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Max Planck Institute for Research on Collective Goods, Kurt-Schumacher-Str. 10, D-53113 Bonn
https://www.coll.mpg.de

Dynamic Inefficiency and Fiscal Interventions in an Economy with Land and Transaction Costs*

Martin F. Hellwig
Max Planck Institute for Research on Collective Goods
Kurt-Schumacher-Str. 10
D - 53113 Bonn, Germany

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Abstract

The paper contributes to the discussion on whether real interest rates smaller than real growth rates can be taken as evidence of dynamic inefficiency that calls for fiscal interventions. A seemingly killing objection points to the presence of land, a non-produced durable asset whose value becomes arbitrarily large as interest rates go to zero. Such an asset, it is claimed, can accommodate any need for a store of value at interest rates above growth rates, so dynamic inefficiency cannot arise. The paper shows that this objection is not robust to the presence of an arbitrarily small per-unit-of-value transaction cost. The paper also gives conditions under which fiscal interventions provide for Pareto improvements even though the interventions themselves are also costly.

Key Words: Dynamic inefficiency, fiscal policy, public debt, overlapping-generations models with land, transaction costs, pay-as-you-go retirement provision.

JEL: D15, D61, E21, E62, H63.

1 Introduction

The experience of low rates of interest over the past decade has revived the notion that, in an economy with an unbounded sequence of overlapping generations, *laissez-faire* allocations might be *dynamically inefficient* and that appropriate government intervention might induce a Pareto improvement. The argument goes back to Allais (1947, Appendix 2),[1] Samuelson (1958), and Diamond (1965). Blanchard (2019) provides it with a new impetus. Based on

*Without implicating them, I thank Peter Diamond, Christoph Engel, Hans-Jürgen Hellwig, and Christian von Weizsäcker for very helpful advice.

[1] I am grateful to Peter Diamond for this reference. A vers brief summary is given by Malinvaud (1987).

a thorough survey of theory and empirical developments, he suggests that the current constellation with real interest rates smaller than real growth rates is likely to persist for some time. Given this expectation, he argues that public borrowing is cheap, in terms of budgetary effects and welfare effects and that a more active fiscal policy is called for.[2]

Before Blanchard, von Weizsäcker (2010, 2014) articulated similar views and triggered a lively debate among German economists. In this debate, Homburg (2014) argued that dynamic inefficiency of *laissez-faire* allocations cannot arise in the presence of a non-produced, durable asset, such as land, that can serve as a store of value.

The argument is based on a theoretical point already madeby von Mises (1924/1953, Part 3, Ch. V, §3), Allais (1947, Appendix 2), Homburg (1991) and Rhee (1991).[3] In models exhibiting dynamic inefficiency, the underlying real structure, as given by preferences, endowments, and technologies, provides scope for indirect exchange between generations, with goods deliveries from generation t to generation t-1, from generation t+1 to generation t, which can only be exploited if there is a store of value that can serve as a medium of exchange between generations, so that any generation t acquires it from generation t-1 and resells it to generation t+1. If this need for a store of value requires asset holdings at levels where real rates of return are lower than real growth rates, *laissez-faire* allocations are not Pareto efficient (Diamond 1965, Tirole 1985). According to Homburg (1991, 2014), however, this phenomenon cannot arise in an economy with land because no matter how large the need for a store of value might be this need can always be met by having a sufficiently high price of land.

On the face of it, this theoretical argument seems to kill any debate about dynamic inefficiency calling for fiscal policy as a means of achieving Pareto improvements. Since land is obviously an important part of wealth, the argument seems to suggest that there is no point in even discussing whether current and past experience supports the proposition that we might be in a dynamically inefficient equilibrium. Blanchard and von Weizsäcker may well be right about the past, the present and the near future, but theory seems to tell us that this constellation cannot last. And therefore, an active fiscal policy now might have dangerous consequences in the future, once interest rates go up again; eventually, someone will have to bear the cost of such a policy.

One can disdain this objection on the grounds that "eventually" may be far away and, anyway, considerations involving the infinite time horizon should not be taken very seriously in policy discussion. However, the accusation that proposed policies come at the expense of later generations can be effective in debate even if "later" may be far in the future.

I therefore want to point out that the theoretical point itself is *non-robust*. For suppose that transactions involving land are costly and the transaction cost

[2] Summers (2014) and Rachel and Summers (2019) also make important contributions to this discussion. They focus on the empirics and their implications for macroeconomic stabilization rather than welfare.

[3] Rhee (1991) actually argues that this point is moot if population growth or technical change cause the income share of the asset in question to become small over time.

is proportional to the value of the transaction. Then, no matter how small the per-unit-of-value transaction cost may be, no general statement about the scope for dynamic inefficiency of *laissez-faire* allocations can be made. The reason is that, if the price of land is high, so is the transaction cost per unit of land. If the price of land is sufficiently high, the transaction cost will actually exceed the value of the produce (or other benefit) from the land and the real rate of return on land, net of the transaction cost, will be negative. Any equilibrium allocation with this property exhibits dynamic inefficiency, i.e. there is some scope for a Pareto improvement.

Can such an improvement be implemented by real-world fiscal interventions? The answer to this question depends on the costs associated with the intervention. In the overlapping-generations model, a simple intervention involving a lump-sum tax on people in the first period and a lump-sum subsidy to people in the second period of their lives in fact will improve on the *laissez-faire* allocation if, in relative terms, the associated dissipation of resources, i.e. the amount by which the subsidy falls short of the tax is smaller than the excess of the per-unit transaction cost over the real rate of return on land, i.e. the amount by which the net rate of return on land is negative. If the fiscal intervention involves no dissipation of resources, it can provide a Pareto improvement whenever the *laissez-faire* outcome is dynamically inefficient, i.e. the net rate of return on land is negative. If the fiscal intervention involves some dissipation of resources, but in relative terms this dissipation is lower than the per-unit transaction cost for land, negativity of the net rate of return on land is not sufficient, but, if the land price rises even higher, at some point, the fiscal intervention will provide a Pareto improvement. Thus, even if fiscal intervention is costly, the existence of a non-produced durable asset like land as such is not sufficient to rule out the possibility that - sufficiently large - dynamic inefficiency might call for such an intervention in order to improve on the *laissez-faire* outcome.

2 The Basic Model

The following simple model will serve to formalize the argument. There is an infinite sequence of periods $t = 1, 2, ...$ In each period, there is a single consumption good; this consumption good is non-storable. There is also land, a non-produced asset, a unit of which provides a units of the consumption good in each period.

In each period, a new generation of people is born. The population is assumed to be constant, so the size N_t of the generation born in period t is given as $N_t = N$ for all t.

A person born in period t lives for two periods and is interested in the consumption good in periods t and $t+1$. The person's preferences are given by the utility function $u(c_1^t)+v(c_2^t)$, where both c_1^t is consumption in the first period and c_2^t is consumption in the second period of the person's life, and $u(\cdot), v(\cdot)$ are twice continuously differentiable, strictly increasing, and strictly concave functions, with $u''(c) < 0$ and $v''(c) < 0$ for all c, and moreover $\lim_{c\to 0} u'(c) = \lim_{c\to 0} v'(c) = \infty$.

A person born in period t has an initial endowment $(e_1^t, e_2^t) = (E, 0)$ of the consumption good in periods t and $t+1$, where $E > 0$, and no initial endowment of land. In period 1, there is also an old generation of size N. Each member of this generation has an endowment $L^0 > 0$ of land and gets utility $v(c_2^0)$ from consuming c_2^0.

In each period t, there is a market in which land can be traded against the consumption good of that period. The price per unit that a buyer has to pay is p_t, the price that the seller receives is $p_t(1-\pi) \in (0, p)$. The wedge $p_t\pi$ between the buyer's price and the seller's price is a transaction cost.

Given the land prices p_t, p_{t+1} in periods t and $t+1$, a person born in period t chooses a consumption plan (c_1^t, c_2^t) and a land purchase L^t in period t so as to maximize the utility

$$u(c_1^t) + v(c_2^t) \tag{1}$$

subject to the budget constraints

$$c_1^t = E - p_t L^t \ \text{ and } \ c_2^t = (a + p_{t+1}(1-\pi))L^t. \tag{2}$$

Members of the old generation in period 1 receive the harvest aL^0 on their land, sell their land at the price p_1, and consume the amount $(a + p_1(1-\pi))L^0$ that is available net of the transaction cost.

An *equilibrium* is given by a land price sequence $\{p_t\}_{t=1}^{\infty}$ and an allocation $\{c_2^{t-1}, c_1^t, L^t\}_{t=1}^{\infty}$ such that (i) $c_2^0 = (a+p_1(1-\pi))L^0$ and, for $t = 1, 2, ...$, the triple (c_1^t, c_2^t, L^t) maximizes the utility (1) under the constraints (2), and moreover (ii) for $t = 1, 2, ...$, the allocation $\{c_2^{t-1}, c_1^t, L^t\}_{\infty}^{t=1}$ satisfies

$$N(c_2^{t-1} + c_1^t) = N(E + (a - \pi p_t)L^{t-1}) \tag{3}$$

and

$$NL^t = NL^{t-1}. \tag{4}$$

The equilibrium is said to be *stationary*, if the land price p_t is the same for all t. By the strict concavity of u and v, in this case, the triple (c_1^t, c_2^t, L^t) that maximizes (1) under the constraints (2) is also the same for all t.

3 Stationary Equilibria

Proposition 1 *For any E, a, π, L^0, there exists a unique stationary equilibrium. The stationary-equilibrium land price, $p^*(E, a, \pi, L^0)$, is increasing in E and decreasing in L^0, with*

$$\lim_{E\to\infty} p^*(E, a, \pi, L^0) = \lim_{L^0\to 0} p^*(E, a, \pi, L^0) = \infty. \tag{5}$$

Proof. I first show that, under the specified assumptions, for any E, a, π, L^0, the equation

$$u'(E - pL^0) \cdot p = v'((a + p(1-\pi))L^0) \cdot (a + p(1-\pi)) \tag{6}$$

has a unique solution $p^*(E, a, \pi, L^0)$. This equation is equivalent to the equation

$$u'(E - pL^0) = v'((a + p(1 - \pi))L^0) \cdot \frac{a + p(1 - \pi)}{p}. \tag{7}$$

For p close to zero, the left-hand side of (7) is close to $u'(E)$, and the right-hand side of (7) becomes arbitrarily large. For p close (but less than) E/L^0, the left-hand side of (7) goes out of bounds, and the right-hand side of (7) is close to $v'(aL^0 + (1 - \pi)E) \cdot (a + (1 - \pi)E/L^0)$. Because both sides of (6) are continuous in p, the intermediate-value theorem implies that (7) has a solution. I further note that, under the given assumptions, the left-hand side of (7) is increasing in p. On the right-hand side, $v'((a + p(1 - \pi))L^0)$ is decreasing in p and so is the ratio $\frac{a+p(1-\pi)}{p}$. Therefore the solution to (7) is unique.

Next I note that, if $p_{t+1} = p_t = p$, then a necessary and sufficient condition for a solution to the maximization problem of generation t is given by the equation

$$u'(E - pL^t) \cdot p = v'((a + p(1 - \pi))L^t) \cdot (a + p(1 - \pi)), \tag{8}$$

in combination with the constraints in (2). Hence if $p_{t+1} = p_t = p^*(E, a, \pi, L^0)$, a necessary and sufficient condition for a solution ot the maximization problem of generation t is to have

$$L^t = L^0, \tag{9}$$

as well as

$$c_1^t = E - p^*(E, a, \pi, L^0)L^0 \tag{10}$$

and

$$c_2^t = (a + p^*(E, a, \pi, L^0)(1 - \pi))L^0. \tag{11}$$

At this point, it is easy to see that the triples (c_2^{t-1}, c_1^t, L^t) that are given by (9) - (11) satisfy conditions (3) and (4) for all t. Hence, by setting $p_t = p^*(E, a, \pi, L^0)$ and using (9) - (11) to specify an allocation, one obtains a stationary equilibrium. Uniqueness follows upon observing that the equilibrium condition (4) implies (9) for all t, so (8) is equivalent to (6), which has $p^*(E, a, \pi, L^0)$ as its only solution.

To prove the claimed comparative-statics properties, I note that under the stated assumptions, an increase in E makes the left-hand side of (7) go down without affecting the right-hand side. Since, by the above argument, the difference between the left-hand side and the right-hand side is increasing in p. Following an increase in E, therefore, an increase in p is required to restore equality. An increase in L^0 makes the left-hand side of (7) go up and the right-hand side go down, so a decrease in p is needed to restore equality.

To see that $\lim_{E\to\infty} p^*(E, a, \pi, L^0) = \infty$, it suffices to observe that, if $p^*(E, a, \pi, L^0)$ were bounded as E goes out of bounds, then, for $p = p^*(E, a, \pi, L^0)$, the left-hand side of (6) would converge to zero and the right-hande side would converge to a positive limit. Similarly, if $p^*(E, a, \pi, L^0)$ were bounded as L^0

becomes small, the left-hand side of (6) would converge to $u'(E)$ and the right-hand side of (6) would go out of bounds. In either case, the validity of (6) at $p = p^*(E, a, \pi, L^0)$ would be violated. Hence $p^*(E, a, \pi, L^0)$ must satisfy (5). ■

To understand this proposition, it is useful to rewrite the equation (6) in the form

$$\frac{u'(c_1^t)}{v'(c_2^t)} = 1 + \frac{a}{p} - \pi. \tag{12}$$

This condition equates the intertemporal marginal rate of substitution in consumption to the gross rate of return on holding land. The net rate of return on holding land, $\frac{a}{p} - \pi$, takes the place of the interest rate in other intertemporal models. From Proposition 1, one trivially obtains the following result about the comparison between the equilibrium interest rate and the growth rate of the economy (which is zero).

Proposition 2 *(a) If $\pi = 0$, the net rate of return $\frac{a}{p^*(E,a,\pi,L^0)} - \pi$ on land in the stationary equilibrium of Proposition 1 is positive. (b) If $\pi > 0$ and either E is very large or L^0 is very small, the net rate of return on land $\frac{a}{p^*(E,a,\pi,L^0)} - \pi$ in the stationary equilibrium of Proposition 1 is negative.*

Statement (a) is the result of Homburg (1991) and Rhee (1991), namely, in a model with zero growth, in the absence of transactions costs, the existence of land as non-produced durable asset ensures that the equilibrium rate of interest must be positive. Statement (b) shows that the finding of statement (a) is non-robust. If π is positive, no matter how close to zero, there exist parameters specifications for which the stationary-equilibrium demand for a store of value is so large and therefore the price of land is so large that the transaction cost outweighs the harvest, and the equilibrium rate of interest is negative. This is the case, in particular, if the endowment E in the first period of life is very large, so participants want to shift a lot of purchasing power to the second period of life. It is also the case if L^0 is close to zero, so a high price of land is needed to enable any substantial transfer of purchasing power from one period to the next.

4 Inefficiency of *Laissez-faire* Allocations

For a welfare assessment of stationary equilibria, I use a very weak concept of efficiency that focuses only on consumption and ignores the possibility of saving on transaction costs for land. An allocation $\{c_2^{t-1}, c_1^t\}_{t=1}^{\infty}$ of consumption levels in all periods is said to be *weakly efficient* if there is no alternative allocation $\{\hat{c}_2^{t-1}, \hat{c}_1^t\}_{t=1}^{\infty}$ of consumption levels that is Pareto preferred to $\{c_2^{t-1}, c_1^t\}_{t=1}^{\infty}$ and satisfies

$$N(\hat{c}_2^{t-1} + \hat{c}_1^t) \leq N(c_2^{t-1} + c_1^t) \tag{13}$$

for all t.

Proposition 3 *(a) If* $\frac{a}{p^*(E,a,\pi,L^0)} - \pi > 0$*, the allocation* $\{c_2^{t-1}, c_1^t\}_{t=1}^\infty$ *of consumption levels associated with the stationary equilibrium of Proposition 1 is weakly efficient.*[4]

(b) If $\frac{a}{p^*(E,a,\pi,L^0)} - \pi < 0$*, the allocation* $\{c_2^{t-1}, c_1^t\}_{t=1}^\infty$ *of consumption levels associated with the stationary equilibrium of Proposition 1 is not weakly efficient.*

Proof of Statement (a). The line of argument is the same as in the proof of the First Welfare Theorem for competitive equilibria in a complete market system. Given that, for any $t \geq 1$, the triple (c_1^t, c_2^t, L^t) maximizes (1) subject to (2), it must also be the case that, for any $t \geq 1$, the pair (c_1^t, c_2^t) maximizes (1) subject to

$$c_1^t + \frac{p_t}{a + p_{t+1}(1-\pi)} c_2^t \leq E. \tag{14}$$

Given the stationarity of the equilibrium, with $p_t = p^*(E, a, \pi, L^0)$ for all t, and given the strict monotonicity of (1), this constraint may be rewritten as

$$c_1^t + (1+r)^{-1} c_2^t = E, \tag{15}$$

where

$$r := \frac{a}{p^*(E, a, \pi, L^0)} - \pi. \tag{16}$$

If statement (a) of the proposition is false, there exists an alternative allocation $\{\hat{c}_2^{t-1}, \hat{c}_1^t\}_{t=1}^\infty$ of nonnegative consumption levels satisfying (13) for all t such that

$$\hat{c}_2^0 \geq c_2^0 \tag{17}$$

and, for $t = 1, 2, ...,$

$$u(\hat{c}_1^t) + v(\hat{c}_2^t) \geq u(c_1^t) + v(c_2^t), \tag{18}$$

and at least one of the inequalities in (17) and (18) is strict. Because, for $t \geq 1$, the pair (c_1^t, c_2^t) maximizes (1) subject to (15), it follows that, for $t \geq 1$,

$$\hat{c}_1^t + (1+r)^{-1} \hat{c}_2^t \geq c_1^t + (1+r)^{-1} c_2^t \tag{19}$$

and at least one of the inequalities in (17) and (19) is strict. The inequalities in (19) are equivalent to the inequalities

$$\frac{1}{(1+r)^{t-1}} \hat{c}_1^t + \frac{1}{(1+r)^t} \cdot \hat{c}_2^t \geq \frac{1}{(1+r)^{t-1}} c_1^t + \frac{1}{(1+r)^t} \cdot c_2^t. \tag{20}$$

Upon adding these inequalities over $t = 1, 2, ...$ and adding (17), using the fact that at least one of the inequalities is strict, one obtains

$$\hat{c}_2^0 + \sum_{t=1}^{\infty} \left[\frac{1}{(1+r)^{t-1}} \hat{c}_1^t + \frac{1}{(1+r)^t} \cdot \hat{c}_2^t \right] > c_2^0 + \sum_{t=1}^{\infty} \left[\frac{1}{(1+r)^{t-1}} c_1^t + \frac{1}{(1+r)^t} \cdot c_2^t \right], \tag{21}$$

[4] The conclusion also holds if $\frac{a}{p^*(E,a,\pi,L^0)} - \pi = 0$, but in this case the infinite sums in (21) and (22) are not well defined so the proof is much more involved. In the interest of brevity, I omit this case.

where the infinite sums are well defined because $r > 0$ and the consumption variables are uniformly bounded by $E + aL^0$. Upon reordering sums, one finds that (21) is equivalent to the inequality

$$\sum_{t=1}^{\infty}\left[\frac{1}{(1+r)^{t-1}}\hat{c}_1^t + \frac{1}{(1+r)^{t-1}} \cdot \hat{c}_2^{t-1}\right] > \sum_{t=1}^{\infty}\left[\frac{1}{(1+r)^{t-1}}c_1^t + \frac{1}{(1+r)^{t-1}} \cdot c_2^{t-1}\right], \tag{22}$$

which is incompatible with (13) holding for all $t \geq 1$. The assumption that statement (a) of the proposition is false has thus led to a contradiction. ∎

Proof of Statement (b). Given the stationarity of the equilibrium, let c_1^*, c_2^* be the common values of c_1^t, c_2^{t-1}, $t = 1, 2, ...$ By the first-order condition (12) and the assumption that $\frac{a}{p^*(E,a,\pi,L^0)} - \pi < 0$, there exists a pair $(\hat{c}_1^*, \hat{c}_2^*)$ such that

$$\hat{c}_1^* + \hat{c}_2^* = c_1^* + c_2^*, \tag{23}$$

$$\hat{c}_1^* < c_1^*, \hat{c}_2^* > c_2^*, \tag{24}$$

and

$$u(\hat{c}_1^*) + v(\hat{c}_2^*) > u(c_1^*) + v(c_2^*). \tag{25}$$

Upon setting $(\hat{c}_2^{t-1}, \hat{c}_1^t) = (\hat{c}_2^*, \hat{c}_1^*)$ for all t, one obtains an allocation $\{\hat{c}_2^{t-1}, \hat{c}_1^t\}_{t=1}^{\infty}$ of consumption levels that satisfies (13) for all t and that Pareto dominates the equilibrium allocation $\{c_2^{t-1}, c_1^t\}_{t=1}^{\infty}$. Therefore the equilibrium allocation $\{c_2^{t-1}, c_1^t\}_{t=1}^{\infty}$ is not weakly efficient. ∎

For the model with transaction costs for land, Proposition 3 confirms standard findings about dynamic efficiency and inefficiency of *laissez-faire* allocations. In particular, in an economy with zero growth, such allocations are dynamically efficient when the equilibrium interest rate, $r := \frac{a}{p^*(E,a,\pi,L^0)} - \pi$, is positive and dynamically inefficient when the equilibrium interest rate is negative.

5 Pareto Improvements by Fiscal Intervention

Whereas the Pareto improvement in Proposition 3 (b) involves a direct intervention in the allocation of consumption, I now show that such an improvement can sometimes be achieved by fiscal policy instruments, without such a direct intervention. In the following, I consider the implications of levying a lump sum tax T on each person when this person is young and providing a lump sum subsidy S to each person when this person is old, leaving all other features of the model unchanged. With a stationary population, of course, feasibility requires that $S \leq T$.

Given the specified fiscal intervention, the budget constraints in (2) take the form

$$c_1^t = E - p_t L^t - T \quad \text{and} \quad c_2^t = (a + p_{t+1}(1-\pi))L^t + S. \tag{26}$$

An *equilibrium for* T is now given by a land price sequence $\{p_t\}_{t=1}^{\infty}$ and an allocation $\{c_2^{t-1}, c_1^t, L^t\}_{t=1}^{\infty}$ such that (i) $c_2^0 = (a + p_1(1-\pi))L^0 + T$ and, for $t = 1, 2, ...,$ the triple (c_1^t, c_2^t, L^t) maximizes the utility (1) under the constraints (26), and moreover (ii) for $t = 1, 2, ...,$ the allocation $\{c_2^{t-1,} c_1^t, L^t\}_{\infty}^{t=1}$ satisfies

$$N(c_2^{t-1} + c_1^t) = N(E + (a - \pi p_t)L^{t-1} - T + S) \tag{27}$$

and

$$NL^t = NL^{t-1}. \tag{28}$$

The equilibrium is again said to be *stationary* if the land price p_t is the same for all t. As before, the associated plans (c_1^t, c_2^t, L^t) that maximize (1) subject to (26) are also the same for all t.

Proposition 4 *For any $T < E$ and $S \leq T$, any E, a, π, L^0, there exists a unique stationary equilibrium for T. The stationary-equilibrium land price, $p^{**}(T, S, E, a, \pi, L^0)$, is increasing in E, as well as decreasing in T, S and L^0, with*

$$\lim_{E\to\infty} p^{**}(T, S, E, a, \pi, L^0) = \lim_{L^0\to 0} p^{**}(T, S, E, a, \pi, L^0) = \infty. \tag{29}$$

Proof. For uniqueness and for the comparative statics with respect to E and L^0, the argument is the same as in the proof of Proposition 1, with equation (7) replaced by the equation

$$u'(E - pL^0 - T) = v'((a + p(1-\pi))L^0 + S) \cdot \frac{a + p(1-\pi)}{p}. \tag{30}$$

The details are left to the reader. For the comparative statics with respect to T and S, it suffices to observe that increases in T and S make the difference between the left-hand side and the right-hand side of (30) go up, so a decrease in p is needed to restore equality in (30). ■

An increase in T reduces resources available to people in the first period of their lives, and an increase in S increases resources available to people in the second period of their lives. Both effects reduce a person's need and desire for transferring resources from the first to the second period of life. Thus the demand for land as a store of value is reduced, the equilibrium price of land is lower, and the level of second-period consumption is higher.

I next consider the welfare effects of a simultaneous increase in T and S. For this purpose, I assume that S takes the form $S = (1-\sigma)T$, for some $\sigma \in [0, 1)$, i.e. I allow for some dissipation of resources by the government. The following result shows that, if the stationary equilibria under *laissez-faire* are dynamically inefficient and if σ is sufficiently small, one can actually use a combination of a first-period lump sum tax and a second-period lump sum subsidy to generate a Pareto superior allocation.

Proposition 5 *If $\frac{a}{p^*(E,a,\pi,L^0)} - \pi < 0$, then for $\sigma < \pi - \frac{a}{p^*(E,a,\pi,L^0)}$, the allocation $\{c_2^{t-1}, c_1^t\}_{t=1}^{\infty}$ of consumption levels associated with the stationary equilibrium of Proposition 1 is Pareto dominated by the allocation $\{\hat{c}_2^{t-1}, \hat{c}_1^t\}_{t=1}^{\infty}$ of* *consumption levels associated with the stationary equilibrium of Proposition 4 for $T > 0$ sufficiently close to zero and $S = (1-\sigma)T$.*

Proof. Dropping the dependence on other parameters, for any T, let $p(T), \hat{c}_1(T)$ and $\hat{c}_2(T)$ be the common values of p_t, $\hat{c}_1^t$ and $\hat{c}_2^{t-1}$, $t = 1, 2, ...$, in the stationary equilibria with parameters $T, S = (1-\sigma)T, E, a, \pi, L^0$. Upon applying the implicit function theorem to (30), one finds that $p(T)$ is actually differentiable, with a derivative given as:

$$p'(T) = -\frac{1}{L^0} \frac{u''(\hat{c}_1(T)) + v''(\hat{c}_2(T))(1-\sigma)\frac{a+p(1-\pi)}{p}}{u''(\hat{c}_1(T)) + v''(\hat{c}_2(T))\frac{a+p(1-\pi)}{p} - v'(\hat{c}_2(T))\frac{a}{p^2}}. \tag{31}$$

Thus, under the given assumptions on u and v, $p'(T)L^0 \in (-1, 0)$ for all T.

Since

$$\hat{c}_1(T) = E - p(T)L^0 - T \tag{32}$$

and

$$\begin{aligned} \hat{c}_2(T) &= (a + p(T)(1-\pi))L^0 + (1-\sigma)T \\ &= aL^0 + (1-\pi)(p(T)L^0 + T) + (\pi - \sigma)T, \end{aligned} \tag{33}$$

it follows that $\hat{c}_1(T)$ is decreasing and $\hat{c}_2(T)$ is increasing in T.

To assess the welfare implications of the fiscal intervention, I first note that people born in date 0 benefit from the increase in $\hat{c}_2(T)$ and are therefore better off. For people born in $t \geq 1$, the welfare assessment depends on the effects of T on the lifetime utility

$$V(T) := u(\hat{c}_1(T)) + v(\hat{c}_2(T)). \tag{34}$$

Using (32) and (33), one obtains:

$$V(T) = u(E - p(T)L^0 - T) + v((a + p(T)(1-\pi))L^0 + (1-\sigma)T) \tag{35}$$

and therefore

$$\begin{aligned} V'(T) = &-u'(\hat{c}_1(T)) + v'(\hat{c}_2(T)) \cdot (1-\sigma) \\ &-u'(\hat{c}_1(T)) \cdot p'(T)L^0 + v'(\hat{c}_2(T)) \cdot (1-\pi) \cdot p'(T)L^0. \end{aligned} \tag{36}$$

Using (30) to substitute for $u'(\hat{c}_1(T))$, one further obtains

$$\begin{aligned} V'(T) = &\; v'(\hat{c}_2(T)) \cdot \left[(1-\sigma) - \frac{a}{p(T)} - 1 + \pi\right] \\ &-v'(\hat{c}_2(T))\frac{a}{p(T)} \cdot p'(T)L^0. \end{aligned} \tag{37}$$

The second term on the right-hand side is positive because $p'(T) < 0$. If $\sigma < \pi - \frac{a}{p^*(E,a,\pi,L^0)}$, then at $T = 0$, the first term on the right-hand side is positive, so (37) implies $V'(0) > 0$. The proposition follows immediately. ■

The imposition of a lump sum tax T on people in the first period of their lives, accompanied by a lump sum subsidy in the second period of their lives, has two effects on the equilibrium allocation. There is a direct effect because the intervention itself lowers $\hat{c}_1(T)$ and raises $\hat{c}_2(T)$. This effect is beneficial if the excess of the marginal utility $v'(\hat{c}_2(T))$ of second-period consumption over the marginal utility $u'(\hat{c}_1(T))$ of first-period consumption is large enough to outweigh the dissipation loss σT. Given that the marginal utilities of second-period and first-period consumption satisfy the equilibrium condition (30), this condition is equivalent to the inequality $\sigma < \pi - \frac{a}{p}$, i.e. the dissipation effect must be smaller than the absolute value of the (negative) net rate of return on land. There is also an indirect effect because the intervention lowers $p(T)$, which raises $\hat{c}_1(T)$ and lowers $\hat{c}_2(T)$. This effect is always beneficial because, by (30), in equilibrium $u'(\hat{c}_1(T))$ always exceeds $v'(\hat{c}_2(T))(1-\pi)$ by $v'(\hat{c}_2(T))\frac{a}{p(T)}$.

If $\sigma = 0$, the specified fiscal intervention provides a Pareto improvement over *laissez-faire* whenever *laissez-faire* induces negative interest rates. This is similar to Proposition 3 (b), except that now the indirect effect of the fiscal intervention lowering the land price provides for an additional improvement.

If $\sigma \in (0, \pi)$, the condition $\sigma < \pi - \frac{a}{p}$ is not necessarily satisfied when *laissez-faire* induces negative interest rates. However, if the *laissez-faire* demand for storing value is large enough, i.e. if E is high enough, the equilibrium land price will be high enough so that the condition $\sigma < \pi - \frac{a}{p}$ is satisfied, i.e. despite the dissipation cost, the specified fiscal intervention will be Pareto-improving.

If $\sigma \geq \pi$, the specified fiscal intervention does not provide a Pareto improvement over *laissez-faire*. Because the dissipation cost is so high, in this case, the direct effect of the fiscal intervention on V is necessarily negative. Moreover, since $p'(T)L^0 \in (-1, 0)$, (37) implies that, in this case, the direct effect always outweighs the indirect effect.

6 Discussion

The theoretical model in the preceding analysis is ridiculously simple, even simpler than those of Homburg (1991) and Rhee (1991), let alone Diamond (1965) and Blanchard (2019). In what sense are the results nevertheless relevant for the policy discussion?

There are several answers to this question. First, some of the simplicity is a matter of exposition rather than substance. Extending the conclusions of the analysis to the settings that have previously been used to discuss dynamic inefficiency and fiscal policy is a routine exercise. Such settings include Homburg's (1991) model in which the supply of the consumption good is the result of contemporaneous production by means of labour and land, rather than an exogenous constant $E + aL^0$. They also include models with real capital serving as an input into production, i.e. an output of production in periods prior to t that serves as an additional input in period t, as in Diamond (1965), Tirole (1985), Homburg (1991), Rhee (1991), and Blanchard (2019). In such models, a transaction cost on land of the sort considered here makes room for dynamic inefficiency in the form of an overaccumulation of capital, as well as an excessive consumption of the young, as in the analysis here. In all these models, the theoretical claim that such inefficiency cannot arise if there is a non-produced durable asset such as land is correct if this asset involves no transaction cost, but is not robust to the introduction of a transaction cost that is a constant fraction of transaction value.

Second, there is a question of how to take account of the fact that in the real world, the most valuable pieces of land are not traded in isolation but in combination with structures that have been built on them. Unbuilt land, in particular land where zoning prohibits building, is much cheaper than built-up land. If one thinks about the structures as being durable, one can simply reinterpret the asset "land" in the analysis here as a package of "land-cum-structure"; the logic of the analysis is unaffected. If one takes account of the endogeneity of the structures, the argument of Diamond (1965) suggests that dynamic inefficiency may involve overinvestment in them. The possibility of industrial decline destroying the usefulness of the structures and possibly even the land puts limits on the assumed durability and therefore on the scope for values to become unbounded as interest rates go to zero. Thus the main point of the analysis may even be strengthened when the durable assets in the model are thought of in terms of "land-cum-structure" packages rather than land.

Land-cum-structure packages are heterogeneous, in terms of quality and size of structures and in terms of location. The heterogeneity of the packages goes along with frictions to tradability. Frictions can arise from matching problems, as well as lemons problems. Matching problems arise from the lack of mobility of participants, limits to divisibility, and the inhomogeneity of different units, where differences in location, neighbourhood and makeup can matter greatly. Lemons problems arise from asymmetries of information about the soundness of structures, or about toxic residues from past activities.

Such frictions contribute substantially to transaction costs for land-cum-structure packages in the real world. In the United States, transaction costs for real-estate are on the order of ten percent of the value of the transaction. More than half of this cost, six percent of the value of the transaction, goes to the real-estate brokers for matching up buyers and sellers. The remainder is spent on legal fees, recording fees, title search and insurance (together 1.5 - 3.5 %) and taxes (up to 1.5 %). In Germany, transaction costs range from nine to sixteen percent of the value of the transaction, with brokerage fees, including value-added tax, ranging from 3.57 % to 7.14 %, depending on bargaining powers, notarization and registration costs ranging from 2 % to 3 %, and taxes ranging from 3.5 % to 6.5 %, depending on regional legislation.[5] The above analysis,

[5] Data from Global Property Guide 2020, https://www.globalpropertyguide.com/faq/guide-transaction-costs, accessed March 9, 2020. In France, total transaction costs can be much higher yet, ranging from 8 % to 29 %, with a 3 % - 10 % range for real estate agents and a 3 % to 10 % range for notarization fees. To some extent, the indicated ranges reflect nonlinearities in schedules relating values to transaction costs. Such nonlinearities may be due to statutes or to greater bargaining powers of parties with large-value transactions. Even

which simply assumes a transaction cost πp can be seen as a way to take account of the implications of the frictions without actually modelling them.[6]

Is it legitimate to link the numbers for real-world transaction costs to the transaction cost term πp in the model? After all, the model has no financial sector that would allow for a separation between the need for a store of value, the ownership of the real assets and the use of the assets. Some of the impediments to tradability in real-estate markets, in particular, the matching frictions and the need for costly real-estate agents, involve the specifics of the use of the assets whole holding them, rather than their ability to store value and make value available through resale at a later time. Financial institutions such as real-estate investment trusts or pension funds provide for a separation of the store-of-value function to investors from the provision of services to users. However, such institutions are not able to avoid the costs of real-estate transactions altogether; moreover, they have their own costs, including the agency costs associated with the management of real-estate portfolios. In some cases, for example with non-traded public real-estate investment trusts, these costs are on the same order of magnitude as the above numbers for real-estate transactions.

In the overlapping-generations model, the need for a store of value arises solely from the desire to provide for retirement. In the real world, there are multiple reasons for why one might need a store of value, for example uncertainty and precautionary saving, or lumpy transactions that are spaced apart in time. Also real-world holding periods are endogenous and differ depending on why value is stored and what assets are held.[7]

The multiplicity of needs for storing value and the development of strategies and institutions for adapting different kinds of assets to these needs may however be more important for our understanding of the financial system than for the policy debate to which this paper is attached. The numbers presented by von Weizsäcker (2014) as well as von Weizsäcker and Krämer (2019) indicate that saving for retirement has contributed a substantial part of the strong worldwide growth of private savings over the past few decades, driven by demographic change as well as increasing incomes, in emerging economies as well as OECD countries. Given these findings, reliance on a model that focuses on retirement

then, the analysis remains valid unless transaction costs per unit of land are actually bounded below the one-period real return although acquisition and resale prices are unbounded.

[6] In the German debate about the potential role of government debt as an antidote to dynamic inefficiency, von Weizsäcker (2014) and von Weizsäcker and Krämer (2019) have also argued that land is not well suited for serving as a store of value whose availability can prevent interest rates from dropping below growth rates. In addition to the issue of inhomogeneity and potential information asymmetry, they point to the risks attached to returns and to political risks, including risks of total or partial expropriation, which are the larger, the higher the value of land is. They sugggest that these risks call for premia on the order of 7 percent p.a., so that observed rates of return on real estate would imply negative interest rates for safe investments. They also suggest that the political risks put bounds on the extent to which price appreciation for land may be sufficient to deal with the entire demand for a store of value.

[7] For example, assets with high transaction costs may be relatively unattractive if holding periods are expected to be short and relatively attractive if holding periods are expected to be long.

provision as the cause of a need for storing value seems quite appropriate.

With a focus on retirement provision, and the costs of alternative arrangements, it is of interest to note that, in relative terms, administrative costs of pay-as-you-go systems tend to be fairly low. In the United States, the share of total administrative costs of old-age and disability insurance in expenditures has declined from some slightly more than 2% in the 1950s and 1960s to 0.7% in recent years, 0.4% for old-age insurance and 1.9% for disability insurance.[8] In Germany, the share of total administrative costs in expenditures is on the order of two percent. These numbers are significantly lower than the per-unit-of-value transaction costs for real estate. Thus the condition in Proposition 5 that the dissipation coefficient σ in a fiscal intervention be less than the transaction cost parameter π for land is not outlandish. The reason is not that the government is so much more efficient but that the pay-as-you-go system, despite the flaws it may otherwise have, does not have to deal with asset quality and asset management.

None of this is to be interpreted as taking a position on the Blanchard-von Weizsäcker proposition that the current macroeconomic environment calls for an expansion of public debt. An assessment of that proposition would have to encompass other forms of fiscal intervention in the provision of stores of value, from tradable debt securities to fiat money. Such an assessment would also have to take into account that, in Proposition 5, the inequality $\sigma < \pi$ is necessary, but not sufficient for fiscal intervention to make for a Pareto improvement.

However, the preceding analysis forestalls the killer argument that, because dynamic inefficiency cannot arise in an economy with land, we know for sure that a state of affairs with interest rates below growth rates cannot last and therefore should not be made a basis for public policy. The killer argument is correct when there are no transaction costs for land, but this finding is not robust to the introduction of even a small per-unit-of-value transaction cost.

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