}\right] v_i \\ \leq\ & (1-\gamma_n)(1-\gamma_{n-1})\tfrac{n-3}{2} + (\gamma_n(1-\gamma_{n-1}) + \gamma_{n-1}(1-\gamma_n))\tfrac{n-2}{2} + \gamma_n\gamma_{n-1}\tfrac{n^2-2n+3}{6(n-2)} p \end{aligned}$$

Taking into account the expected market clearing condition, the left hand side of the inequality is maximized at $\gamma_n = \gamma_{n-1} = \frac{n}{n+1}$, and the right hand side is minimized at $\gamma_n = \gamma_{n-1} = \frac{n}{n+1}$.[36] Therefore it is sufficient to prove that:

$$\begin{aligned} &\left[(2\theta-1)n^3 - (4\theta-2)n^2 + (6(\theta-\phi)-3)n - (12(\theta-\phi)-6)\right] v_i \\ \leq\ & \left[n^4 + 4n^3 - 18n^2 + 9n + 18\right] p \end{aligned}$$

But given $p \geq \frac{n+2-4\phi}{n^2+3n-2} v_{n-1}$ and $v_i \leq v_{n-1}$, a sufficient condition is

$$\begin{aligned} 0 \leq\ & 2(1-\theta)n^5 + 2(4-2\phi-\theta)n^4 - (11-10(\theta-\phi))n^3 \\ & - (38+14\theta-78\phi)n^2 + 21(1-4\phi+2\theta)n + 6(11-8\phi-4\theta) \end{aligned}$$

Using $\theta \in \left[\frac{3}{4}, 1\right]$ and $\phi \in \left[0, \frac{1}{4}\right]$ a sufficient condition is $5n^4 - 6n^3 - 52n^2 + \frac{63}{2}n + 30 \geq 0$, which is verified for all $n \geq 5$.

The Lemma is proven and Theorem 1 follows immediately. ■

Proposition 2. *Suppose* $(v_1, .., v_n)$ *are independent draws from a Uniform distribution with support* $[0,1]$. *Then* $W_{MR} > W_{VM}$ *for all* n.

[36] First, consider $\gamma_n = x$, $\gamma_{n-1} = y$, and the constraint $x+y = s$. Plugging this constraint into the function $\alpha xy + \beta(x(1-y) + y(1-x)) + \eta(1-x)(1-y)$ it is easy to see that $d(x) = 0 \Leftrightarrow x = \frac{s}{2}$. In our case, this means $\gamma_n = \gamma_{n-1} = \frac{n}{n+1}$. Moreover, this point is a maximum (minimum) whenever $2\beta - \alpha - \eta$. is positive (negative).

Consider the left-hand side of the inequality, with:.$\alpha = \frac{\theta+1}{3}, \beta = \frac{1}{2}, \eta = (\theta - \phi)$. It is easy to see that in this case $2\beta - \alpha - \eta < 0$ and, therefore, the LHS is maximized at $\gamma_n = \gamma_{n-1} = \frac{n}{n+1}$. Similarly, the right-hand side of the inequality corresponds to the equation above with: $\alpha = \frac{n^2-2n+3}{6(n-2)}, \beta = \frac{n-2}{2}, \eta = \frac{n-3}{2}$. In this case $2\beta - \alpha - \eta > 0$ and therefore there is a minimum at $\gamma_n = \gamma_{n-1} = \frac{n}{n+1}$.

Proof. If $(v_1, .., v_n)$ are independent draws from a Uniform distribution, we can write:

$$\begin{aligned} W_{VM} &< n! \int_0^1 \int_0^{v_n/\mu_n} .. \int_0^{v_2} \left(\frac{v_n}{2n} + \frac{v_1 + .. + v_n}{2n} \right) dv_1..dv_n + \\ &+ n! \int_0^1 \int_{v_n/\mu_n}^{v_n} .. \int_0^{v_2} [1 - (1-\gamma_n)(1-\gamma_{n-1})] \left(\frac{v_n}{2n} + \frac{v_1 + .. + v_n}{2n} \right) dv_1..dv_n + \\ &+ n! \int_0^1 \int_{v_n/\mu_n}^{v_n} .. \int_0^{v_2} (1-\gamma_n)(1-\gamma_{n-1}) \left(\frac{v_1 + .. + v_n}{n} \right) \left(\frac{1}{2} + \phi \right) dv_1..dv_n \end{aligned} \tag{19}$$

$$\begin{aligned} W_{MR} &= n! \int_0^1 \int_0^{v_n/\mu_n} .. \int_0^{v_2} \left(\frac{v_1 + .. + v_n}{n} \right) \left(\frac{1}{2} + \phi \right) dv_1..dv_n + \\ &+ n! \int_0^1 \int_{v_n/\mu_n}^{v_n} .. \int_0^{v_2} [1 - (1-\gamma_n)(1-\gamma_{n-1})] \left(\frac{v_1 + .. + v_n}{n} \right) \left(\frac{1}{2} + \phi \right) dv_1..dv_n + \\ &+ n! \int_0^1 \int_{v_n/\mu_n}^{v_n} .. \int_0^{v_2} (1-\gamma_n)(1-\gamma_{n-1}) \left(\frac{v_1 + .. + v_n}{n} \right) \left(\frac{1}{2} + \phi \right) dv_1..dv_n \end{aligned} \tag{20}$$

where, as earlier, $\phi \equiv 2^{-n} \binom{n-1}{\frac{n-1}{2}}$, and $\mu_n \geq 1$ is the threshold such that for $v_n < \mu_n v_{n-1}$ voter n offer his vote for sale with positive probability ($\gamma_n < 1$). The threshold μ_n is defined in the proof of Lemma 1: $\mu_n = (n-1)(n+5) \left[(n+1) \left(n + 3 - \binom{n-1}{\frac{n-1}{2}} 2^{-(n-3)} \right) \right]^{-1}$

Suppose first $n = 3$. Then $\mu_n = 1$, and $W_{VM} < W_{MR}$ if

$$n! \int_0^1 \int_0^{v_n} .. \int_0^{v_2} \left(\frac{v_n}{2n} + \frac{v_1 + .. + v_n}{2n} \right) dv_1..dv_n \leq n! \int_0^1 \int_0^{v_n} .. \int_0^{v_2} \left(\frac{v_1 + .. + v_n}{n} \right) \left(\frac{1}{2} + \phi \right) dv_1..dv_n,$$

or, solving the integrals:

$$W_{VM} < W_{MR} \text{ if } \frac{n+3}{4(n+1)} \leq \frac{1}{2} \left(\frac{1}{2} + \phi \right)$$

At $n = 3$, $\phi = 1/4$, and the sufficient condition is satisfied with strict equality.

Suppose now $n \geq 5$. Note that, by Lemma 1, if $v_{n-1} > v_n/\mu_n$, $[1 - (1-\gamma_n)(1-\gamma_{n-1})] \in$

$[1 - 1/(n+1)^2, 1]$. Hence, by (19) and (20) above:

$$W_{VM} < W_{MR} \Longleftarrow n! \int_0^1 \int_0^{v_n/\mu_n} .. \int_0^{v_2} \left(\frac{v_n}{2n} + \frac{v_1 + .. + v_n}{2n} \right) dv_1..dv_n +$$
$$+ n! \int_0^1 \int_{v_n/\mu_n}^{v_n} .. \int_0^{v_2} \left(\frac{v_n}{2n} + \frac{v_1 + .. + v_n}{2n} \right) dv_1..dv_n <$$
$$< n! \int_0^1 \int_0^{v_n/\mu_n} .. \int_0^{v_2} \left(\frac{v_1 + .. + v_n}{n} \right) \left(\frac{1}{2} + \phi \right) dv_1..dv_n +$$
$$+ n! \int_0^1 \int_{v_n/\mu_n}^{v_n} .. \int_0^{v_2} \left(1 - \frac{1}{(n+1)^2} \right) \left(\frac{v_1 + .. + v_n}{n} \right) \left(\frac{1}{2} + \phi \right) dv_1..dv_n$$

or:

$$n! \int_0^1 \int_0^{v_n} .. \int_0^{v_2} \left(\frac{v_n}{2n} + \frac{v_1 + .. + v_n}{2n} \right) dv_1..dv_n <$$
$$< n! \int_0^1 \int_0^{v_n} .. \int_0^{v_2} \left(\frac{v_1 + .. + v_n}{n} \right) \left(\frac{1}{2} + \phi \right) dv_1..dv_n +$$
$$- n! \int_0^1 \int_{v_n/\mu_n}^{v_n} .. \int_0^{v_2} \left(\frac{1}{(n+1)^2} \right) \left(\frac{v_1 + .. + v_n}{n} \right) \left(\frac{1}{2} + \phi \right) dv_1..dv_n$$

A still stronger, but simpler, sufficient condition is then:

$$n! \int_0^1 \int_0^{v_n} .. \int_0^{v_2} \left(\frac{v_n}{2n} + \frac{v_1 + .. + v_n}{2n} \right) dv_1..dv_n <$$
$$< n! \left(1 - \frac{1}{(n+1)^2} \right) \int_0^1 \int_0^{v_n} .. \int_0^{v_2} \left(\frac{v_1 + .. + v_n}{n} \right) \left(\frac{1}{2} + \phi \right) dv_1..dv_n$$

or:

$$\frac{n+3}{4(n+1)} < \left(1 - \frac{1}{(n+1)^2} \right) \frac{1}{2} \left(\frac{1}{2} + \phi \right) \quad \text{for } n > 3$$

It is not difficult to verify that if the inequality is satisfied at some value n_0, then it is satisfied at $n_0 + 2$. It can be checked immediately that it is satisfied at $n = 5$; hence by induction it is satisfied at all n odd larger than 5. Together with the previous result for $n = 3$, this concludes the proof. ∎

Proposition 3. *Suppose* $u(\cdot) = -e^{-\rho(\cdot)}$ with $\rho > 0$*; R1 is the rationing rule. Then for all our experimental treatments the set of strategies in Theorem 1 together with the price*

$p = \frac{2}{\rho(n+1)} \ln\left(\frac{1}{2} + \frac{1}{2}e^{\rho v_{n-1}}\right)$ *constitute an ex ante competitive Equilibrium.*

Proof of Proposition 3. The Proposition follows from the following Lemma:

Lemma 2. *Suppose* $\eta_i = \frac{1}{2}$, $w_i = 1$, $m_i = 0$ $\forall i$, *and* $u(\cdot) = -e^{-\rho(\cdot)}$ with $\rho > 0$; *R1 is the rationing rule. Then for all* n *there exists a finite threshold* $\mu'_n \geq 1$ *such that if* $v_n \geq \mu'_n v_{n-1}$ *the set of actions presented in Theorem 1 together with* $\gamma_{n-1} = (n-1)/(n+1)$, $\gamma_n = 1$, and $p = \frac{2}{\rho(n+1)} \ln\left(\frac{1}{2} + \frac{1}{2}e^{\rho v_{n-1}}\right)$ *constitute an ex ante competitive Equilibrium.*

Proof of Lemma 2. In the Ex Ante Competitive Equilibrium described in Lemma 2, voter n demands $(n-1)/2$ votes with probability 1; voters 1 to $n-2$ offer their vote for sale, and voter $n-1$ demands $(n-1)/2$ votes with probability $(n-1)/(n+1)$, and offers to sell his vote with complementary probability $2/(n+1)$. As in the proof of Lemma 1, consider the incentives of the different voters in turn.

Voter $\mathbf{n-1}$. As in Lemma 1, p must be such that individual $n-1$ is indifferent between selling his vote or demanding a majority of votes. If voter $n-1$ offers to sell his vote, he is rationed with probability 1/2; whether he is rationed or not, the decision is made by voter n, who owns a majority of votes and agrees with voter $n-1$ with probability 1/2. Thus:

$$U^{n-1}(-1) = \frac{1}{4}\left(u(0) + u(v_{n-1}) + u(p) + u(v_{n-1}+p)\right)$$

If voter $n-1$ demands $\frac{n-1}{2}$ votes, he is again rationed withe probability 1/2; if he is not rationed, he is dictator, if he is rationed, the dictator is voter n who agrees with $n-1$ with probability 1/2. Hence:

$$U^{n-1}\left(\frac{n-1}{2}\right) = \frac{1}{4}\left(u(0) + u(v_{n-1})\right) + \frac{1}{2}u\left(v_{n-1} - \frac{n-1}{2}p\right).$$

Thus the price at which $n-1$ is indifferent must solve:

$$u(p) + u(v_{n-1}+p) = 2 \cdot u\left(v_{n-1} - \frac{n-1}{2}p\right) \tag{21}$$

In the case of a CARA utility, the price that makes voter $n-1$ indifferent is computable and equal to $p = \frac{2}{\rho(n+1)} \ln\left(\frac{1}{2} + \frac{1}{2}e^{\rho v_{n-1}}\right)$.

As in Lemma 1, demanding other quantities is strictly dominated because it is either equivalent to demanding $\left(\frac{n-1}{2}\right)$ if voter $n-1$ is rationed, or strictly worse, if he is not.

Voter n. In equilibrium, voter n's expected utility from demanding $\left(\frac{n-1}{2}\right)$ votes, is given by:

$$
\begin{aligned}
U^n\left(\frac{n-1}{2}\right) &= \frac{2}{n+1}u\left(v_n - \frac{n-1}{2}p\right) + \\
&+\frac{n-1}{2(n+1)}\left[\frac{1}{2}u(0) + \frac{1}{2}u(v_n) + u\left(v_n - \frac{n-1}{2}p\right)\right]
\end{aligned}
$$

If voter n deviates and offers his vote for sale, his expected utility is

$$
\begin{aligned}
U^n(-1) &= \frac{2}{n+1}\left[\phi^n\left(\frac{n-1}{2}\right)u(v_n) + \left(1 - \phi^n\left(\frac{n-1}{2}\right)\right)u(0)\right] + \\
&+\frac{n-1}{2(n+1)}\left[\frac{1}{2}u(0) + \frac{1}{2}u(v_n) + \frac{1}{2}u(p) + u(v_n + p)\right]
\end{aligned}
$$

where, as defined earlier, $\phi^n\left(\frac{n-1}{2}\right) = \sum_{i=(n-1)/2}^{n-1}\binom{n-1}{i}\cdot\left(\frac{1}{2}\right)^{n-1}$ is the probability that at least $\frac{n-1}{2}$ other voters agree with him, in the event that no trade has occurred.

Finally, if voter n deviates and demands $\left(\frac{n-1}{2} - g\right)$ votes, his expected utility is:

$$
\begin{aligned}
U^n\left(\frac{n-1}{2} - g\right) &= \left(\frac{2}{n+1}\phi^n(g) + \frac{n-1}{2(n+1)}\right)u\left(v_n - \left(\frac{n-1}{2} - g\right)p\right) + \\
&+\left(\frac{2}{n+1}(1 - \phi^n(g)) + \frac{n-1}{2(n+1)}\right)u\left(-\left(\frac{n-1}{2} - g\right)p\right)
\end{aligned}
$$

where again $\phi^n(g) = \sum_{i\geq g}^{\frac{n-1}{2}+g}\binom{\frac{n-1}{2}+g}{i}\cdot\left(\frac{1}{2}\right)^{\frac{n-1}{2}+g}$ is the probability that at least g other voters agree with him, when voter $n-1$ has offered his vote for sale and $\frac{n-1}{2} + g$ voters in all retain their vote.

All three expected utilities are continuous in p; $U^n\left(\frac{n-1}{2}\right)$ and $U^n\left(\frac{n-1}{2} - g\right)$ are everywhere strictly decreasing in p, $U^n(-1)$ is everywhere strictly increasing in p Notice that at $p = 0$ (and thus $v_{n-1} = 0$), $U^n\left(\frac{n-1}{2}\right)|_{p=0} > U^n(-1)|_{p=0}$, because $\phi^n\left(\frac{n-1}{2}\right) < 1$, and $U^n\left(\frac{n-1}{2}\right)|_{p=0} > U^n\left(\frac{n-1}{2} - g\right)|_{p=0}$, because $U^n\left(\frac{n-1}{2} - g\right)|_{p=0}$ is increasing in $\phi^n(g)$ and $\phi^n(g)$ is maximal at $g = 0$. Thus for any v_n $U^n\left(\frac{n-1}{2}\right) > U^n(-1)$ and $U^n\left(\frac{n-1}{2}\right) > U^n\left(\frac{n-1}{2} - g\right)$ if v_{n-1} (and thus p) is sufficiently low. Equivalently there must exist a number μ'_n such that if $v_n \geq \mu'_n v_{n-1}$, $U^n\left(\frac{n-1}{2}\right) > U^n(-1)$ and $U^n\left(\frac{n-1}{2}\right) > U^n\left(\frac{n-1}{2} - g\right)$. This is the condition identified in the Proposition.

Voters $\mathbf{1, 2, ..., n-2}$. As in Lemma 1, for voter $i \in \{1, ..., n-2\}$ deviation can be profitable only if the voter demands $\frac{n-1}{2}-1$ or $\frac{n-1}{2}$ votes. (1) Consider first deviation to demanding $\frac{n-1}{2}$ votes. If $n>3$, the possible outcomes and their probabilities are represented in the following Table:

Offer		**Demand $\frac{n-1}{2}$ Votes**	
Outcome	Prob	Outcome	Prob
0	$\frac{1-\delta}{2}$	0	$\frac{1-\lambda}{2}$
p	$\frac{1}{2}\delta$		
v_i	$\frac{1-\delta}{2}$	$v_i - \frac{n-1}{2}p$	λ
v_i+p	$\frac{1}{2}\delta$	v_i	$\frac{1-\lambda}{2}$

where we define $\delta=\frac{1}{n+1}+\frac{(n-1)^2}{2(n+1)(n-2)}$ and $\lambda=\frac{n+2}{3(n+1)}$. Thus:

$$\begin{aligned} U^i(-1) &> U^i\left(\frac{n-1}{2}\right) \Longleftrightarrow \\ \delta\cdot\left[u\left(p\right)+u\left(v_i+p\right)\right] &\geq 2\lambda\cdot u\left(v_i-\frac{n-1}{2}p\right) \\ &\quad+\left(\delta-\lambda\right)\cdot\left[u\left(v_i\right)+u\left(0\right)\right] \end{aligned} \tag{22}$$

Note that $v_i\leq v_{n-1}$. At $v_i=v_{n-1}$, given equation (21), the fact that $u\left(\cdot\right)$ is increasing and the fact that $\delta>\lambda$, equation (22) holds with strict inequality. We can show that if equation (22) holds at $v_i=v_{n-1}$, it must hold at all $v_i<v_{n-1}$. Denote:

$$\Delta=\delta\cdot\left[u\left(p\right)+u\left(v_i+p\right)\right]-2\lambda\cdot u\left(v_i-\frac{n-1}{2}p\right)-\left(\delta-\lambda\right)\cdot\left[u\left(v_i\right)+u\left(0\right)\right]$$

Then

$$\frac{\partial\Delta}{\partial v}=\delta\cdot\left[u'\left(v_i+p\right)-u'\left(v_i\right)\right]-\lambda\cdot\left[2\cdot u'\left(v_i-\frac{n-1}{2}p\right)-u'\left(v_i\right)\right]$$

But the concavity of u then implies $\frac{\partial\Delta}{\partial v}<0$, and the result is established.

If $n=3$, selling is preferred to buying one vote if $2u(p)+2u(v+p)\geq 3u(v)+u(0)$. Note first that because $2u'(v+p)-3u'(v)<0$ by concavity, we only need to check the condition at $v=v_{n-1}$. Using the specific functional form of CARA utility simplifies the rest of the proof. Recall that the price is given by $p=\frac{1}{2\rho}ln\left(\frac{1+e^{\rho v_{n-1}}}{2}\right)$. Hence, $e^{-\rho p}=\left(\frac{2}{1+e^{\rho v_{n-1}}}\right)^{\frac{1}{2}}$ and

$e^{-\rho(v+p)} = \left(\frac{2}{1+e^{\rho v_{n-1}}}\right)^{\frac{1}{2}} e^{-\rho v}$. The inequality that we need to verify reduces to

$$2\sqrt{2}\left(1+e^{\rho v_{n-1}}\right)^{\frac{1}{2}} \leq e^{\rho v_{n-1}} + 3$$

Define $x = 1 + e^{\rho v_{n-1}}$. Then, we want to show that $2\sqrt{2}\sqrt{x} \leq 2 + x$. But $2 + x - 2\sqrt{2}\sqrt{x}$ has a minimum at $x = 2$, which is 0. Hence, the condition is always satisfied.

(2) We now show that if $n > 3$, selling one's vote dominates demanding $\frac{n-3}{2}$ votes. If $n > 3$, the difference of utilities is given by:

$$\begin{aligned} U^k(-1) - U^k(\frac{n-3}{2}) &= -\frac{n^2-6n+11}{12(n+1)(n-2)}(u(v_k)+u(0)) \\ &+\frac{n^2-3}{4(n+1)(n-2)}(u(p)+u(v_k+p)) \\ &- \left[\frac{n-1}{3(n+1)}\left(\frac{1}{2}\right)^{\frac{n+1}{2}}\right] u\left(-\frac{n-3}{2}p\right) \\ &- \left[\frac{n-1}{3(n+1)}\left(1-\left(\frac{1}{2}\right)^{\frac{n+1}{2}}\right)\right] u\left(v_k - \frac{n-3}{2}p\right) \\ &- \frac{1}{n+1}\left[u\left(v_k - \frac{n-3}{2}p\right) + u\left(-\frac{n-3}{2}p\right)\right] \end{aligned} \tag{23}$$

It is somewhat cumbersome but not difficult to show that the expression is decreasing in v.[37] Hence it is minimal at $v = v_{n-1}$; if 23 is positive then, it is positive for all $v_k \leq v_{n-1}$. Again, we make use of the CARA functional form. Define $m = \frac{1+e^{\rho v_{n-1}}}{2}$ so that $p = \frac{2}{(n+1)\rho} ln\,(m)$. Thus:

$$\begin{aligned} e^{-\rho p} &= m^{-\frac{2}{n+1}} & e^{-\rho(v+p)} &= e^{-\rho v} m^{-\frac{2}{n+1}} \\ e^{\rho\frac{n-3}{2}p} &= m^{\frac{n-3}{n+1}} & e^{-\rho\left(v-\frac{n-3}{2}p\right)} &= e^{-\rho v} m^{\frac{n-3}{n+1}} \end{aligned}$$

[37] The proof is available upon request.

Substituting in equation 23, we can write:

$$\begin{aligned}
e^{\rho v_{n-1}}(U^k(-1)-U^k(\frac{n-3}{2})) &= \frac{n^2-6n+11}{12(n+1)(n-2)}(1+e^{\rho v_{n-1}}) \\
&\quad -\frac{n^2-3}{4(n+1)(n-2)}m^{-\frac{2}{n+1}}(1+e^{\rho v_{n-1}}) \\
&\quad + \left[\frac{n-1}{3(n+1)}\left(\frac{1}{2}\right)^{\frac{n+1}{2}}\right]e^{\rho v_{n-1}}m^{\frac{n-3}{n+1}} \\
&\quad + \left[\frac{n-1}{3(n+1)}\left(1-\left(\frac{1}{2}\right)^{\frac{n+1}{2}}\right)\right]m^{\frac{n-3}{n+1}} \\
&\quad + \frac{1}{n+1}m^{\frac{n-3}{n+1}}(1+e^{\rho v_{n-1}})
\end{aligned}$$

First, because $e^{\rho v_{n-1}} \geq 1$, it is sufficient to show that

$$\begin{aligned}
G &= \frac{n^2-6n+11}{12(n+1)(n-2)}(1+e^{\rho v_{n-1}}) - \frac{n^2-3}{4(n+1)(n-2)}m^{-\frac{2}{n+1}}(1+e^{\rho v_{n-1}}) \\
&+ \frac{n-1}{3(n+1)}m^{\frac{n-3}{n+1}} \\
&+ \frac{1}{n+1}m^{\frac{n-3}{n+1}}(1+e^{\rho v_{n-1}})
\end{aligned}$$

is positive. Note that $1+e^{\rho v_{n-1}} = 2m$. Hence, we can write

$$G = \frac{m^{\frac{n-3}{n+1}}}{n+1}\left[\frac{n^2-6n+11}{6(n-2)}m^{\frac{4}{n+1}} - \frac{n^2-3}{2(n-2)}m^{\frac{2}{n+1}} + \frac{n-1}{3} + 2m\right]$$

Denote $\Gamma = \frac{n^2-6n+11}{6(n-2)}m^{\frac{4}{n+1}} - \frac{n^2-3}{2(n-2)}m^{\frac{2}{n+1}} + \frac{n-1}{3} + 2m$. We want to show that $\Gamma \geq 0$. Note that:

$$\begin{aligned}
\frac{\partial\Gamma}{\partial m} &= 2(\frac{n^2-6n+11}{3(n+1)(n-2)})m^{\frac{3-n}{n+1}} - \frac{n^2-3}{(n+1)(n-2)}m^{\frac{1-n}{n+1}} + 2 \\
\frac{\partial^2\Gamma}{\partial m^2} &= 2(\frac{(3-n)(n^2-6n+11)}{3(n+1)^2(n-2)})m^{\frac{2(1-n)}{n+1}} - \frac{(1-n)(n^2-3)}{(n+1)^2(n-2)}m^{\frac{-2n}{n+1}}
\end{aligned}$$

Therefore:

$$\frac{\partial^2\Gamma}{\partial m^2} \geq 0 \;\;\Leftrightarrow\;\; 2(n^2-6n+11)m^{\frac{2}{n+1}} \leq 3(n-1)(n+3)$$
$$\Leftrightarrow\;\; m \leq m^* = \left(\frac{3(n-1)(n+3)}{2(n^2-6n+11)}\right)^{\frac{n+1}{2}} \;\; (>1)$$

Note that by construction, $m \in [1,+\infty]$. Hence, $\frac{\partial\Gamma}{\partial m}$ has a maximum at m^*. But:

$$\frac{\partial\Gamma}{\partial m}_{|m=1} = \frac{5n^2-18n+19}{3(n+1)(n-2)}$$

which is always positive for $n \geq 3$. Moreover, as $m \to \infty$, we can see that for $n > 3$,

$$\frac{\partial\Gamma}{\partial m} \sim_{m\to\infty} 2 > 0$$

Therefore, $\frac{\partial\Gamma}{\partial m} \geq 0$ for any n and ρ. Hence, we only need to show that at $m=1$, $\Gamma \geq 0$. But $\Gamma_{|m=1} = 0$. Thus: $\Gamma \geq 0$, which implies $G \geq 0$, which implies $U^k(-1) - U^k(\frac{n-3}{2}) \geq 0$, concluding the proof for $n > 3$.

Finally, we need to show that demanding $\frac{n-3}{2}$ is also dominated when $n = 3$, a condition that amounts to showing $3u(v+p) - 4u(v) - 2u(0) + 3u(p) \geq 0$. But $3u'(v+p) - 4u'(v) \leq 0$, and thus we only need to check the inequality at $v = v_{n-1}$. Redefining $m = \frac{1+e^{rv_{n-1}}}{2}$, the condition becomes $-6m + 2\sqrt{m} + 4m\sqrt{m} \geq 0$. The RHS is increasing in m and is 0 at $m = 1$. Hence, it is always satisfied. □

As in Lemma 1, the equilibrium where voter n always demands a majority of votes is supported if there is a sufficient gap between v_n and v_{n-1}. The required gap is small: Figure 7 shows the minimum required gap with CARA utility, for the two cases of $\rho = 1$ (the dashed line) and $\rho = 2$ (the dotted line), and as reference for risk neutrality (the solid line), as function of n. The minimum gap is always smaller than 1 percent if $\rho = 1$ and smaller than half a percent if $\rho = 2$, and disappears asymptotically as n increases. Using a grid of size 0.001, we have verified numerically that the condition is satisfied by our experimental parameters for all $\rho \in (0, 1000]$. The Proposition then follows.

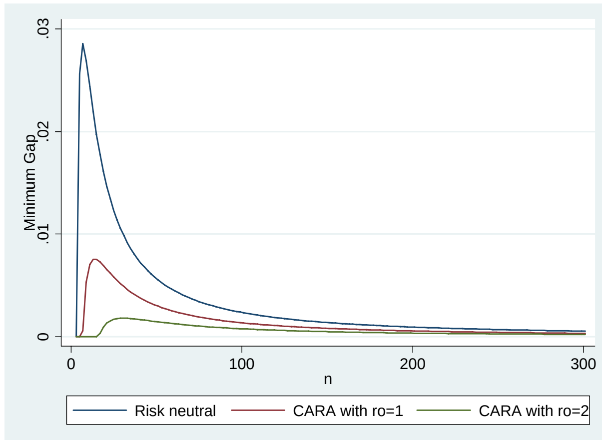

Figure 7: Minimum percentage gap with risk neutrality and with CARA utility funcion: $u(x) = -e^{-\rho x}$, with $\rho = 1$ and $\rho = 2$.

Linear regression: $log(p_{Mmt}) = \alpha_M + \beta_{Mm}(1/t) + \varepsilon_{Mmt}$

$Parameters$	$n = 5$	$n = 9$	$Parameters$	$n = 5$	$n = 9$
α_{HB}	-0.0375	-0.0238	$\beta_{LT,match0}$	-27.04	$82.47*$
	(0.210)	(0.161)		(38.80)	(43.03)
α_{LT}	-0.403	-0.344	$\beta_{LT,match1}$	$79.89**$	$15.73*$
	(0.293)	(0.232)		(36.35)	(8.223)
α_{LB}	$-0.729*$	$-0.391**$	$\beta_{LT,match2}$	12.60	$8.206*$
	(0.437)	(0.189)		(11.78)	(4.252)
$\beta_{HT,match0}$	$58.52**$	$44.25**$	$\beta_{LT,match3}$	$-156.9**$	5.706
	(26.79)	(19.81)		(80.07)	(5.648)
$\beta_{HT,match1}$	$34.38*$	4.058	$\beta_{LB,match0}$	9.444	$-36.43*$
	(20.00)	(3.969)		(11.42)	(19.79)
$\beta_{HT,match2}$	3.829	2.493	$\beta_{LB,match1}$	35.85	$29.49*$
	(10.24)	(2.208)		(35.38)	(17.03)
$\beta_{HT,match3}$	$-5.769**$	-0.833	$\beta_{LB,match2}$	-10.35	11.14
	(2.647)	(2.068)		(7.691)	(9.667)
$\beta_{HB,match0}$	$74.08**$	$41.52**$	$\beta_{LB,match3}$	$43.35*$	6.629
	(33.62)	(19.05)		(22.96)	(8.737)
$\beta_{HB,match1}$	$-4.697**$	$24.78**$	$Constant$	$5.490***$	$5.304***$
	(2.244)	(11.64)		(0.175)	(0.147)
$\beta_{HB,match2}$	$-59.36*$	$2.486*$	N	178	435
	(31.80)	(1.348)	$\overline{R}^2$	0.419	0.455
$\beta_{HB,match3}$	-0.566	$-2.938**$	Standard errors in parentheses		
	(0.563)	(1.463)	$***p < 0.01, **p < 0.05, *p < 0.1$		

Table 8: Linear regression of the log of the transacted price on a market dummy and the inverse of the time, in seconds, since the beginning of each match. The time coefficients are allowed to vary across both markets and matches. For each market, the data are clustered by session and match. Because of the small number of clusters, the standard erros are estimated through a cluster-robust bootstrap estimator, with 2000 repetitions (Cameron, Gelbach and Miller, 2006). The regression does not include four price realizations just above zero that were observed after a dictator had emerged.

Appendix II

Alternative Rationing Rule (R2)

The rationing rule is part of an Ex Ante Competitive Equilibrium. We show here that the analysis has some robustness to the choice of rationing rule. While we cannot consider every possible rule, here we discuss one that we think is the most obvious alternative among anonymous rules that do not exploit information that is private to the voters (such as valuations or direction of preference). Consider the following rule, which we call *rationing-by-vote*, or $R2$: if voters' orders result in excess demand, any vote supplied is randomly allocated to one of the individuals with outstanding purchasing orders, with equal probability. An order remains outstanding until it has been completely filled. When all supply is allocated, each individual who put in an order must purchase all units that have been directed to him, even if the order is only partially filled. If there is excess supply, the votes to be sold are chosen randomly from each seller, with equal probability.

Contrary to rationing-by-voter, or $R1$, $R2$ guarantees that the short side of the market is never rationed, but forces individuals to accept partially filled orders. The two rationing rules have both strong and weak points. For our purposes, the important result is that they support equilibria with very similar structure. We can show:

Proposition 4 *Suppose $\eta_i = \frac{1}{2}$, $w_i = 1$, and $m_i = 0\ \forall i$, agents are risk neutral, and $R2$ is the rationing rule. Voters are ordered according to increasing valuation: $v_1 < v_2 < ... < v_n$. Then there exist $\overline{n}$ and a finite threshold $k(n) \geq 1$ such that for all $n \leq \overline{n}$ and $v_{n-1} \geq k(n)v_{n-2}$ the following set of price and actions constitute an ex ante competitive Equilibrium:*

1. *Price $p^* = \frac{v_{n-1}}{2(n-1)}$.*
2. *Voters 1 to $n-2$ offer to sell their vote with probability 1.*
3. *Voter n demands $\frac{n-1}{2}$ votes with probability 1.*
4. *Voter $n-1$ offers his vote with probability $\frac{2}{n+1}$, and demands $\frac{n-1}{2}$ votes with probability $\frac{n-1}{n+1}$.*

Proof. If $n = 3$, in equilibrium $R2$ is identical to $R1$, and Lemma 1 applies here. immediately. The proof considers $n > 3$.

Voter $\mathbf{n-1}$. In the candidate equilibrium, voter $n-1$ has expected utility $U^{n-1}(-1) = 1/2(p + v_{n-1}/2) + 1/2(v_{n-1}/2) = v_{n-1}/2 + p/2$. (a) Demanding a number of votes $x \in (0, (n-1)/2)$ cannot be a profitable deviation. Any such demand is satisfied with probability 1, causing an expenditure of $px > 0$ while leaving the probability of obtaining the desired outcome at 1/2, and thus results in expected utility $U^{n-1}(x) = v_{n-1}/2 - px$. (b) Demanding more than $(n-1)/2$ votes cannot be a profitable deviation: as long as voter $n-1$ has received less than $(n-1)/2$ votes, $n-1$'s order is outstanding whether his demand is $(n-1)/2$ or higher, and thus the deviation does not affect the probability of individual n being rationed; once voter $n-1$ has received $(n-1)/2$ votes, he controls the final outcome, and any further expenditure is wasted. (c) Finally, doing nothing ($U^{n-1}(0) = v_{n-1}/2$) is dominated by offering to sell.

Voter n. (a) Doing nothing is again dominated by selling: it is identical to selling if $n-1$ sells, and it is strictly dominated if $n-1$ buys. (b) Selling is dominated by demanding $(n-1)/2$ votes. If $n-1$ offers to buy, then n must prefer demanding $(n-1)/2$ to selling because in the identical circumstance $n-1$, with smaller valuation, is indifferent between the two options. If $n-1$ offers to sell, again n must prefer to buy $(n-1)/2$: when $n-1$ sells, buying yields expected utility $v_n - (n-1)/2p$, while offering to sell means that no trade takes place (all voters try to sell) and n wins with probability $\phi^n(\frac{n-1}{2}) \equiv \sum_{k=(n-1)/2}^{n-1} \binom{n-1}{k}(1/2)^{n-1}$ (the probability that at least $(n-1)/2$ of the other voters agree with him). But μ is declining in n, and thus is maximal at $n = 3$, where it equals 3/4. Hence when $n-1$ sells, n's expected utility from offering to sell has upper bound $(3/4)v_n$. But $v_n - (n-1)/2p = v_n - v_{n-1}/4 > (3/4)v_n$ for all $v_n > v_{n-1}$. Hence the only deviation to consider is demanding a quantity of votes x different from $(n-1)/2$. (c) Demanding a quantity x larger than $(n-1)/2$ cannot be profitable. If voter $n-1$ is selling, the order will be filled and is less profitable than demanding $(n-1)/2$; if voter $n-1$ is buying, the argument is identical to point 1b above. (d) Demanding a quantity x smaller than $(n-1)/2$ is not a profitable deviation either. In the candidate equilibrium, n has expected utility equal to:

$$
\begin{aligned}
U^n\left(\frac{n-1}{2}\right) &= \left(\frac{2}{n+1}\right)(v_n - p\frac{n-1}{2}) + \\
&+ \left(\frac{n-1}{n+1}\right)\left[\frac{1}{2}\left(v_n/2 - p\frac{n-3}{2}\right) + \frac{1}{2}\left(v_n - p\frac{n-1}{2}\right)\right] = \\
&= \frac{v_n(5+3n) - nv_{n-1}}{4(n+1)}
\end{aligned}
$$

where the second expression is obtained by substituting for p. If n offers to buy $x < (n-1)/2$, his demand is always satisfied. His expected utility is $v_n/2 - px$ if $n-1$ is buying, and $\eta(x)v_n - px$ if $n-1$ is offering to sell, where $\eta(x) \equiv \sum_{k=(n-1)/2-x}^{n-1-x} \binom{n-1-x}{k}(1/2)^{n-1-x}$ is the probability that n obtains his preferred outcome when owning $x+1$ votes (while everyone else has one vote). Thus:

$$U^n(x) = \left(\frac{2}{n+1}\right)(\eta(x)v_n - px) + \left(\frac{n-1}{n+1}\right)(v_n/2 - px)$$

With $x < (n-1)/2$, the difference $U^n\left(\frac{n-1}{2}\right) - U^n(x)$ is minimal when p is highest, i.e. when $v_{n-1} = v_n$. But:

$$\begin{aligned} U^n\left(\frac{n-1}{2}\right)|_{v_{n-1}=v_n} &= \frac{5+2n}{4(n+1)} \\ U^n(x)|_{v_{n-1}=v_n} &= (1/2)\left(\frac{n-1+4\eta(x)}{n+1} - \frac{x}{n-1}\right) \end{aligned}$$

It then follows immediately that $U^n\left(\frac{n-1}{2}\right)|_{v_{n-1}=v_n} > U^n(x)|_{v_{n-1}=v_n}$ for all $x > 0$ and all $\eta(x) \leq 1$. Hence $U^n\left(\frac{n-1}{2}\right) > U^n(x)$: deviation is not advantageous.

Voters $\mathbf{1, 2, ..., n-2}$. (a) One possible deviation is for voter $i \in \{1, .., n-2\}$ to do nothing. When voter $n-1$ demands $(n-1)/2$ votes, supply is $n-3$ and it is then possible for neither $n-1$ nor n to be dictator. Call $r_e(n)$ the probability that votes supplied are allocated equally to n and $n-1$, when $n-1$ demands $(n-1)/2$ votes. I.e.:

$$r_e(n) = \binom{n-3}{(n-3)/2}(1/2)^{n-3}. \tag{24}$$

Then i's expected utility from doing nothing, U_i^0, is given by:

$$U^i(0) = \frac{2}{n+1}\left(\frac{v_i}{2}\right) + \frac{n-1}{n+1}\left[(1-r_e(n))\frac{v_i}{2} + r_e(n)\frac{3}{4}v_i\right]. \tag{25}$$

Voter i's expected utility from selling, $U^i(-1)$, is:

$$U^i(-1) = \frac{2}{n+1}\left(\frac{v_i}{2} + \frac{p}{2}\right) + \frac{n-1}{n+1}\left(\frac{v_i}{2} + p\right) \tag{26}$$

Comparing (25) to (26) and substituting (24), we derive:

$$\begin{aligned} U^i(-1) &\geq U^i(0) \Longleftrightarrow \\ \frac{v_{n-1}}{v_i} &\geq \frac{(n-1)^2}{n}\binom{n-3}{(n-3)/2}(1/2)^{n-2} \end{aligned} \tag{27}$$

The right hand side of equation (27) is smaller than 1 for all $n < 9$, and equals 10/9 at $n = 9$. Thus deviation to doing nothing is never advantageous for any i at $n = 5$ or 7; at $n = 9$, we need to impose $v_{n-1} \geq (10/9)v_{n-2}$. It will be shown below that this is not the binding restriction. However:

$$\lim_{n->\infty} \frac{(n-1)^2}{n}\binom{n-3}{(n-3)/2}(1/2)^{n-2} = \infty$$

Thus unless $v_i = 0$ for all $i < n-1$, there must always exist a number $\overline{n}$ such that for all $n > \overline{n}$ voter $n-2$ prefers to do nothing than selling. The actions and price described in the Proposition can only be an equilibrium for $n \leq \overline{n}$.

(b) The other possible deviation for voter i is demanding a positive number of votes x, with $x \in \{1, 2, .., (n-1)/2\}$. As before, demanding more than $(n-1)/2$ votes is never advantageous. Consider first i's expected utility from demanding $(n-1)/2$ votes. Call $\delta_i(n, x_{n-1})$ the probability that i becomes the dictator, i.e. the probability that he obtains $(n-1)/2$ votes, as function of n and of voter $n-1$'s demand, x_{n-1}. When $n-1$ demands $(n-1)/2$ votes, the total supply of votes is $n-3$, and $\delta_i(n, (n-1)/2)$ is the probability that at least $(n-1)/2$ votes are randomly allocated to voter i:

$$\delta_i(n, (n-1)/2) = \sum_{i=(n-1)/2}^{n-3} \sum_{z=0}^{n-3-i} \frac{(n-3)!}{i!z!(n-3-z-i)!}(1/3)^{n-3}$$

Similarly, $\delta_{-i}(n, (n-1)/2))$ is the probability that either n or $n-1$ become dictator, i.e. the probability that at least $(n-1)/2$ votes are randomly allocated to one of them:

$$\delta_{-i}(n, (n-1)/2) = 2\sum_{z=(N-1)/2}^{n-3} \sum_{y=0}^{n-3-z} \frac{(n-3)!}{z!y!(n-3-z-y)!}(1/3)^{n-3}$$

Thus i's expected utility, when $n-1$ demands $(n-1)/2$ votes, either n or $n-1$ become dictator and i demands $x_i = (n-1)/2$ votes is given by:

$$Z^i_{d(-i)}\left(\frac{n-1}{2}\right) = 2\sum_{z=(n-1)/2}^{n-3}\sum_{y=0}^{n-3-z}\frac{(n-3)!}{z!y!(n-3-z-y)!}(1/3)^{n-3}$$
$$\left[\frac{v}{2} - p\left(n-3-z-y+\sum_{i=1}^{z-(n-1)/2}\binom{z-(n-1)/2}{i} i(1/2)^{z-(n-1)/2}\right)\right]$$

Finally, there is the probability that no dictator arises:

$$1-\delta_i-\delta_{-i} = \sum_{z=0}^{(n-3)/2}\sum_{y=(n-3)/2-z}^{(n-3)/2}\frac{(n-3)!}{z!y!(n-3-z-y)!}(1/3)^{n-3}$$

and the corresponding expected utility:

$$Z^i_{nod}\left(\frac{n-1}{2}\right) = \sum_{z=0}^{(n-3)/2}\sum_{y=(n-3)/2-z}^{(n-3)/2}\frac{(n-3)!}{z!y!(n-3-z-y)!}(1/3)^{n-3}\left[\frac{3v}{4} - p(n-3-z-y)\right]$$

We can then write:

$$U^i\left(\frac{n-1}{2}\right) = \frac{n-1}{n+1}\left[Z^i_{d(-i)}\left(\frac{n-1}{2}\right) + Z^i_{nod}\left(\frac{n-1}{2}\right) + \delta_i\left(v - \frac{n-1}{2}p\right)\right] +$$
$$\frac{2}{n+1}\left[\frac{1}{2}\left(v - \frac{n-1}{2}p\right) + \frac{1}{2}\left(\frac{v}{2} - \frac{n-3}{2}p\right)\right]$$

Comparing $U^i\left(\frac{n-1}{2}\right)$ to i's expected utility from selling, and substituting p, we find that $U^i(-1) \geq U^i\left(\frac{n-1}{2}\right)$ for all $i < n-1$ if and only if: $v_{n-1} \geq (25/24)v_{n-2}$, if $n = 5$; $v_{n-1} \geq (11/10)v_{n-2}$, if $n = 7$; and $v_{n-1} \geq 1.15v_{n-2}$, if $n = 9$.

(c) For $n > 3$, demanding less than $(n-1)/2$ votes can in principle be advantageous if $n-1$ demands $(n-1)/2$ votes, and neither $n-1$ nor n emerge as dictators. The calculations are somewhat cumbersome, but follow the logic just described, and we do not report them here (they are available from the authors upon demand). They show: (1) $U^i(-1) \geq U^i\left(\frac{n-3}{2}\right)$ for all i as long as $v_{n-1} \geq v_{n-2}$, satisfied by definition. For $n = 5$, this concludes the proof. (2) $U^i(-1) \geq U^i\left(\frac{n-5}{2}\right)$ for all i as long as $v_{n-1} \geq v_{n-2}$, satisfied by definition. For $n = 7$, this completes the proof. (3) For $n = 9$, $U^i(-1) \geq U^i\left(\frac{n-7}{2}\right)$ for all i as long as $v_{n-1} \geq 1.04v_{n-2}$. The condition is not binding, because $v_{n-1} \geq 1.15v_{n-2}$ is required to prevent $n-2$ to deviate to doing nothing.

Summarizing the conditions derived in (a), (b) and (c), we conclude that the actions and price described in the proposition are an equilibrium for $n = 5$ if and only if $v_{n-1} \geq (25/24)v_{n-2}$; for $n = 7$, if and only if $v_{n-1} \geq (11/10)v_{n-2}$; and for $n = 9$, if and only if $v_{n-1} \geq 1.15v_{n-2}$. With $1.15 > 11/10 > 25/24$, the latter condition is sufficient for $n = 3, 5, 7$ and necessary and sufficient for $n = 9$. This is the statement in the Proposition. ■

In particular, 1.–4. constitute an equilibrium if $n \leq 9$ and $v_{n-1} \geq 1.15v_{n-2}$, conditions satisfied in our experimental treatments. Note that with risk neutrality the constraint $m_i = 0$ $\forall i$ is again irrelevant, and imposed for simplicity of notation only.

The equilibrium is almost identical to the equilibrium with $R1$ in which the highest valuation voter demands $(n-1)/2$ votes with probability 1, with two differences. First, because orders can be filled partially, rationing can be particularly costly. In equilibrium, if voter $n-1$ attempts to buy, either he or voter n will be required to pay for votes that are strictly useless (since the other voter will hold a majority stake). To support voter $n-1$'s mixed strategy, if there are more than three voters the equilibrium price is lower than with the original rationing rule: $\frac{v_{n-1}}{2(n-1)}$ instead of $\frac{v_{n-1}}{n+1}$. And because the price is lower, voter n always demands a majority of votes: the small wedge between $v_n = v_{n-1}$ that with $R1$ is required to prevent voter n from selling his vote with positive probability now disappears. Second, and again because of the possibility of filling partial orders, lower valuation voters can use their own orders strategically, refraining from selling to increase the probability that no dictator emerges. To rule out the possibility of such a deviation, the price must be high enough, relative to their valuation. This is the reason for requiring $v_{n-1} \geq k(n)v_{n-2}$.

With these two caveats, the change in rationing rule has little effect. In particular, equilibrium vote trading results in dictatorship, and the welfare implications are similar.