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Notes on Endogenous Change of Tastes

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1. INTRODUCTION

Economists have always acknowledged the fact that tastes of consumers change (e.g., Marshall in his *Principles*). But the overwhelming majority took the attitude that it is not their business to be concerned with these changes of taste. As one example, we may quote Friedman's *Price Theory* [3]: "Despite these qualifications, economic theory proceeds largely to take wants as fixed. This is primarily a case of division of labour. The economist has little to say about the formation of wants; this is the province of the psychologist. The econmist's task is to trace the consequences of any given set of wants. The legitimacy of and justification for this abstraction must rest ultimately, in this case as with any other abstraction, on the light that is shed and the power to predict that is yielded by the abstraction" (p. 13). There is no doubt that the analysis of the formation of wants or tastes needs the help of the psychologist, perhaps also the help of the biologist, the physician, the sociologist, etc. But as these other sciences give insight into the process of the formation of tastes, economic theory may have to take the results into account, at least some of them.

There are influences on tastes which may have little to do with those variables economists are usually concerned about. From the point of view of the economist we can call them exogenous influences. We may acknowledge their existence, but we need not investigate them very much further. But other influences on tastes may depend more or less directly on certain economic variables like, for instance, the tendency of firms to increase their profits through advertising, or the consumption patterns of other consumers, or the consumer's past experience with consumer goods. If these influences on tastes are not taken into account, we clearly would be in the danger of making false predictions due to misspecification of certain parameters of our model. To this extent, the abstraction would fail to be congruent with one of the criteria of a legitimate abstraction as Friedman has formulated it in the paragraph quoted above. But it is also a legitimate question to ask whether this abstraction gives the proper insight into economic problems. The efficiency or Pareto optimality properties of general equilibrium models rest on the assumption of fixed tastes. Are these efficiency theorems relevant to a world with changing tastes? Surely, purely exogenous changes of tastes (from the point of view of the economist) will essentially not alter these theorems. But what about Pareto optimality if endogenous variables of the economic system influence tastes?

I think it is fair to say that this question has not yet been investigated in sufficient depth by economists. Yet it appears to be one of the important problems in the discussion about the social relevance of economic theory. Does the model with fixed tastes give insights into the problems of an economy with endogenously changing tastes? One of the reasons why economists did not very deeply discuss this question may be that their present concepts of Pareto optimality and efficiency possibly are not flexible enough to cope with endogenously changing tastes. It may become necessary to change the conceptual framework of our theory; we may be forced to ask almost philosophical questions about the concepts we use. In every discipline, there is always a reluctance to engage into this kind of activity.

In the following I want to discuss one example of endogenously changing tastes; I presume it is the conceptually simplest example. I shall assume that the tastes of a consumer depend on his past consumption. The model of consumer behavior which I have in mind has been formulated by Marshall who, in discussing the law of decreasing marginal utility, writes: "There is, however, an implicit condition in this law which should be made clear. It is that we do not suppose time to be allowed for any alteration in the character or tastes of the man himself. It is, therefore, no exception to the law that the more good music a man hears, the stronger is his taste for it likely to become; that avarice and ambition are often insatiable; or that the virtue of cleanliness and the vice of drunkenness alike grow on what they feed upon. For in such cases our observations range over some period of time; and the man is not the same at the beginning as at the end of it" [7, p. 79].

There exists some literature on models in which past consumption influences present consumption. But as far as I can see their central theme never seems to be the welfare implications of this phenomenon. I would like to mention [2, 6, 8, 9].

2. Short Run and Long Run Demand Functions

For simplicity, I shall assume that there are only two goods. Also for simplicity I shall assume that tastes are only influenced by the consumption vector of the last period. Influences from periods before the last one are neglected. Although the mathematics would become more complicated, I presume that the relaxation of these two assumptions would not change the substance of the argument.

Let $q_1{}^t$ be the quantity of good 1 consumed in period t. Let $q_2{}^t$ be the quantity of good 2 consumed in period t. Prices are denoted by p_1, p_2. Income is denoted by y. Tastes together with prices and income determine demand. Hence we can write down the following demand equations

$$q_1{}^t = f_1(p, y, q^{t-1}), \tag{1}$$

$$q_2{}^t = f_2(p, y, q^{t-1}), \tag{2}$$

where p and q are the price and quantity vectors, respectively. We shall assume the budget equation

$$p_1 q_1 + p_2 q_2 = y \tag{3}$$

to hold in every period.

The first question to ask is the question of stability of the process of adaptation of demand to given prices and to a given income. We introduce the following notation, referring to the demand functions (1) and (2). Let

$$a_{ij} = \frac{\partial f_i{}^t}{\partial q_j^{t-1}}.$$

Then we have, keeping prices and income constant.

$$dq_1{}^t = a_{11}\, dq_1^{t-1} + a_{12}\, dq_2^{t-1},$$
$$dq_2{}^t = a_{21}\, dq_1^{t-1} + a_{22}\, dq_2^{t-1}.$$

From the budget Eq. (6), we get

$$p_1 dq_1{}^t + p_2 dq_2{}^t = 0.$$

Since we can vary dq_1^{t-1}, dq_2^{t-1} freely, it follows that

$$p_1 a_{11} + p_2 a_{21} = 0, \tag{4}$$

$$p_1 a_{12} + p_2 a_{22} = 0. \tag{5}$$

We now can prove the following result:

If $| a_{11} + a_{22} | < 1 - \epsilon$ everywhere for some $\epsilon > 0$, then the process of adaptation is stable and has a unique equilibrium point.

Proof. By the mean value theorem and the inequality

$$| a_{11} + a_{22} | < 1 - \epsilon,$$

there exist α_{11}, α_{12}, α_{21}, and α_{22} such that

$$\Delta q_1{}^t = q_1^{t+1} - q_1{}^t = \alpha_{11}\, \Delta q_1^{t-1} + \alpha_{12}\, \Delta q_2^{t-1},$$

$$\text{with } | \alpha_{11} + \alpha_{22} | < 1 - \epsilon \qquad \text{and} \quad \alpha_{12} = \frac{p_2}{p_1}\, \alpha_{22}\,;$$

hence we get

$$p_1\, \Delta q_1{}^t = \alpha_{11} p_1\, \Delta q_1^{t-1} - \alpha_{22} p_2\, \Delta q_2^{t-1}.$$

But for $t - 1 \geqslant 1$, we have $p_1 \Delta q_1^{t-1} + p_2 \Delta q_2^{t-1} = 0$ (due to the budget constraint), then

$$p_1\, \Delta q_1{}^t = (\alpha_{11} + \alpha_{22})\, p_1\, \Delta q_1^{t-1} \tag{6}$$

which implies

$$p_1\, | q_1{}^T - q_1{}^t | \leqslant p_1 (1 - \epsilon)^{t-1} \frac{| q_1{}^2 - q_1{}^1 |}{\epsilon}\,, \; T \geqslant t,$$

which implies convergence of $q_1{}^t$. Then $q_2{}^t$ also converges, being linearly related to $q_1{}^t$. The equilibrium point is unique for, by the continuity of the demand functions, it must satisfy the equations

$$q_1{}^* = q_1(p, y, q^*),$$
$$q_2{}^* = q_2(p, y, q^*),$$

Let $\bar{q}$ be any other point; then we know by a similar argument as in the stability proof that

$$p_1(q_1(p, y, \bar{q}) - q_1(p, y, q^*)) = (\alpha_{11} + \alpha_{22})\, p_1(\bar{q}_1 - q_1{}^*) \text{ with}$$
$$| \alpha_{11} + \alpha_{22} | < 1 - \epsilon \text{ which proves } q_1(p, y, \bar{q}) \neq \bar{q}_1 \text{ for } \bar{q}_1 \neq q_1{}^*.$$

The same is true of q_2. This proves uniqueness of the equilibrium point.

Let us then assume that for any set of prices and of income there exists a unique equilibrium demand vector $q^* = F(p, y)$ to which demand q converges, if prices p and income y remain constant. We can interpret $F(p, y)$ as a demand function. We could call it the long-run demand function as opposed to the short-run demand function $f(p, y, q^{t-1})$. What are the properties of the long-run demand function? Using the equation

$$F(p, y) = f(p, y, F(p, y)) \tag{6}$$

and introducing the notation f_{ij} (or F_{ij}) for the partial derivative of demand for good i with respect to price j and f_{iy} (or F_{iy}) for the partial derivative of demand for good i with respect to income, we get, by differentiation of the above equation,

$$\begin{aligned} dq_1 &= F_{11}\,dp_1 + F_{12}\,dp_2 + F_{1y}\,dy \\ &= f_{11}\,dp_1 + f_{12}\,dp_2 + f_{1y}\,dy + a_{11}\,dq_1 + a_{12}\,dq_2 \,. \end{aligned}$$

We first put dp_1, dp_2 equal to zero and then we observe

$$p_1\,dq_1 + p_2\,dq_2 = dy,$$

$$a_{12} = -\frac{p_2}{p_1}\,a_{22}\,.$$

By multiplying with p_1, we get

$$\begin{aligned} p_1\,dq_1 = p_1F_{1y}\,dy &= p_1f_{1y}\,dy + a_{11}p_1\,dq_1 - a_{22}p_2\,dq_2 \\ &= p_1f_{1y}\,dy + a_{11}p_1\,dq_1 + a_{22}(p_1\,dq_1 - dy) \\ &= \frac{p_1f_{1y} - a_{22}}{1 - a_{11} - a_{22}}\,dy \qquad \text{or} \quad F_{1y} = \frac{f_{1y} - \dfrac{a_{22}}{p_1}}{1 - a_{11} - a_{22}}\,. \end{aligned}$$

Similarly,

$$F_{2y} = \frac{f_{2y} - \dfrac{a_{11}}{p_2}}{1 - a_{11} - a_{22}}\,.$$

If we vary price p_1 only, we have

$$p_1\,dq_1 + p_2\,dq_2 = -q_1\,dp_1\,,$$

which implies

$$\begin{aligned} p_1\,dq_1 = p_1F_{11}\,dp_1 &= p_1f_{11}\,dp_1 + p_1a_{11}\,dq_1 - a_{22}p_2\,dq_2 \\ &= p_1f_{11}\,dp_1 + p_1a_{11}\,dq_1 + a_{22}(p_1\,dq_1 + q_1\,dp_1) \\ &= \frac{p_1f_{11} + a_{22}q_1}{1 - a_{11} - a_{22}}\,dp_1\,, \qquad \text{or} \quad F_{11} = \frac{f_{11} + \dfrac{a_{22}q_1}{p_1}}{1 - a_{11} - a_{22}}\,. \end{aligned}$$

In a similar way, we prove

$$F_{21} = \frac{f_{21} + \dfrac{a_{11}q_1}{p_1}}{1 - a_{11} - a_{22}}, \qquad F_{22} = \frac{f_{22} + \dfrac{a_{11}q_2}{p_2}}{1 - a_{11} - a_{22}},$$

$$F_{12} = \frac{f_{12} + \dfrac{a_{22}q_2}{p_1}}{1 - a_{11} - a_{22}}.$$

We are then able to derive the slopes of demand curves keeping real income constant. In that case, the relation $dy = q_j\, dp_j$ holds for $j = 1, 2$. Denoting the slopes of the compensated demand curves with g_{ij} for the short-run and with G_{ij} for the long-run demand function, we have the Slutsky equations

$$g_{ij} = f_{ij} + q_j f_{iy}, \qquad i, j = 1, 2,$$
$$G_{ij} = F_{ij} + q_j F_{iy}.$$

They yield

$$G_{11} = \frac{f_{11} + \dfrac{a_{22}q_1}{p_1}}{1 - a_{11} - a_{22}} + q_1 \frac{f_{1y} - \dfrac{a_{22}}{p_1}}{1 - a_{11} - a_{22}}$$
$$= \frac{f_{11} + q_1 f_{1y}}{1 - a_{11} - a_{22}} = \frac{g_{11}}{1 - a_{11} - a_{22}}.$$

or in general

$$G_{ij} = \frac{g_{jj}}{1 - a_{11} - a_{22}}.$$

The long-run direct and cross price elasticities of demand are proportional to the equilibrium short-run price elasticities. The factor of proportionality is given by $1/(1 - a_{11} - a_{22})$ which by virtue of our assumption is always positive. If $a_{11} + a_{22}$ is positive, the price elasticities are higher in the long run than in the short run. If $a_{11} + a_{22}$ is negative, the short run price elasticities are higher. It is not necessarily true that the long-run elasticities are higher than the short-run elasticities. Here the analogy with the Marshallian theory of the firm is not as close as perhaps one would have hoped.

3. A Theorem on Long Run Indifference Curves

Since the short-run demand function is derived from a utility function, we know that $g_{ij} = g_{ij}$. It follows from the equation above that the integrability condition $G_{ji} = G_{ji}$ is also fulfilled for the long-run demand function. We are then able to derive a consistent set of "indifference curves" corresponding to the long-run demand function $F(p, y)$. We can draw these indifference curves in the q_1, q_2-space. For a given income y we also can draw them in the p_1, p_2-space. They obey the differential equation

$$\frac{dp_2}{dp_1} = -\frac{F_1(p, y)}{F_2(p, y)}$$

in price space and

$$\frac{dq_2}{dq_1} = -\frac{p_1}{p_2}$$

in quantity space. The question arises: What is the economic interpretation of these long-run indifference curves. The axiom of revealed preference is satisfied. Thus we could give these indifference curves a revealed preference interpretation.

We shall investigate two cases. Case I is characterized by

$$0 \leqslant a_{11} + a_{22} \leqslant 1 - \epsilon, \qquad \text{Case II by } -1 + \epsilon \leqslant a_{11} + a_{11} \leqslant 0.$$

We will not have to say very much about cases where at certain points one of these inequalities and other points the other inequality holds.

Case I. $0 \leqslant a_{11} + a_{22} < 1$. Here the long-run "indifference curve" can be characterized in a way so that it appears to be a reasonable concept from an economic point of view. We restrict our attention to the regular case of long-run indifference curves; the system of differential equations

$$\frac{dq_2}{dq_1} = r(q)$$

describing the long-run indifference curves is such that only one solution of the differential equation passes through any point q.

Let us introduce the following notation: $q'I(q)\, q''$ means q' and q'' are indifferent, given that consumption of past period was q. $q'K(q)\, q''$ means q' is preferred or indifferent to q'', given that consumption of past period was q. $q'L(q)\, q''$ means q' is preferred to q'', given that consumption of past period was q.

Continuous differentiability of the short-run demand functions with respect to their arguments implies a property of the short-run indifference curves which we call continuity of indifference curves: If $q'L(q)\, q''$, then there exist neighborhoods N of q, N' of q' and N'' of q'' such that for any $\bar{q} \in N, \bar{q}' \in N', \bar{q}'' \in N''$ the relation $\bar{q}'L(\bar{q})\, \bar{q}''$ holds.

We then can prove the following theorem: Let a long-run demand vector $q^0 = F(p^0, y^0)$ be given. Let $A(q^0)$ be the long-run "indifference curve" going through q^0. Let $\bar{q}$ be another point in the commodity space.

Let the short run indifference curve have the continuity property. There exists a finite sequence of vectors

$$q^0, q^1, q^2, \ldots, q^n = \bar{q}$$

such that $q_1 L(q^0)\, q^0, q^2 L(q^1)\, q^1, \ldots, \bar{q} = q^n L(q^{n-1})\, q^{n-1}$ if and only if $\bar{q}$ lies above the indifference curve $A(q^0)$.

In other words, it is possible for the consumer to go from q^0 to $\bar{q}$ in a finite number of periods and always feel improved compared to the already attained status quo of the last period if and only if $\bar{q}$ lies above the indifference curve going through q^0. In this sense the long-run indifference curves exhibit the "long-run preference structure" of the person.

Proof of the Theorem. We first prove the necessity of the condition that $\bar{q}$ lies above $A(q^0)$. Let $A(q^*)$ be the long-run indifference curve going through any point q^* in quantity space. Let $B(q, q^*)$ be the short-run indifference curve corresponding to past consumption q^* and going through point q in quantity space. Correspondingly, let $\tilde{A}(p^*)$ be the long-run indifference curve in price space going through p^* for a fixed income level Y. For the same income level Y, denote by $\tilde{B}(p, q^*)$ the short-run indifference curve going through p for past consumption q^*. Consider now any p^* and the corresponding long-run demand vector $q^* = F(p^*)$. As is well known, the slope of any indifference curve in price space is given by the demand ratio $-q_1(p)/q_2(p)$ at that point.

Consider any point $p \neq p^*$ with $p \in \tilde{A}(p^*)$.

We are interested in the slope of $\tilde{A}(p^*)$ and $\tilde{B}(p, q^*)$ at the point p. They are given by

$$-\frac{F_1(p)}{F_2(p)} = -\frac{f_1(p, F(p))}{f_2(p, F(p))} \text{ in the case of } \tilde{A}(p^*),$$

and

$$-\frac{f_1(p, q^*)}{f_2(p, q^*)} = -\frac{f_1(p, F(p^*))}{f_2(p, F(p^*))} \text{ in the case of } \tilde{B}(p, q^*).$$

We compute the difference between $f_1(p, F(p))$ and $f_1(p, F(p^*))$. By Taylor's theorem, we can write

$$\begin{aligned} &f_1(p, F_1(p), F_2(p)) - f_1(p, F_1(p^*), F_2(p^*)) \\ &\qquad = (F_1(p) - F_1(p^*))\, \alpha_{11} + (F_2(p) - F_2(p^*))\, \alpha_{12}\,, \end{aligned}$$

where α_{ij} are the values of a_{ij} at some point p' on the line between p and p^*.

As observed earlier, we always have $\alpha_{12} = -(p_2/p_1)\,\alpha_{22}$, and then we can write

$$\begin{aligned} &p_1(f_1(p, F(p) - f_1(p, F(p^*))) \\ &= p_1(F_1(p) - F_i(p^*))\,\alpha_{11} - p_2(F_2(p) - F_2(p^*))\,\alpha_{22} \\ &= p_1(\alpha_{11} + \alpha_{22})(F_1(p) - F_1(p^*)), \end{aligned}$$

since for a constant nominal income Y the equation

$$p_2(F_2(p) - F_2(p^*)) = -p_i(F_1(p) - F_1(p^*))$$

holds. Since $\alpha_{11} + \alpha_{22} \geqslant 0$, we have shown that

$$(F_1(p) - f_1(p, q^*))(F_1(p) - F_1(p^*)) \geqslant 0.$$

In a similar way we can show

$$(F_2(p) - f_2(p, q^*))(F_2(p) - F_2(p^*)) \geqslant 0.$$

For $p_1 > p_1{}^*$, this implies

$$F_1(p) - f_1(p, q^*) \leqslant 0, \text{ since } F_1(p) - F_1(p^*) \leqslant 0.$$

(Demand decreases as price increases as we move along an indifference curve.) Similarly, we have

$$F_1(p) - f_1(p, q^*) \geqslant 0 \qquad \text{for} \quad p_1 < p_1{}^*.$$

Consider now a utility index $U_{q*}(p)$ representing the indifference curves in price space, given past consumption q^*. We have

$$dU_{q^*} = \frac{\partial U_{q^*}}{\partial p_1}\,dp_1 + \frac{\partial U_{q^*}}{\partial p_1}\,dp_2\,,$$

where

$$\frac{\dfrac{\partial U_{q^*}}{\partial p_2}}{\dfrac{\partial U_{q^*}}{\partial p_1}} = -\,\frac{f_2(p, q^*)}{f_1(p, q^*)}\,.$$

If we move along $\tilde{A}(p^*)$, we have

$$\frac{dp_2}{dp_1} = -\,\frac{F_1(p)}{F_2(p)}$$

hence

$$\frac{dU_{q^*}}{dp_1} = \frac{\partial U_{q^*}}{\partial p_1}\left(1 - \frac{f_2(p, q^*)}{f_1(p, q^*)}\,\frac{F_1(p)}{F_2(p)}\right),$$

where the expression in square brackets is nonnegative for $p_1 > p^*$, and $(\partial U_{q^*}/\partial p_1) \leqslant 0$; hence $(dU_{q^*}/dp_1) \leqslant 0$. Utility decreases as one moves along $\tilde{A}(p^*)$ away from p^*, as long as $p_1 > p_1^*$.

Similarly, we prove that utility also decreases as one moves along $\tilde{A}(p^*)$ away from p^* for $p_1 < p_1^*$. This implies that p^* is the point which maximizes U_q^* within the set $\tilde{A}(p^*)$. Hence the short-run indifference curve corresponding to past consumption q^* lies below $\tilde{A}(p^*)$ in price space and above $A(q^*)$ in quantity space.

Consider now any sequence $q^0, q^1, \ldots, q^n = \bar{q}$ such that

$$\bar{q}L(q^{n-1})\, q^{n-1}L(q^{n-2})\, q^{n-2} \cdots q^1L(q^0)\, q^0.$$

Then q^1 lies above $A(q^0)$, hence $A(q^1)$ is above $A(q^0)$.

q^2 is above $A(q^1)$; hence above $A(q^0)$, hence $A(q^2)$ is above $A(q^0)$, and so on. This shows that $\bar{q}$ lies above $A(q^0)$. This completes the necessity part of the proof.

To prove sufficiency, we define the set $S(q^0)$ to be the set of all points $\bar{q}$ which can be reached from q^0 by a finite sequence of points $q^0, q^1, \ldots, q^n = \bar{q}$ with $q^1L(q^0), \ldots, q^nL(q^{n-1})\, q^{n-1}$. Clearly, if $\bar{q}$ is in $S(q^0)$, then any point $\bar{\bar{q}}L(\bar{q})$ is contained in $S(q^0)$: we simply add $\bar{\bar{q}}$ to the finite sequence whose last element was $\bar{q}$. Let $S'(q^0)$ be the lower boundary of $S(q^0)$: $S'(q^0)$ is the set of vectors $q = (q_1, q_2)$, such that any neighborhood of q intersects with $S(q^0)$ and such that for any positive number z the point $(q_1, q_2 - z)$ is not contained in $S(q^0)$. It is easily seen that $S'(q^0)$ is the graph of a function $q_2(q_1)$, defined over the projection of $S(q^0)$ onto the q_1-axis : for any q_1 let $V(q_1)$ be the set of points with first component q_1. Then if $S(q^0) \cap V(q_1)$ is not empty, there exists a unique q_2 such that $q_2 = \inf(S(q^0) \cap V(q_1))$ and, of course,

$$(q_1, \inf(S(q^0) \cap V(q_1))) \in S'(q^0).$$

If q' and q'' are both in $S'(q^0)$, then $q''K(q'')\, q'$. For otherwise we would have $q'L(q'')\, q''$, and then by the continuity of indifference curves, we can find a vector $z = (0, z_2)$, $z_2 > 0$ and a neighborhood $N(q'')$ such that $(q' - z)\, L(q)\, q$ for all $q \in N(q'')$; hence, in particular, for some $q \in N(q'')$ with $q \in S(q^0)$. But $(q' - z)\, L(q)\, q$ and $q \in S(q^0)$ imply $q' - z \in S(q^0)$ which contradicts $q' \in S'(q^0)$. This shows that $q_2(q_1)$ is a continuous function; let $\hat{q}_1$ be given. Let $q_1^1, q_1^2, \ldots$ be any sequence converging to $\hat{q}_1$. If a subsequence of the sequence $q_2(q_1^1), q_2(q_1^2), \ldots$ would converge to a point $\tilde{q}_2 < q_2(\hat{q}_1)$, then we could find i such that

$$(\hat{q}_1, q_2(\hat{q}_1))\, L(q_1^i, q_2(q_1^i))(q_1^i, q_2(q_1^i))$$

by continuity of the indifference curves. In a similar way, we exclude the existence of a subsequence of the sequence $q_2(q_1^1)$, $q_2(q_1^2)$,... converging to $\hat{q}_2 > q_2(q_1)$. This proves continuity.

Now let q^1, q^2,... be any sequence of points in $S'(q^0)$ such that $\lim_{i\to\infty} q^i = \hat{q}$ with $\hat{q} \in S'(q^0)$. Since for any q^i in the sequence

$$\hat{q}K(\hat{q})\, q^i \text{ and } q^iK(q^i)\, \hat{q},$$

we have

$$(q_1^i - \hat{q}_1)\, S(\hat{q}, q^i) \leqslant q_2^i - \hat{q}_2 \leqslant (q_1^i - \hat{q}_1)\, S(q^i, \hat{q}),$$

where $S(q^a, q^b)$ is the slope of the indifference curve passing through q^a with past consumption q^b. Because of the continuity property of indifference curves, $\lim_{i\to\infty} q^i = \hat{q}$ implies

$$\lim_{i\to\infty} S(\hat{q}, q^i) = \lim_{i\to\infty} S(q^i, \hat{q}) = S(\hat{q}, \hat{q}) = r(\hat{q}),$$

where $r(\hat{q})$ is the slope of the long-run indifference curve passing through $\hat{q}$. We can write

$$\lim_{i\to\infty} \frac{q_2^i - \hat{q}_2}{q_1^i - \hat{q}_1} = r(\hat{q}).$$

The function $q_2(q_1) = \inf(S(q^0) \cap V(q_1))$ is differentiable and fulfills the differential equation $(dq_2/dq_1) = r(q)$.

By the necessity part of the proof we know that q^0 is not contained in $S(q^0)$. On the other hand, for any point $(q_1^0, q_2^0 + z)$ with $z > 0$, we have $(q_1^0, q_2^0 + z)\, L(q^0)\, q^0$. Thus $q^0 \in S'(q^0)$. The unique solution of this differential equation passing through q^0 is identical with $A(q^0)$ and hence $S'(q^0) = A(q^0)$. For any $\bar{q}$ which lies above $A(q^0)$ we can find $\hat{q} \in A(q^0) = S'(q^0)$ such that $\bar{q} - \hat{q}$ is positive in both components, which implies there exists a neighborhood $N(\hat{q})$ such that for $q' \in N(\hat{q})$ $\bar{q} - q'$ is positive in both components and hence $\bar{q}L(q')\, q'$. Since some such $q' \in S(q^0)$, it is clear that $\bar{q} \in S(q^0)$ which completes the proof.

There exists a symmetric theorem for case II $(a_{11} + a_{22} \leqslant 0)$. But it is of no particular economic interest. We only state it here without proving it: Let a point q^0 be given, let $A(q^0)$ be the long-run indifference curve going through q^0. Let $\bar{q}$ be another point in the commodity space. There exists a finite sequence of vectors $q^0, q^1, q^2, \ldots, q^n = \bar{q}$ such that

$$q^0L(q^0)\, q^1, q^1L(q^1)\, q^2, \ldots, q^{n-1}L(q^{n-1})\, q^n = \bar{q}$$

if and only if $\bar{q}$ lies below $A(q^0)$. But we are not interested in moving from one point to another through a sequence of ever deteriorating points.

Case I implies a certain inertia or conservatism of the consumer. A point different from the present one may be inferior from the present point of view, but after this other point has been reached it may be considered superior to the present one; in both cases the person likes to stay at the position where he has arrived.

Case II implies a preference for change. The person may move back and forth between two points and always consider the change from one point to the other superior to staying at the point where he has arrived.

This last phenomenon leads to the following odd consequence: We may be able to find a sequence of points q^0, q^1,... such that

$$q^1L(q^0)\, q^0,\ldots\; q^{i+1}L(q^i)\, q^i,\ldots$$

with the property that $\lim_{i\to\infty} q^i = 0$. This becomes plausible by looking at the following diagram. The indifference curve going through q^0 is drawn under the assumption of past consumption q^0. Similarly, for the other points.

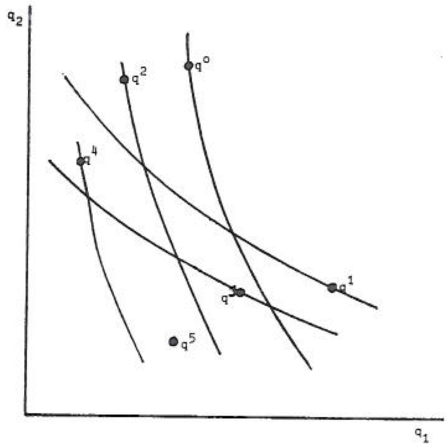

FIGURE 1

We see that we move always in the direction of an improvement, given present tastes. Yet our consumption of both goods is less than it was two periods before. My myopic preference for change makes me blind to the global properties of my consumption path, and I will thereby be led towards starvation. Such a situation suggests the description: subjectively the path is considered an improvement but in reality it is not. The consumer suffers from an insufficient degree of rationality. But, of course, we have to be careful with such an interpretation.

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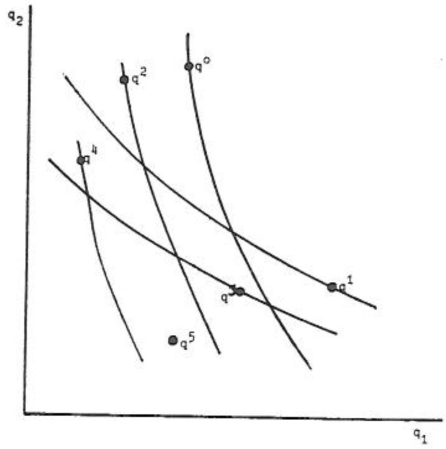

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We can find a corresponding phenomenon in Case I. There we can construct a sequence of points converging to infinity in both components, yet the consumer will consider a change from q^i to q^{i+1} as a deterioration of his situation for all i. Here again the distinction between the perceived and the "real" quality of a change suggests itself. In both cases, the existence of consistent long-run indifference curves suggests that we take them as the criterion for the "real" quality of a change in the commodity basket.

4. Formal Analogy With Revealed Preference Theory

We can try to draw an analogy to the theory of revealed preference. The theory starts from the observed demand behavior of the individual. Given that this demand behavior is consistent in that it obeys the axiom of revealed preference, we can infer whether a certain change of prices and demand can be considered an improvement for the consumer or not. We are able to describe the implicit preference structure of a person by observing his behavior. In our theory we have abandoned the assumption of a constant demand structure. We have replaced it by the assumption that there are constant laws which determine how the demand function changes. (The next stage then probably is the assumption that there are constant laws which determine how the laws change which determine how the demand function changes.) The axiom of revealed preference is a postulate on the consistency of demand behavior. If $q^0, q^1,\dots q^n$ is a sequence such that q^{i+1} is revealed preferred to q^i, then $q^n = q^0$ should be ruled out. The corresponding consistency postulates for the law governing the change in demand function can be formulated in exactly the same way by changing from the relation "revealed preferred to" to the relation $q^{i+1}L(q^i)\, q^i$. We formulate the axiom of changing preferences: If $q^0, q^1,\dots q^n$ is a sequence of commodity vectors such that

$$q^nL(q^{n-1})\, q^{n-1}L(q^{n-2})\, q^{n-2} \cdots q^1L(q^0)\, q^0, \text{ then } q^n \neq q^0.$$

From the axiom of revealed preference follows that there exists a consistent preference structure such that the consumer behaves as if he would maximize utility given this preference structure. From the axiom of changing preferences follows the following theorem: Continuity of indifference curves and the axiom of changing preferences imply that there exists a transitive and complete preference relation on the commodity space such that for any two vectors $q^0, \bar{q}$ with $\bar{q} \neq q^0$ there exists a finite sequence $q^0, q^1,\dots q^n = \bar{q}$ with

$$q^nL(q^{n-1})\, q^{n-1}L(q^{n-2})\, q^{n-2} \cdots q^1L(q^0)\, q^0$$

if and only if $\bar{q}$ is preferred to q^0 according to this preference relation.

Proof. The proof is a slight modification of the proof of our first theorem. In the notation of that proof, the axiom of changing preferences implies that q^0 is not contained in $S(q^0)$. On the other hand, any neighborhood of q^0 intersects with $S(q^0)$ and q^0, therefore, belongs to $S'(q^0)$. We define: q is preferred to q^0, if $q \in S(q^0)$ and q is indifferent to q^0, if $q \in S'(q^0)$. We have shown that $S'(q^0)$ is characterized by the differential equation $(dq_2/dq_1) = r(q)$, where $r(q)$ is the slope of the indifference curve passing through q, given past consumption q. Through every point q passes a unique solution of the differential equation $dq_2/dq_1 = r(q)$. The solutions of this differential equation are the indifference curves of the preference relation. Since these solutions do not intersect, transitivity is implied. Moreover, any two points q^1 and q^2 are comparable since through both pass solutions of the differential equation. One of them lies above the other, if they are not identical. This system of preference relations has the required property by construction: If $S'(q^0)$ is the indifference curve passing through q^0, then any point $\bar{q}$ which lies above $S'(q^0)$ is preferred to q^0 and, by definition of $S'(q^0)$, there exists a finite sequence with

$$\bar{q}L(q^{n-1})\, Lq^{n-2} \cdots q^1L(q^0)\, q^0.$$

Moreover, a point q on $S'(q^0)$ other then q^0 cannot be reached by such a sequence. For, if it could, we would find $q^{n-1} \in S(q^0)$ such that qLq^{n-1} and, by continuity of preferences, we could find $z_2 > 0$ such that the vector $q - (0, z_2)\, L(q^{n-1})\, q^{n-1}$ which is inconsistent with $q \in S'(q^0)$. This shows that indifference excludes preference and hence the defined preference relation fulfills all required properties. As our above argument shows, the axiom of changing preferences is equivalent to the condition $a_{11} + a_{22} \geqslant 0$ everywhere. For if at some point $a_{11} + a_{22} < 0$, a local argument corresponding to the discussion of the global case II shows that we can find a sequence

$$q^0 = q^nL(q^{n-1})\, q^{n-1} \cdots q^1L(q^0)\, q^0.$$

Our first theorem implies that the preference relation of the theorem just proved corresponds to the long-run demand function of the individual.

5. The Information Problem of the Consumer and Satisficing Behavior

We now want to try to find out whether this theory of endogenous change of tastes sheds new light on old questions. We first give a few reasons why the long-run demand behavior should differ from the short-run demand behavior.

A realistic theory of consumer behavior should take account of the fact that in making his purchasing decision the consumer does not consider all possible alternatives open to him. He relies on his past experience and—if only easily divisible goods are involved—changes his behavior usually only slightly from one period to the next. This may be a perfectly rational way of action. In every period he saves a lot of time when he does not review all possible consumption alternatives. Moreover, he may be aware of the fact that a theoretical consideration of a specific alternative may yield a result which is not consistent with what experience would tell him about this alternative. He may not fully trust his power of imagination. If he is a risk averter, he may shrink away from large changes in his consumption pattern. But as time goes on, he gains experience after having explored new consumption patterns and thereby moves further and further away from his initial point. We could say the long-run price elasticity of demand is higher than the short-run price elasticity of demand because the price change causes a learning process to start which will gradually shift demand towards a new equilibrium point where learning comes to an end. Under this interpretation the long-run preference structure is the "real" preference structure prevailing under the assumption of perfect information and perfect imaginative powers. The short-run preference structure takes account of the fact of limited information and limited imaginative powers: the less favourable short-run preference structure (in the sense that commodity baskets which are prefered to the present one in the long run, but are considered, if considered, inferior in the short run) reflects the disadvantages of imperfect information and imagination.

But this interpretation has important policy implications. Let us start at a certain income y^0 and with certain prices p^0. We now may have the chance to change income and prices to y^1 and p^1 so that the long-run consumption equilibrium corresponding to (y^1, p^1) lies above the long-run but below the short-run indifference curve passing through the initial consumption vector. If we interpret the long-run preference structure as the real preference structure we should decide for this change even though short-run preferences indicate we should resist it. The theorem proved above provides the possibility to reconcile the short-run inflexibility of the consumer with his long-run real preferences as long as gradual changes are possible.

Another possible interpretation between short-run and long-run demand changes could be given by the theory of satisficing instead of maximizing behavior. This theory runs as follows: People set themselves certain levels of aspiration, then they compare these levels with their present real situation. If this situation is inferior to the aspired level of satisfaction, they try to find ways to improve their present situation. After they have tried for a while they will either have achieved their goal or failed to do so. In the latter case, they will tend to lower their standards, i.e., the desired level of aspiration, and they will try again to attain this lower goal. If they have achieved the goal set by themselves, they become more ambitious, and they will revise upwards the goal to be achieved. It is not difficult to see that this method of satisfacing—given the feedback between setting and achieving goals—will, in the long run, yield similar results to maximization. In the case of our consumer, the long-run utility function may be considered to be the intrinsic criterion which the consumer is interested in. He then starts out at a certain level U_0 and makes some changes in his consumption in order to achieve a higher utility U_1. He ends up at a certain level U_1' which may be equal U_1 if he was successful; otherwise $U_1' < U_1$. He now revises his goal: upwards if he has succeeded in achieving U_1, downwards if he was unsuccessful. Then he changes his consumption policy again in order to achieve the new goal U_2, and so on. Under the condition that the utility function $U(q)$ is quasiconcave it is not difficult to show that such a policy will lead the consumer to the utility maximum attainable within his budgetary limits. Here again it is clear that the long-run indifference curves rather than the short-run indifference curves should underlie normative judgements about price and income changes. The short-run indifference curve is essentially meaningless, having been derived under the assumption that utility is maximized in every period, whereas under the satisfacing model it is just the limited amount of searching to obtain a given goal which makes the short-run demand behavior different from long-run demand behavior. Only the latter is consistent with maximizing behavior.

6. Application to the Problem of Regional Mobility

Let us now draw certain welfare economic conclusions from the model. Consider a farmer whose present position is point q^0 in diagram 2. His budget constraint is the straight line B. His long-run indifference curve is $A(q^0)$, his short-run indifference curve is $A'(q_0)$. He has the chance to leave his farm and to move into the city where he can work, say, in a factory. The budget constraint corresponding to his life in the city is B'. Price ratios are different in the city and in the countryside. His income differs too. Hence B' usually will not coincide with B. Should he live in the city, his equilibrium consumption point in the long run would become $\bar{q}$. To this corresponds the long-run indifference curve $A(\bar{q})$ and the short-run indifference curve $A'(\bar{q})$. As we have drawn the curves, the farmer will not decide to go to the city. The point $\bar{q}$ is considered inferior to q^0 if one applies the preference structure prevailing at q^0. Yet if we interpret the long-run indifference curves to reflect his "real" preferences, $\bar{q}$ would be superior to q^0 and it is only his way of restricting himself to marginal changes which prevents him from seeing that the city is better for him than the countryside. But this interpretation of long indifference curve is not the only possible one. We could think of $A'(q^0)$ as properly representing preferences at point q^0 and assume that changes in demand induce proper changes of tastes. There is no way of telling from the demand functions which interpretation is correct. In case of the last interpretation, we are not allowed to neglect the short-term preference structure. We would otherwise do something which is similar to a direct interpersonal utility comparison, since a change in consumption changes the consumer. As Marshall says: "The man is not the same at the beginning as at the end" of a certain consumption process. But the person with past consumption q^0 and the person with past consumption $\bar{q}$ or $q^1, q^2,...$ is still the same person even though his tastes may have changed. It is, therefore, reasonable at least to say that compared to a constant consumption q^0 he is better off with a sequence of consumption vectors $q^0, q^1, q^2,..., \bar{q}$ if he always, after having consumed q^i prefers q^{i+1} to q^i and, in addition, in the end prefers $\bar{q}$ to any other member of the sequence, in particular q^0. Let us, therefore, introduce this axiom which is of course much more cautious than the acceptance of the long-run "indifference curves" as the proper manifestation of his utility. Under a regime of *laissez faire* the farmer would remain a farmer. And he would remain a factory worker if he happend to begin his life in the city. There is a certain degree of immobility in such a state of affairs. It is the consequence of the fact that with a *laissez faire* policy the status quo receives the benefit of the doubt, if the status quo is a state of laissez faire. I now want to show that under certain specifications the status quo is not Pareto-optimal even though none of the usual kinds of externalities are present.

We assume that the income the man can earn in agriculture and in industry corresponds to his marginal product. The differences in the price ratios of good 1 (say textiles) and good 2 (say wheat) are due to transport costs incurred by shipping textiles to the countryside and wheat to the city. The budget constraint BB, therefore, corresponds to the social marginal product of the man as long as he works as a farmer, the budget constraint $B'B'$ corresponds to his social marginal product as an industrial worker. Let the interest rate r in the economy be given. In an efficient state of the economy, this interest rate reflects the marginal rate of time preference in the sense that everyone is indifferent between an additional dollar now and $(1 + r)^t$ additional dollars in period t. Let $\bar{q}$ be some point which lies vertically below $\bar{q}$ but above the long-run "indifference curve"

$A(q^0)$. By our theorem we are able to construct a finite sequence of points $q^0, q^1, q^2, ..., q^n = \bar{q}$ such that the man under consideration will prefer q^{i+1} to q^i if he had consumed q^i in the last period. Such a sequence will of course lie above $A(q^0)$. At least the initial part of the sequence will lie above both budget lines BB and $B'B'$. The man can only afford it if he is subsidized by the government. Let the total discounted value of the subsi-

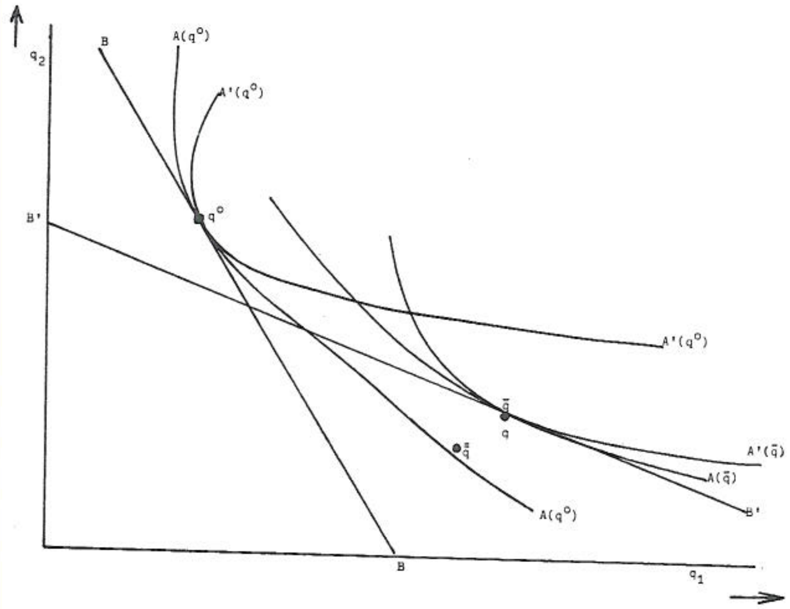

FIGURE 2

dies he receives be $S(r)$. From a certain point onwards, the sequence will lie below the budget line $B'B'$. If the man is transferred to industry as soon as this happens, he will now produce more than what he receives. If after n periods he stays at the point $\bar{q}$ and if his son does the same after his death and so on, the family produces each year a surplus above their level of consumption. Let the total present value of the surplus be denoted by $T(r)$. It is clear that, for any sufficiently small interest rate, T will be greater than S. For S is confined to a finite number of initial periods whereas T gets contributions of a given size from each sufficiently distant period. Thus $\lim_{r\to 0} S(r)$ is finite whereas $\lim_{r\to 0} T(r)$ is infinite.

Hence for an interest rate sufficiently small, the man and his family will under this policy not get their total dicounted marginal product. Society is richer than under the status quo; that is, nobody else will be worse off if such a policy towards our farmer is adopted. He himself is also better off, if we adhere to our valuation axiom. Hence the status quo is not Pareto optimal. There is an obvious criticism against this argument. We have endowed the government with the knowledge of the whole preference structure of the individual, which, as his behavior shows, he does not know himself. No wonder that the government should do better than the individual. This argument is valid for the single case. The situation may be different if, due to structural changes in the economy, there is a strong likelihood that for many people employed in agriculture the situation is similar to the one discussed above. The relative immobility of people working in agriculture may, in the process of a shrinking share of agricultural products in total demand, lead to a relative if not absolute deterioration in the standard of living of farmers. Of course, this deterioration will cause a major outflow of workers out of agriculture, but it is usually accompanied by huge subsidies to agriculture, by political dissatisfaction, etc. If such a situation were predicted early enough by the government it could, by essentially subsidizing mobility out of the declining industry, avoid, politically and financially, more expensive measures later on. The welfare economics of changing tastes could provide a rationale, along the lines of the above argument, for subsidization of mobility and prove the superiority of such policy to other measures like the simple redistribution of income (for a given size of funds available) in favor of low income groups or like the simple subsidization of agriculture which frequently reduces mobility. Of course, the same argument applies to any kind of declining industry.

7. Social Indifference Curves under Changing Tastes

We now want to work with social indifference curves which replace the individual ones used so far. Under conditions of given tastes, Samuelson has shown that the usual continuity and convexity conditions for individual utility functions, and for a Bergson social welfare function, imply the existence of social indifference curves which have the usual properties and which connect points corresponding to equal maximum social welfare (10). Given past consumption q^0, the curve $A'(q^0)$ can be interpreted to be such a social indifference curve. If we allow tastes to become variable, the concept of a social indifference curve appears to be more difficult. Arrow's impossibility theorem suggests that there exists no completely satisfactory general rule how to change the social welfare function when individual tastes change. But, in my opinion, this additional difficulty of the social welfare function, as we introduce changing tastes, is only apparent. We either have to admit that even under given tastes the Bergson social welfare function is not a useful concept (except in certain trivial and unrealistic cases), or we have found a satisfactory general rule for the dependence of the social welfare function on individual tastes which of course would have to violate one of Arrow's axioms (1). The Bergson social welfare function is usually written in the form

$$W = W(U_1, U_2, \ldots, U_n),$$

where the U_i are the utility functions of the individuals in the economy. Everybody knows that in this form the function is not operational since the concept of a payoff between x utility units of person i and y utility units of person j is nonsense. In economic theory, interpersonal utility comparisons of this naive type have been ruled out for decades. We, therefore, have to transform the arguments of the social welfare function into measurable quantities. Keeping other influences on individual utilities constant, they are determined by the income and the market prices of the goods traded in the economy. Let us, therefore, write

$$W = W(y_1, y_2, \ldots, y_n, p_1, p_2, \ldots, p_m),$$

where y_i is the income of person i, and p_j is the price of good j. In order to exclude arbitrariness or corruption, we may have to assume that certain symmetry properties hold. If all persons are alike otherwise—let us assume this for simplicity—it should only be their individual tastes, with respect to the different traded goods, which, together with certain general principles not referring to a specific individual, determine the structure of this function W. For instance, if all persons have the same tastes, the function W should be symmetric with respect to $y_1, \ldots, y_n$. Moreover, if we want to avoid ad hoc arguments in favour of certain people who happen to have certain tastes, we need general principles which tell us how to construct a social welfare function for any possible constellation of tastes. Otherwise nobody can refute the contention that any specific Bergson welfare function which is being adopted is just a clever instrument of exploitation of one group of people by another group of people. Then of course the concept of a social welfare function is just as useful or useless as the concept of Pareto optimality. And we know that the latter is insufficient to solve the problems of an optimum economic policy.

Thus, unless we want to refute the concept of a social welfare function, we have to derive it from principles general enough to encompass the possibility of deriving a social welfare function for any reasonable constellation of tastes. But, then, changing tastes cause no problem for the existence of a social welfare function. Given the usual regularity conditions, we then can derive Samuelson's social indifference curves for any given state of tastes.

Assuming now that all inhabitants of the country or at least the overwhelming majority of them change their tastes in the same direction as a consequence of changes of past consumption, we can derive properties of these social indifference curve systems which are similar to the individual indifference systems. For this purpose we introduce a few postulates about the influence of tastes on the social welfare function.

In its general form, we may now write the social welfare function as

$$W = W(y_1, y_2, ..., y_n, T_1, ..., T_n, p_1, ..., p_m),$$

where T_i stands for the tastes or the indifference curve structure of person i. We can assume that this expression is symmetric with respect to the different persons; interchanging y_1 and y_2 and at the same time interchanging T_1 and T_2 does not change the value of W. We introduce the following postulates:

1) $\partial W/\partial y_i > 0$.

2) $(\partial W/\partial y_i)/(\partial W/\partial y_j)$ is independent of y_k and T_k for $k \neq i, k \neq j$. The welfare function can then be written as an additive function.

3) $(\partial W/\partial y_i)/(\partial W/\partial y_j)$ decreases with increasing y_i. The welfare function is quasiconcave in the incomes.

Postulate No. 4 will only be formulated for the case of two goods. Consider two situations: I and II. In both situations, prices are the same and incomes and tastes of all persons except one are also the same. Only tastes of person i are different in situation I and situation II. If y_i (I) and y_i (II) and T_i (I) and T_i (II) are such that

$$q_{i1}(\mathrm{I}) > q_{i1}(\mathrm{II}),$$
$$q_{i2}(\mathrm{I}) > q_{i2}(\mathrm{II});$$

then we postulate that

$$\left(\frac{\partial W}{\partial y_i}\right)_{\mathrm{I}} \Big/ \left(\frac{\partial W}{\partial y_j}\right)_{\mathrm{I}} < \left(\frac{\partial W}{\partial y_i}\right)_{\mathrm{II}} \Big/ \left(\frac{\partial W}{\partial y_j}\right)_{\mathrm{II}}$$

for all persons $j \neq i$.

In other words, we try to make situations with different tastes comparable. By any reasonable standards, person i has a lower real income in situation II than in situation I, since he consumes less of both goods. Then we assume that the relative weight of an additional dollar he gets for social welfare is higher in situation II than in situation I. That is, we treat his influence on social welfare in a similar way as if his tastes were the same in both situations and only his income were lower in situation II.

It is now convenient to work with social indifference curves in the price space. We assume that a certain national income $y = \sum_{i=1}^{n} y_i$ is given. Then for any given prices p_1, p_2 and given tastes, we maximize the social welfare function

$$W(y_1, \dots, y_n, T_1, \dots, T_n, p_1, p_2)$$

subject to the constraint

$$\sum_{i=1}^{n} y_i = y.$$

If this is the only constraint, then of course at the optimum point we have $\partial W/\partial y_i = \partial W/\partial y_j$ for i and $j = 1, 2, \dots, n$. It is easy to see that the slope of the social indifference curve going through any point p_1, p_2 is given by the equation $dp_2/dp_1 = -q_1/q_2$, where q_1 is total demand for good 1 and q_2 is total demand for good 2. For, a change dp_1 of the first price would require a change $dy_i = q_{1i}\, dp_1$ of person i's income to keep his utility the same. Hence national income would have to change by $\sum_i q_{1i}\, dp_i = q_1\, dp_i$. A similar relation holds for good 2. Since we keep y constant we must have $0 = dy = q_1\, dp_1 + q_2\, dp_2$ if we want to keep everybody at the same utility level. Whenever dp_2/dp_1 takes on a value different from q_1/q_2, we could without changing y make somebody better off without making anyone worse off by making the expression $q_1\, dp_1 + q_2\, dp_2$ negative. This would contradict the constancy of the social welfare function.

In what follows, we assume that with each individual both goods have a positive income elasticity of demand. Assume now that the preferences of person 1 change in such a way that he buys more of good 1 and less of good 2 with given income y_1 and given prices. This change in tastes may require a redistribution of income if the social welfare function is to be maximized. But even after this redistribution of incomes will person 1 consume not less of good 1 than he did before and he will consume not more of good 2 than he did before. If y_1 has been reduced, it has not been reduced by so much that consumption of good 1 has declined; for otherwise, by our axioms and by the assumption that both goods are normal goods, $(\partial W/\partial y_1)/(\partial W/\partial y_i)$ will have increased for any person i whose income did not decline. But since total income remained the same and since person 1 has lost income, there exists at least one person i whose income did not decline. But the optimum requires that $\partial W/\partial y_1 = \partial W/\partial y_i$ for all i, hence $(\partial W/\partial y_1)/(\partial W/\partial y_i)$ cannot have become larger after the change in tastes. In a similar way we prove our proposition for the case where y_1 has increased.

Since the social welfare function is quasi-concave (axiom 3) and essentially additive (axiom 2), the redistribution of income in consequence of a change of tastes of person 1 will move the income of person i and person j for $i, j \neq 1$ in the same direction: otherwise the equation $\partial W/\partial y_i = \partial W/\partial y_j$ cannot be maintained. And of course y_i will move in the opposite direction compared to y_1. If y_1 decreases, it follows that y_i increases and hence q_{i1} increases. But then q_1 increases and therefore q_2 decreases. Thus we have shown that a change in tastes of person 1 such that good 1 becomes more wanted and good 2 becomes less wanted will induce a change of relative social demand for the two goods in the same direction.

We can of course in a similar way deduce that taste changes of a number of people such that their demand for good 1 increases or remains the same with constant tastes of all others implies that social demand for good 1 increases or remains the same. The proof is essentially the same as above. But this implies in particular: changing tastes such that, at the prevailing prices and with the prevailing personal incomes, personal demand does not change will not induce changes in social demand at the prevailing prices.

Given these axioms about the social welfare function, we can try to compare social indifference curves corresponding to the short-run and long-run demand behavior of the individuals. Operating in price space with given national income, the above argument implies that the slope of the long-run indifference curve of any given point is equal to the slope of the short-run indifference curve corresponding to past consumption equal to present consumption. The same is then true in quantity space. But it appears to be difficult to prove that the elasticity of substitution of the long-run social indifference curve is large than the elasticity of substitution of the short-run indifference curve, even if this is true for all individuals.

This difficulty can be made plausible when we contrast it with the case of Scitovsky social indifference curves. Given an initial supply q^0 and an initial distribution, the Scitovsky social indifference curve going through q^0 connects all points which just enable society to make everyone as well off as at the initial position q^0. Take any other point q' on the Scitovsky indifference curve going through q^0, corresponding to, say, a higher supply of good 1 and a lower supply of good 2 and corresponding to the short-run tastes: the new allocation keeps everybody on the same short-run indifference curve as before and, therefore, shifts everyone up to a higher long-run indifference curve, which implies that the point q' must lie above the long-run Scitovsky indifference curve going through q^0.

But in the case of a Bergson–Samuelson social indifference curve, not everybody will be on the same indifference curve at q' as at q^0. These changes in real income make computations quite complicated, so that I was not able to prove the result I was looking for.

On the other hand, in most cases we would expect the long-run social indifference curve to lie below the corresponding short-run social indifference curve. First, we note that for any given set of tastes, the Samuelson social indifference curve going through q^0 lies below the Scitovsky social indifference curve going through q^0 and corresponding to the distribution if we maximize welfare at the point q^0. For social welfare cannot have declined, if we are able to keep everyone as well off as before. Hence maximum social welfare on any point of the Scitovsky indifference curve is at least as high as at q^0. But this implies that any redistribution of real income as we move along a social indifference curve helps to slow down the change of the slope of the indifference curve: the equilibrium price ratio will be closer to the original price ratio than if no redistribution of real income had taken place: the redistribution of real income will be such as to slow down the rise in the relative price of the good which becomes scarcer. This means that people with an above average propensity to spend additional income on good 1 will lose real income, as good 1 becomes scarcer and people with an above average propensity to spend additional income on good 2 will gain real income under such circumstances. This interpersonal substitution (through redistribution) between good 1 and good 2 explains the difference between the elasticity of substitution of a Bergson–Samuelson social indifference curve and the corresponding Scitovsky indifference curve (which only allows for intrapersonal substitution). Because of the redistribution we are not able to say that, every person will consume more of the good which became more plentiful and less of the good which became scarcer. But the redistribution will (loosely speeking) work in the direction of enhancing the relative weight of the more plentiful good in the consumption basket of most persons. Therefore, we can expect that most persons will change their tastes in the direction of a higher preference of the now more plentiful good. Thus in the long run, the equilibrium prices will be closer to the original prices than in the short run: the long-run social indifference curve exhibits a higher elasticity of substitution than the short-run social indifference curve. This intuitive argument makes it less arbitrary, when we assume below that the long-run social indifference curve lies below the corresponding short-run curves.

8. Application to the Problem of Innovations

We have stated above that a *laissez faire* economy contains a bias towards the status quo if judged by the long-run indifference curves which lie below short-run indifference curves. But this picture would be incomplete if we did not discuss the problem of innovations, inventions, and public investments. Figure 3, which is similar to figure 2, shows on one

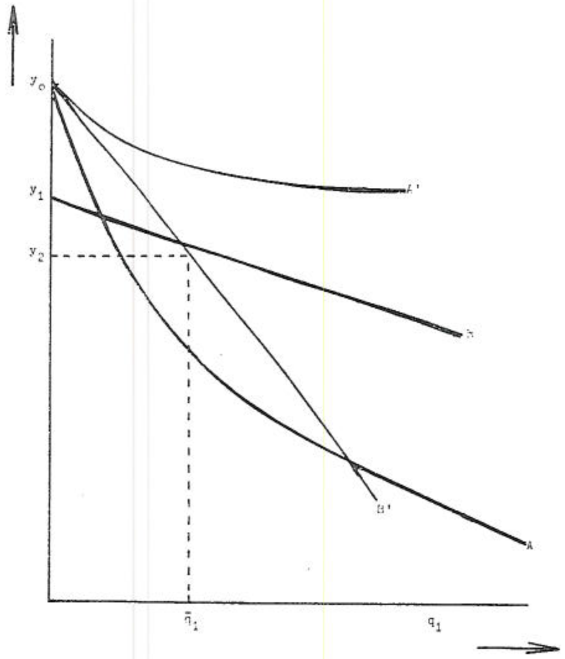

FIGURE 3

axis the quantity q_1 produced annually of a new product. Before this new product is developed, national income is y_0. The costs of developing the product are $(y_0 - y_1)/r$, where r is the interest rate. This corresponds to an annual loss of income of $y_0 - y_1$. After the product is developed, its unit cost of production is constant, say c. Then the straight line B beginning at y_1 with slope c gives the set of possible efficient combinations of the new product and national income exclusive of q_1. Let A' be the short-run social indifference curve passing through y^0. If a central authority would have to decide whether or not to go ahead with the development of the new product and if it would follow the criterion of present preferences it would decide no. This may be different under conditions of decentralized decision making. If a firm develops the new product and sells it at the market price $p > c$, it may in the long run be able to make a profit and at the same time push the rest of society to a higher long-run indifference curve. If B' is the straight line through y_0 with slope $-p$, then the choice open to society (minus the firm) is any point on B'. If in the long run, the market decides to choose the point $(y_2, \bar{q}_1)$, then society benefits in terms of long-run preferences, and of course, after having arrived at this point short-run preferences are such that now this point is preferred to the status quo ante. Moreover, if the monopolist decides to sell his product from the beginning at the same price, then there is never a moment where consumers can complain about a deterioration of their situation. Since the point $(y_2, \bar{q}_1)$ lies below the line B, the monopolist makes a profit which more than compensates his development costs.

To recover his development costs, the innovator has to produce an innovation which shifts the rest of society to a higher long-run indifference curve. To prove this, we have to investigate the maximization problem of the innovator-monopolist. He wants to maximize the present value of his profits which is at time T given by

$$\sum_{t=T}^{\infty} q_t(p_t - c)\, \gamma^{t-T},$$

where γ is the discount factor. Given the initial q this expression is independent of T. Hence the maximum of discounted future profits at any time T only depends on consumption of the good in the last period which together with the present price determines demand today, which, in turn, together with tomorrow's price will determine demand tomorrow and so on.[1] We can give this model the interpretation of an accumulation model, where q_{T-1} is the capital stock, $q_t(p_t - c)$ is consumption, and the value of $q_t(p_t - c)$ under the condition of a constant capital stock is the net output. If we want to maximize the discounted stream of consumption, the optimal path will converge to some equilibrium value q^* and stay there. But, clearly, if the initial value of q is zero the total sum of discounted profits is less than if the initial value had been q^*. Total discounted profits if $q = q^*$ in every period corresponds to an annuity of $q^*(p^* - c)$ which covers development cost only if the corresponding point $(q^*, y_0 - p^*q^*)$ lies below the line BB. On the other hand, the slope $-p^*$ of the line passing through y_0 and $(q^*, y^0 - p^*q^*)$ is also equal to the slope of the long-run indifference curve at this point. But this is only possible if this point lies above the long-run indifference curve passing

[1] A similar problem was investigated and solved mathematically in (5).

through y_0, which was to be proved. (A straight line through y_0 can never be tangential to a lower indifference curve.)

9. Conclusion

We have seen that democratic or bureaucratic decisions may hinder innovations more than a market decision process. But our analysis is only valid if we assume that single innovators are able to foresee changes in tastes of other people. This is quite frequently the case. A single firm may often be able to push through an innovation at its own risk and thereby change its environment to such a degree that it will at last accept the innovation and reward the innovator for it. But many decisions cannot be decentralized in such a fashion. We therefore have to ask whether it is reasonable for a society to build its decisions on its present preferences or on some other criterion. If present preferences are strongly influenced by myopic thinking, by lack of imagination how a different world would look, we should not accept these preferences as the last word. This is true for the individual as well as for the society. What is necessary in such a situation is a broadening of the horizon of the people; in other words: education. This leads us to a very important aspect of the educational process: One of the functions of education seems to be to reduce peoples' mental and emotional dependence on the continuation of the status quo, or in economic terms to increase the elasticity of substitution of their preferences. An adult person may have sufficient insight to educate himself, to work on his own preferences. If this is so, he makes today decisions in anticipation of his changed preferences. It will require another paper to discuss this problem of intertemporal decision making in view of anticipated and planned changes of tastes. Society has the same problems. To remain viable or to become progressive it may have to opt for a metapreference in favour of challenging the prevailing preferences of its members and to ask if and how these preferences can perhaps be improved. Clearly the role of education in this context is specially important. This does not, of course, mean that there should be some person or group of persons who from their "superior" point of view dictate the values of society. Society's decisions must rest on the preferences of all its members. But we have to acknowledge and make use of the fact that preferences are partly the product of peoples' environment.

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