Liquidity provision, banking, and the allocation of interest rate risk*

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The paper studies the efficient allocation of technology-induced interest rate risk and the implications of such risk for efficient liquidity provision. Complete immunization against interest rate risk is shown to be undesirable as it precludes the exploitation of favourable reinvestment opportunities. Under the assumptions of Diamond-Dybvig (1983), interest-induced valuation risks of long-term assets should be born by early withdrawers, reinvestment opportunity risks of short-term assets by late withdrawers. Efficient liquidity provision thus entails no shifting of interest rate risk. In the absence of additional moral hazard, second-best allocations can be implemented through unregulated competition among banks.

Key words: Liquidity provision; Interest rate risk; Securitization; Banking regulation
JEL classification: G21; G28

1. Introduction

In this paper I study the efficient allocation of interest rate risk in an economy. Interest rate risk plays a role whenever agents are choosing between assets of different maturities. If future market rates of interest are uncertain, an agent who invests in long-term assets is subject to a valuation risk because the market value of a long-term asset prior to maturity depends on the prevailing rate of interest. At the same time, an agent who invests in short-term assets is subject to a reinvestment-opportunity risk as he does not know what rate of return he will obtain once his current investment has matured.

Interest rate risk poses an important problem for banks and other financial intermediaries that take in short-term funds and finance long-term

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investments.\(^1\) If the returns on the long-term investment are given and subsequently the market rate of interest turns out to be very high, it is quite possible that at some point refinancing costs may exceed investment returns, or, equivalently, that the market value of the initial investment may fall below the repayment obligation to the initial financiers. A case lesson is provided by the crisis of the savings and loans industry in the United States. Having granted very-long-term, fixed-rate mortgages until the early seventies, American savings and loans institutions found themselves squeezed by the high interest rates of the mid seventies and early eighties. Indeed if one relies on market values rather than book values of assets, it appears that a substantial portion of the industry was insolvent even before interest rates peaked in 1980/81 and the industry was deregulated [see, e.g., Kane (1985, 1989), Benston et al. (1991)]. As for Europe, the difficulties experienced by banks in several countries in the early nineties can at least in part be ascribed to the high interest rates that prevailed in the period 1989–92 and to the depressed real estate prices that went with high interest rates.

The exposure of banks and other depository institutions to interest rate risk is a matter of concern to bank supervisors. The Basle Committee on Banking Supervision has recently presented a proposal to extend capital adequacy requirements under the Basle agreement so as to take account of interest rate risk as well as other forms of market risks [Basle Committee (1993), see also Carosio (1990)]. Underlying this proposal is a presumption that statutory regulation is needed to control the interest rate risk exposure of banks. However no argument is given to justify this presumption.

The bank supervisor who is concerned with the safety and soundness of banks [see, e.g., Key and Scott (1990)] may feel that no further argument is needed to justify regulatory interventions to limit interest rate risks in banking. For the economist who is concerned with the efficiency of the overall allocation of risks, the matter is less clearcut. Whereas the regulatory literature pays no attention to the distinction between diversifiable and nondiversifiable risks, the economist will note that fluctuations in interest rates affect the economy as a whole, so that interest rate risk is not diversifiable. The interest-induced valuation risks of long-lived real assets can be shifted from one agent to another, or they can be shared between agents, but they cannot be diversified away.\(^2\) The view that interest rate risks in banking need to be controlled by regulation can therefore not be based on the notion that these risks are otherwise insufficiently diversified; such a view

\(^1\)In this context, it is useful to think of savings deposits with terms of up to seven years and real estate investments with terms of over twenty years.

\(^2\)To be sure the problem of interest rate risk may be reduced if interest rates are correlated with inflation, and asset returns go up with inflation. However, this leaves the problem of exposure to fluctuations in real interest rates, as experienced by English building societies and their insurance companies in the early nineties when the decline in nominal interest rates fell short of the decline in inflation rates.
requires either that the economy as a whole ought to limit its exposure to interest rate risk, or that parties other than banks are better qualified to bear these risks. The underlying issue then is (i) what is an optimal level of aggregate exposure to interest rate risk and (ii) how are these risks efficiently shared? This issue cannot be settled by a routine invocation of the safety and soundness of banks; it requires a systematic welfare analysis.

In this context, one quickly realizes that the very terminology which is used by bank supervisors is unsuitable for the problem. For instance, the Basle Committee makes a distinction between market risks and credit risks, proposing to treat them independently and subsuming interest rate risk under market risks. The distinction between market risks and credit risks reflects the traditional separation of credit departments and market investment departments in banks, but it obscures the economic structure as the impact of interest rate risk has little to do with the fungibility of claims. The discounted present value of returns on a fixed-rate nonfungible loan or mortgage is just as much subject to interest rate risk as the market value of a fungible asset with the same return pattern. The absence of market quotations for the former may give an illusion of safety, but as the American S&L's and their supervisors found out, the losses in present values are real even if book values are not adjusted to reflect them.

The Basle Committee's separation of interest rate risks from credit risks is the more baffling since sometimes the two are direct substitutes. A bank that issues loans or mortgages on a floating-rate basis may find that it has merely replaced 'interest rate risk' by credit risk. When the interest rate goes up, the borrower may suddenly be unable or unwilling to service his debt. If the lender then tries to foreclose on the borrower's assets, he may find that because of high interest rates the market value of these assets is depressed. This mechanism has played a role in some of the difficulties that banks in Europe as well as the United States have had with real estate and property development finance as well as in some of the more spectacular bankruptcies in recent years. Given this experience, the proposed separate treatment of 'interest rate risk' and credit risk or, in the case of interest rate derivatives, of interest rate risk and counterparty or settlement risks seems problematic. The present study will use the term interest rate risk to encompass all risks that are directly or indirectly induced by uncertainty about future interest rates.3

For a welfare economic analysis, even this definition has the weakness that it seems to treat the interest rate as an exogenous parameter. From the

3By this definition, interest rate risk also comprises the liquidity risks to which intermediaries are subjected if deposit rate regulation prevents them from adjusting their interest rates to market conditions, see the experience of depository institutions in the United States in the late seventies. Given this experience, it is worrisome to see a recent OECD publication suggesting that deposit rate deregulation 'from the 1970s onwards made interest rate volatility increase' and 'exposed banks to a greater degree to market risks (interest rate risks . . .)' [OECD (1992, pp. 10, 14)].
perspective of individual market participants, this may be appropriate, but from the perspective of the economy as a whole, the interest rate is endogenous. Variations in future interest rates must be caused by variations in exogenous data, at least if one neglects sunspot phenomena. Any welfare analysis of the allocation of interest rate risk must take account of the exogenous source of this risk.

Leaving aside the role of monetary and fiscal policy, one may distinguish between interest rate risk that is induced by uncertainty about future technologies and interest rate risk that is induced by uncertainty about future preferences. The former concerns the productivity of real investments in the future, the latter concerns the cross-section distribution of tastes for consumption in different periods. For the allocation of resources both types of uncertainty are important because they affect the choice between short-term and long-term real investments in the economy.

In the final analysis, the problem of interest rate risk is one that concerns the relation between the maturity structure of the real assets of the economy and the time pattern of aggregate consumption. From the theory of the term structure of interest rates, it is well known that the interest rate risk exposure of an individual is reduced to zero if the maturity structure of his assets is perfectly matched to the time structure of his payment needs. In this case there is never any need for either reinvestments or premature liquidations, so neither reinvestment-opportunity risks nor valuation risks play a role. However, as was pointed out by Stiglitz (1970), such maturity matching presumes that the time structure of payment needs is known from the beginning. This precludes the possibility that the individual's consumption plan may provide for a dependence of future consumption on observed interest rates in order to take advantage of changes in relative intertemporal prices. Typically though, a dependence of consumption on interest rates is desirable for the individual; from the perspective of the economy as a whole, such a dependence may reflect a desirable response of the allocation to the variations in technologies or preferences that underlie the changes in interest rates. An allocation involving perfect maturity matching with no response of consumption to changes in technologies or preferences is unlikely to be efficient.

Matters are complicated by the possibility that agents may be uncertain about the timing of their future consumption needs or, more generally, about their future time preferences. Such uncertainty leads to a demand for liquidity, i.e., for the flexibility to adapt the timing of asset liquidations to the timing of consumption needs as it is realized. As discussed by Diamond and Dybvig (1983), one of the functions of banks is precisely to provide consumers with liquidity, giving them the right to withdraw their funds 'on demand'. From the banks' point of view, this may be feasible because in the aggregate at least some of the individual uncertainty about the timing of
consumption needs washes out, so that the time path of aggregate consumption may be fairly predictable even though individual consumption paths are not. But then there is a discrepancy between the time pattern of aggregate consumption and the maturity structure of the depositors’ legal claims on their banks. Given the ‘on demand’ clause of the deposit contract, banks must in principle regard all deposits as short-term funding even though they may expect a certain portion of deposits to stay with them. How then does the discrepancy between the time pattern of actual consumption and the time pattern of legal claims affect the assessment of interest rate risk as one compares the pattern of payment needs to the maturity structure of asset returns? This is a key question not only for the determination of bank behaviour and market equilibrium, but also for the welfare analysis of the relation between interest rate risk and liquidity provision.

Building on the work of Diamond and Dybvig (1983) and von Thadden (1991), the present paper develops a simple model for studying the efficient allocation of interest rate risk as well as the interdependence between the allocation of interest rate risk and the provision of liquidity. The analysis proceeds under the assumption that the source of interest rate risk is purely technological; however the general case of uncertainty about future preferences as well as technologies would lead to the same conclusions [Hellwig (1993)].

After developing the basic model in section 2, sections 3 and 4 characterize Pareto-efficient allocations under different information assumptions. Section 5 discusses the implementation of second-best allocations under asymmetric information through various institutions, focusing in particular on the role of banks in the provision of liquidity and on the implications of the analysis for the interest rate risk exposure of banks. The implications of the analysis for the regulation of banking are considered in section 7.

2. The basic model

Like Diamond and Dybvig (1983), I consider an economy going through a sequence of three periods, $t=0,1,2$. In each period there is a single good, which may be used for investment as well as consumption. There are altogether three investment opportunities, all with constant returns to scale:

- A short-term investment at date 0 of one unit of the good yields $\theta_1$ units of the good at date 1.
- A long-term investment at date 0 of one unit of the good yields $\eta$ units of the good at date 2; premature liquidation at date 1 is not altogether infeasible, but then the rate of return is only $\epsilon<\theta_1$.
- A short-term investment at date 1 of one unit of the good yields $\theta_2$ units of the good at date 2. The rate of return $\theta_2$ is the realization of a random
variable $\tilde{\theta}_2$; it is known at date 1, but not at date 0. As of date 0, only the distribution function $F$ of the random variable $\tilde{\theta}_2$ is known; $F$ is assumed to be a continuous function.

On the household side of the economy, I assume that there is a continuum of unit mass of ex ante identical consumers. Each consumer has an initial endowment $k_0$ of the good at date 0 and zero of the good at dates 1 and 2. Like Diamond and Dybvig (1983), I assume that households are uncertain about the timing of their consumption needs: each household faces a probability $p$ of needing to consume at date 1 and a probability $1-p$ of needing to consume at date 2. These needs are inexorable, so once their incidence is known, there is no more question of any substitution between dates 1 and 2. However, from the ex ante point of view, i.e., before the time incidence of consumption is known, there is scope for substitution, as in any insurance problem. Preferences are characterized by a von Neumann–Morgenstern utility function $u(\cdot)$, where $u(c)$ is the household's von Neumann–Morgenstern utility of consuming $c$ at the date when consumption is needed, regardless of whether this is date 1 or date 2. The function $u(\cdot)$ is assumed to be continuously differentiable, strictly increasing and strictly concave.

The uncertainty about the time incidence of consumption needs is restricted to the level of individuals. At the aggregate level, this uncertainty is assumed to wash out, i.e., the law of large numbers works so that with probability one a fraction $p$ of all consumers needs to consume at date 1 and a fraction $1-p$ of all consumers needs to consume at date 2.

The specification of the basic data of the economy, preferences, endowments, and technologies, is now complete. From an abstract point of view, the economy must deal with the following allocation problem:

- At date 0, the initial endowment $k_0$ must be divided between short-term investments $k_{01}$ and long-term investments $k_{02}$.
- At date 1, the liquidation $L \leq k_{02}$ of date 0 long-term investments must be determined; this may depend on the observed realization of $\tilde{\theta}_2$.
- At date 1, the available returns from date 0 short-term investments and possibly from liquidated date 0 long-term investments must be divided between current consumption and new short-term investments; this decision also may depend on the observed realization of $\tilde{\theta}_2$.
- For any one consumer, it must be determined how much he consumes if he happens to need to consume at date 1 and how much he consumes if he happens to need to consume at date 2.

In the following two sections I study this allocation problem in the abstract, without any concern for the institutional setting. The objective is to characterize efficient allocations and to see how the initial uncertainty about $\tilde{\theta}_2$, the rate of return on date 1 short-term investments, affects (i) the initial
investment choice at date 0 and (ii) the allocation of consumption across date 1 and date 2 households. Subsequently, in section 5, I will discuss the implementation of efficient allocations through specific institutions, with particular attention to the implications of the analysis for the role of banking.

3. Efficient risk allocations

The analysis of efficient allocations is restricted to *equal-treatment* allocations under which all date 1 consumers have the same consumption and all date 2 consumers the same consumption. Thereby, I abstract from distributional issues, and I can use the representative consumer's ex ante expected utility for welfare assessments. An efficient equal-treatment allocation then is any solution to the

**First-Best Welfare Problem:**

\[
\text{max } E [p u(c_1) + (1-p) u(c_2)],
\]

subject to:

\[
k_{01} + k_{02} = k_0, \quad (2a)
\]

\[
p c_1 \leq \theta_1 k_{01} + \varepsilon \bar{L}, \quad (2b)
\]

\[
(1-p)c_2 = \eta (k_{02} - \bar{L}) + \bar{d}_2 (\theta_1 k_{01} + \varepsilon \bar{L} - p c_1), \quad (2c)
\]

\[
0 \leq \bar{L} \leq k_{02}; \quad (2d)
\]

here $c_1$ is a prior plan indicating the consumption allocated to any household whose consumption needs happen to arise at date 1, $c_2$ is a prior plan indicating the consumption allocated to any household whose consumption needs happen to arise at date 2, and $\bar{L}$ is a prior plan concerning possible liquidations of long-term investments at date 1.

As discussed by Diamond and Dybvig (1983) and von Thadden (1991), the feasibility constraints (2b) and (2c) reflect the assumption that the proportions of date 1 and date 2 consumers in the population are nonrandom and equal to the probabilities $p$ and $1-p$. Per capita of the economy as a whole, aggregate consumption at date 1 is just $p c_1$. Condition (2b) requires that this amount should not exceed the aggregate resources per capita that are available from short-term and liquidated long-term investments. Similarly, condition (2c) requires that per capita of the economy as a whole, aggregate consumption at date 2, $(1-p)c_2$, should be covered by the returns that are available from nonliquidated long-term investments at date 0 as well as from short-term reinvestments of unused resources at date 1.
In the following, I write \((c_1^*(q,b), c_2^*(q,b))\) to denote the solution to the maximization problem
\[
\max_{c_1, c_2} \left[ pu(c_1) + (1 - p)u(c_2) \right]
\]
subject to:
\[
qc_1 + (1 - p)c_2 = b
\]
for given \(q > 0\) and \(b \geq 0\). I also assume that
\[
\varepsilon \theta_2 < \eta \quad \text{with probability one}
\]
so that at date 1 it is never desirable to liquidate long-term investments in order to make room for additional short-term investments. Given this assumption as well as the condition \(\varepsilon < \theta_1\), one obtains:

**Proposition 1.** Let \((k_{01}, k_{02}, \tilde{c}_1, \tilde{c}_2, \tilde{L})\) be a solution to the first-best problem and define \(\theta_2^* := u'(\theta_1 k_{01}/p)/u'((\eta k_{02}/(1 - p))\). Then
\[
\tilde{c}_1 = \begin{cases} 
\frac{\theta_1 k_{01}}{p} & \text{if } \theta_2 \leq \theta_2^*, \\
\tilde{c}_1^*(\tilde{\theta}_2, \tilde{\theta}_2 \theta_1 k_{01} + \eta k_{02}) & \text{if } \theta_2 \geq \theta_2^*, 
\end{cases}
\]
\[
\tilde{c}_2 = \begin{cases} 
\eta k_{02} \frac{1 - p}{p} & \text{if } \theta_2 \leq \theta_2^*, \\
\tilde{c}_2^*(\tilde{\theta}_2, \tilde{\theta}_2 \theta_1 k_{01} + \eta k_{02}) & \text{if } \theta_2 \geq \theta_2^*, 
\end{cases}
\]
and
\[
\tilde{L} = 0
\]
with probability one.

Proposition 1 characterizes the first-best consumption and liquidation policies in terms of the initial investments \(k_{01}, k_{02}\) and the exogenous parameters \(\eta, \theta_1, \theta_2\) and \(p\). Since \(k_{01}\) and \(k_{02}\) are themselves endogenous this characterization may seem awkward, but it is easily understood in terms of dynamic-programming considerations. Once the investments \(k_{01}, k_{02}\) and the liquidation policy \(\tilde{L}\) are given, the consumption policies \(\tilde{c}_1, \tilde{c}_2\) must be maximizing the objective (1) under the constraints (2b) and (2c). Eqs. (5a) and (5b) reflect the results of this optimization subroutine for the given \(k_{01}, k_{02}\) and the liquidation policy \(\tilde{L} = 0\) that is given by (5c).

Liquidations of long-term investments are always zero under the first-best allocation. This contrasts with a situation of autarky in which the consumer is subject to the constraints \(\tilde{c}_1 = \theta_1 k_{01} + \varepsilon k_{02}\) and \(\tilde{c}_2 = \theta_2 \theta_1 k_{01} + \pi k_{02}\), requiring him to liquidate long-term investments if he needs to consume at date 1 and to reinvest short-term investments if he needs to consume at date 2.
Here the inefficiencies associated with premature liquidations of long-term investments are altogether avoided as the law of large numbers ensures that there are always enough date 2 consumers whose short-term investments can be used to provide for the needs of date 1 consumers. This observation has previously been made by von Thadden (1991) in a simpler model without uncertainty about the rate of return on new investments at date 1. It does not appear in the original analysis of Diamond and Dybvig (1993) because – in the notation of this paper – they have \( \varepsilon = \theta_1 = 1 \), so in their analysis there is no efficiency gain from avoiding premature liquidations of long-term investments.

While long-term investments are earmarked for consumption at date 2, short-term investments are not necessarily earmarked for consumption at date 1. Given the return \( \theta_1 k_{01} \) on short-term investments, the choice between consumption and investment at date 1 depends on the rate of return \( g_2 \) on investments at date 1. Upon rewriting (2c), in the form

\[
\tilde{\vartheta}_2 pc_1 + (1-p)c_2 = \eta(k_{02} - L) + \tilde{\vartheta}_2(\theta_1 k_{01} + \varepsilon L),
\]

one sees that \( \tilde{\vartheta}_2 \) determines the tradeoff between aggregate consumption \( pc_1 \) at date 1 and aggregate consumption \( (1-p)c_2 \) at date 2. From the ex ante point of view, it is desirable to exploit this tradeoff whenever the realization of \( \tilde{\vartheta}_2 \) is sufficiently favourable. More precisely, if the realization \( \tilde{\vartheta}_2 \) exceeds the critical level \( \theta_2^* \), it is desirable to reinvest some of the return \( \theta_1 k_{01} \) that is available at date 1 in order to exploit the high rate of return on new short-term investments.

Thus, under a first-best allocation, the uncertainty about \( \tilde{\vartheta}_2 \) should affect consumption at date 1 as well as consumption at date 2. This contrasts with the autarky allocation where consumption at date 1 is unaffected by the uncertainty about \( \tilde{\vartheta}_2 \). The important observation is that from an ex ante point of view, the uncertainty about \( \tilde{\vartheta}_2 \) is seen as a source of opportunities rather than a threat. As it exploits the law of large numbers with respect to the incidence of the timing of consumption needs, the economy could in principle immunize itself altogether from the uncertainty about \( \tilde{\vartheta}_2 \), simply by setting \( \tilde{c}_1 = \theta_1 k_{01} / p \) and \( \tilde{c}_2 = \eta k_{02} / (1-p) \). However such an avoidance of risks associated with the prior uncertainty about \( \tilde{\vartheta}_2 \) is undesirable because it requires the economy to forego the benefits of new investments at date 1 when the rate of return on these investments happens to be high.\(^4\)

To be sure, from the ex post point of view, if \( \theta_2 > \theta_2^* \), the consumer who finds that he needs to consume at date 1 is rather unhappy if he gets \( c_1(\theta_2, \theta_1 k_{01} + \eta k_{02}) \) rather than \( \theta_1 k_{01} / p \). But this is the sort of regret that arises in any insurance problem if one finds that one has paid a premium

\(^4\)For \( \varepsilon \) close to zero, the bound \( \eta/\varepsilon \) on \( \tilde{\vartheta}_2 \) that is given by (4) exceeds the critical level \( \theta_2^* \). The relevance of the above considerations is therefore not mooted by the boundedness of \( \theta \) under (1).
and then there was no accident after all. Given that the problem of liquidity provision is treated as a problem of insurance against the uncertainty about the timing of consumption needs, welfare assessments must be based on ex ante expected utility without regard to any regrets that might arise ex post.

To conclude this discussion, I turn to the first-best levels of the initial investments $k_{01}$ and $k_{02}$ at date 0. By standard dynamic programming arguments, the first-best investment levels at date 0 must solve the problem

$$\max_{k_{01}, k_{02}} \left[ \int_0^\infty \left[ pu(\theta_1 k_{01}/p) + (1-p)u(\eta k_{02}/(1-p)) \right] \, dF(\theta_2) \right]$$

subject to:

$$k_{01} + k_{02} = k_0,$$

where $V(\cdot)$ is the indirect utility function for problem (3), i.e.,

$$V(q, b) = pu(c^*_q(q, b)) + (1-p)u(c^*_b(q, b))$$

for any $q$ and $b$. The analysis of problem (6) is standard and is left to the reader. I merely note the following features of the solution:

- The first-best level of short-term investment, $k_{01}$, is not less than and usually exceeds the level that maximizes the expected utility $pu(\theta_1 k_{01}/p) + (1-p)u(k_0 - k_{01})/(1-p)$ when reinvestments at date 1 are ruled out. The attractiveness of short-term investments thus is enhanced by the option to reinvest the returns of such investments for another period at date 1. If $u'(0) = \infty$, one can immediately infer that $k_{01} > 0$.

- The first-best level of short-term investment is not less than and usually exceeds the level that maximizes the expected indirect utility $EV(\theta_2, \theta_2 \theta_1 k_{01} + \eta k_{02})$ in the case when long-term investments can always be liquidated at date 1 for the present value $\eta k_{02}/\theta_2$. The attractiveness of short-term investments is enhanced by the illiquidity of long-term investments, i.e., the inability to liquidate them for their present value at date 1 if $0 < \theta_1 E\theta_2$.

- If $\theta_2 > 0$ with probability one, and if $\eta \geq \theta_1 E\theta_2$, then $k_{02} > 0$. Indeed if $\eta < \theta_1 E\theta_2$, one still has $k_{02} > 0$ if the difference $\theta_1 E\theta_2 - \eta$ is not too large. Long-term investments at date 0 are attractive if the rate of return $\eta$ on these investments is sufficiently high. Even if $\eta$ is not so high, they may still be attractive because the represent a safe way to provide for consumption at date 2. In contrast, under successive short-term investments, the uncertainty about $\theta_2$ would make consumption at date 2 uncertain.

5 A sufficient condition for this is that $u'(0) = \infty$. 

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4. Second-best allocations under asymmetric information

I now abandon the assumption that there are no information asymmetries in the economy. Following Diamond and Dybvig (1983), I assume that the realization of the timing of consumption needs is the private information of the consumer in question. If a consumer reports that he happens to need to consume at date 1, nobody else is able to check whether this report is true or not. I also assume that nobody else observes at what date he actually consumes. Nor is anybody else able to check if he takes resources at date 1 and reinvests them in his backyard for consumption at date 2. The rate of return on such backyard investments at date 1 is taken to be $\theta_2$, as on all other investments at this date.

Given this information asymmetry, an allocation $(k_{01}, k_{02}, \bar{c}_1, \bar{c}_2, \bar{L})$ can only be implemented if it is incentive compatible, i.e., if it gives no consumer an incentive to lie about when he actually wants to consume. Under the plan $(k_{01}, k_{02}, \bar{c}_1, \bar{c}_2, \bar{L})$, an agent who claims a need to consume at date 1 obtains $\bar{c}_1$ units of the good at this date. If in fact he reinvests these $\bar{c}_1$ units in his backyard, he obtains $\bar{c}_2 \bar{c}_1$ units of good at date 2. Incentive compatibility requires that with probability one the $\bar{c}_2$ units which he gets at date 2 if he is honest be at least as large as the $\bar{c}_2 \bar{c}_1$ units that he gets if he lies and claims that he wants to consume at date 1 when in fact he wants to consume at date 2.

In the absence of a backyard technology for converting date 2 consumption goods into date 1 consumption goods, there is no other incentive compatibility condition to be considered, so one now faces the:

Second-Best Welfare Problem:

Max $\mathbb{E}[pu(\bar{c}_1) + (1-p)u(\bar{c}_2)]$, \hspace{1cm} (7)

subject to: \hspace{1cm} $k_{01} + k_{02} = k_0$, \hspace{1cm} (8a)

$p\bar{c}_1 \leq \theta_1 k_{01} + \varepsilon \bar{L}$, \hspace{1cm} (8b)

$(1-p)\bar{c}_2 = \eta (k_{02} - \bar{L}) + \bar{c}_2 (\theta_1 k_{01} + \varepsilon \bar{L} - p\bar{c}_1)$ \hspace{1cm} (8c)

$\bar{c}_2 \bar{c}_1 \leq \bar{c}_2$, \hspace{1cm} (8d)

$0 \leq \bar{L} \leq k_{02}$. \hspace{1cm} (8e)

Relying on dynamic-programming considerations as before, one now obtains:

$^6$See, however, the discussion of market-related incentive-compatibility conditions in footnote 14.
Proposition 2. Assume that \( \overline{\theta}_2 \geq 1 \) with probability one and that the function \( c \rightarrow u'(c)c \) is decreasing. Let \( (k_{o1}, k_{o2}, c_1, c_2, L) \) be a solution to the second-best welfare problem.

(a) If \( e u'(\theta_1 k_{o1}/p) \leq \eta u'(\eta k_{o2}/(1 - p)) \), then

\[
\tilde{c}_1 = \min \left[ \frac{\theta_1 k_{o1}}{p}, \theta_1 k_{o1} + \frac{\eta}{\overline{\theta}_2} k_{o2} \right],
\]

and

\[
\tilde{c}_2 = \max \left[ \frac{\eta k_{o2}}{1 - p}, \overline{\theta}_2 \theta_1 k_{o1} + \eta k_{o2} \right],
\]

with probability one.

(b) If \( e u'(\theta_1 k_{o1}/p) > \eta u'(\eta k_{o2}/(1 - p)) \), then

\[
\tilde{c}_1 = \min \left[ \frac{1}{\epsilon} \left( \frac{\eta}{\epsilon} \theta_1 k_{o1} + \eta k_{o2} \right), \frac{1}{\epsilon} \frac{(\eta/\epsilon) \theta_1 k_{o1} + \eta k_{o2}}{(1 - p) \overline{\theta}_2 + p(\eta/\epsilon)}, \theta_1 k_{o1} + \frac{\eta}{\overline{\theta}_2} k_{o2} \right]
\]

and

\[
\tilde{L} = \max \left[ 0, (p \tilde{c}_1 - \theta_1 k_{o1})/\epsilon \right]
\]

with probability one.

In Proposition 2, the assumption that \( \overline{\theta}_2 \geq 1 \) with probability one can be interpreted in the sense that at the very least, one can always store the good without costs. The assumption about the utility function is equivalent to the condition that \( c_1^*(q, q) \) be increasing and \( c_2^*(q, b) \) be decreasing in \( q \), i.e., that for the solutions to problem (3) the income effects of a change in \( q \) always dominate the substitution effects. Both assumptions are taken from Diamond and Dybvig (1983), but even so one must acknowledge that they are restrictive. The interest of Proposition 2 is not in its generality, but in the remarkable strength of its conclusions.

For instance, in the isoelastic case, \( u'(c) = c^{\gamma - 1} \), the exponent \( \gamma \) must be nonpositive, so, e.g., \( u(c) = 2e^{c^{1.5}} \) is excluded. More generally, the assumption on the utility function requires that the coefficient of relative risk aversion of consumers be larger than one.
Three aspects of Proposition 2 are remarkable. First, the incentive compatibility constraint (8d) plays a key role in determining consumption. If \( u'(c)c \) is decreasing in \( c \), one has \( c^*_1(q,b) \geq b/q \) and \( c^*_2(q,b) \geq b \) as \( q \geq 1 \) and hence \( q \cdot c^*_1(q,b) > c^*_2(q,b) \) whenever \( q > 1 \). Given the assumption that \( \bar{\theta}_2 \geq 1 \) with probability one, it follows that the incentive constraint (8d) must be binding whenever there is active reinvestment at date 1 and the tradeoff between \( \bar{c}_1 \) and \( \bar{c}_2 \) is given by \( \bar{\theta}_2 \). In this case consumption is determined by the interplay between incentive compatibility and feasibility conditions rather than the details of the utility function. The details of the utility function affect the initial investment choice \( (k_{01},k_{02}) \), but once \( k_{01} \) and \( k_{02} \) are given, the function \( u(\cdot) \) matters only to the extent that it drives the choice of consumption \( (\bar{c}_1, \bar{c}_2) \) against the incentive compatibility constraint (8d).

Secondly, the incentive compatibility constraint (8d) provides a new reason for having a negative dependence on \( \bar{c}_1 \) on \( \bar{\theta}_2 \). In Proposition 1, the dependence of date consumption on \( \bar{\theta}_2 \) was presented as a way to take advantage of the opportunities provided by high realizations of \( \bar{\theta}_2 \). Here a dependence of date 1 consumption on \( \bar{\theta}_2 \) may simply be imposed by incentive compatibility considerations. To see the difference, note that under (8d), one can no longer set \( \bar{c}_1 = \theta_1 k_{01}/p, \bar{c}_2 = \eta k_{02}/(1-p) \) in order to immunize the economy against the uncertainty about \( \bar{\theta}_2 \). If \( \bar{\theta}_2 \) exceeds a critical value \( \bar{\theta}_2 \) defined as

\[
\bar{\theta}_2 = \frac{p}{1-p} \frac{\eta k_{02}}{\theta_1 k_{01}},
\]

then \( \bar{\theta}_2 \theta_1 k_{01}/p > \eta k_{02}/(1-p) \) so that \( (\bar{c}_1, \bar{c}_2) = (\theta_1 k_{01}/p, \eta k_{02}/(1-p)) \) is not incentive compatible.

Thirdly, the second-best allocation may provide for premature liquidations of long-term investments at date 1. As shown by Proposition 1, under the first-best allocation, such liquidations never occur; the level of short-term investments, \( k_{01} \), is always so high that a further increase of date 1 consumption by liquidations of long-term investments is undesirable. However, when the constraint (8d) is taken into account, the level of short-term investment at date 0 may be reduced in order to diminish the incentive-compatibility-imposed exposure to uncertainty when \( \bar{\theta}_2 \) exceeds the critical value \( \bar{\theta}_2 \) in (11). Thus the second-best allocation may satisfy \( \theta_1 u'((\theta_1 k_{01}/p) > \eta u'(\eta k_{02}/(1-p))) \) and even, for \( \epsilon \) sufficiently close to \( \theta_1 \), \( \epsilon u'((\theta_1 k_{01}/p) > \eta u'(\eta k_{02}/(1-p))) \), so that premature liquidations at date 1 are in principle...
desirable. With \( \varepsilon < \theta_1 \), this may appear to be an inefficient way to provide for consumption at date 1. However, since liquidations can be conditioned on \( \theta_2 \), the productivity losses from relying on premature liquidations may be outweighed by the gains from a less rigid incidence of the incentive compatibility constraint (8d).

As the premise of statement (b) in Proposition 2 has not yet been shown to be ever satisfied, these considerations may appear to be a bit speculative. The terms \( u'(\theta_1 k_0/(1-p)) \) and \( \eta u'((k - k_0)/(1-p)) \) in the premises of statements (a) and (b) of Proposition 2 involve the endogenous variables \( k_0 \) and \( k_2 \), so it is not clear what constellations of the exogenous parameters of the model would actually correspond to the two cases. To deal with this issue, consider the problem of maximizing (7) subject to the constraints (8a)–(8d) and \( L = 0 \). Let \( (k_0, k_2, \tilde{c}_1, \tilde{c}_2, \tilde{L}) = 0 \) be a solution to this problem and note that this solution is independent of the parameter \( \varepsilon \). It follows that

\[
\tilde{\varepsilon} = \frac{\eta u'((k_0 - k_0)/(1-p))/u'(\theta_1 k_0/p)}{} \tag{12}
\]

is also independent of \( \varepsilon \) (though it does depend on the other model parameters). One easily verifies that the optimal zero-liquidation allocation \( (k_0, k_2, \tilde{c}_1, \tilde{c}_2, \tilde{L}) \) is a solution to the second-best welfare problem if and only if \( \varepsilon \leq \tilde{\varepsilon} \). For \( \varepsilon \leq \tilde{\varepsilon} \), it follows that any solution to the second-best welfare problem is almost surely equal to \( (k_0, k_0, \tilde{c}_1, \tilde{c}_2, \tilde{L}) \) and hence satisfies (9a)–(9c). For \( \varepsilon > \tilde{\varepsilon} \), it follows that any solution to the second-best welfare problem must satisfy (10a)–(10c); a solution satisfying (9a)–(9c) would be almost surely equal to the optimal zero-liquidation allocation \( (k_0, k_0, \tilde{c}_1, \tilde{c}_2, \tilde{L}) \), contrary to the nonoptimality of \( (k_0, k_0, \tilde{c}_1, \tilde{c}_2, \tilde{L}) \) for the second-best welfare problem in this case. Now it is clear that statement (a) of Proposition 2 is always applicable if \( \varepsilon \) is sufficiently small. As for statement (b), its relevance depends on whether the condition \( \varepsilon > \tilde{\varepsilon} \) is compatible with the assumption \( \varepsilon < \theta_1 \). This in turn depends on the other parameters of the model, as indicated in

**Proposition 3.** Assume that \( \tilde{\theta}_2 \geq 1 \) with probability one, and that the function \( c \mapsto u'(c) c \) is decreasing. Let \( \tilde{\varepsilon} \) be defined by (12). If \( \varepsilon \in [0, \tilde{\varepsilon}] \), any solution to the second-best welfare problem satisfies (9a)–(9c); if \( \varepsilon \in (\tilde{\varepsilon}, \theta_1) \), any solution to the second-best welfare problem satisfies (10a)–(10c). The interval \( (\tilde{\varepsilon}, \theta_1) \) is nonempty if \( F(\tilde{\theta}_2) < 1 \) and

\[
\int_{\tilde{\theta}_2}^{\infty} (\theta_2 - \theta_1 - \eta) \, dF(\theta_2) \leq 0, \tag{13}
\]

where

\[\text{Recall that } u(\cdot) \text{ is strictly concave and the constraint set defined by (8a)–(8e) is convex.}\]
With \( u'(c)c \) decreasing in \( c \), one has \( \bar{\theta}_2 < \eta/\theta_1 \), so (13) is actually compatible with \( F(\bar{\theta}_2)<1 \). The inequality \( F(\bar{\theta}_2)<1 \) ensures that with positive probability one has \( \bar{\theta}_2 > \bar{\theta}_2 \) and for initial investments satisfying \( \theta_1 u'(\theta_1 k_{01}/p) \leq \eta u'(\eta k_{02}/(1 - p)) \), the incentive-compatibility constraint (8d) is binding. The inequality (13), ensures that long-term investments at date 0 are not too unattractive for there to be anything to be prematurely liquidated at date 1. Thus in the case \( F(\eta/\theta_1)=1 \), with \( \bar{\theta}_2 \theta_1 \leq \eta \) almost surely, Proposition 3 asserts that in a situation in which long-term investments dominate successive short-term investments, if \( \bar{\theta}_2, \theta_1 \in [\bar{\theta}_2, \eta/\theta_1] \) with positive probability, the incentive constraint (8d) creates a strict incentive to raise \( k_{02} \) and lower \( k_{01} \) relative to the first-best levels; for \( \varepsilon \) sufficiently close to \( \theta_1 \) this effect outweighs the efficiency argument against premature liquidations.

To conclude this section, I note that here as in the first-best welfare problem, \( k_{02} \), must be strictly positive if \( \theta_1, \eta \in [\theta_2, \eta/\theta_1] \) with positive probability. The proof is left to the reader.

5. The implementation of second-best allocations: A role for banks?

So far I have neglected the question of what institutions would implement first-best or second-best allocations. Along the lines of Diamond and Dybvig (1983), I now discuss the implementation of second-best allocations by a competitive system of financial intermediation. Abstracting from problems of moral hazard and fraud on the side of the intermediary, I assume that an intermediary can actually commit itself to an investment-and-liquidation policy which can be verified – and if necessary enforced through court action – by the intermediary’s financiers. A contract between an intermediary and a household will therefore specify

- the amount of initial capital which the household surrenders to the intermediary,
- the allocation of this initial capital to short-term and long-term investments,
- a plan for state-dependent liquidations of long-term investments at date 1,
- a system of state- and date-contingent promises of return payments from the intermediary to the household.

Intermediaries compete by making contract offers to the different households. On the basis of the intermediaries’ promises, the households form expectations about the returns that they will actually get under the different contracts. They will choose the contracts whose outcomes they like best.

The standard argument of Bertrand suggests that in this setting compe-
tition forces the intermediaries to offer the households the best contracts that are at all feasible. If we suppose that as of date 1, the intermediaries can distinguish whether a given household is a date 1 consumer or a date 2 consumer, this implies that a system of unregulated (Bertrand) competition among intermediaries will actually implement the first-best allocation. If instead we suppose that as of date 1 intermediaries cannot distinguish between date 1 and date 2 consumers, the pressures of competition should still force intermediaries to offer contracts that will implement a second-best allocation.

Actually, the matter is slightly more complicated because under asymmetric information households as well as the intermediaries have to worry about what happens at date 1. If at date 1 all consumers claim the date 1 consumption $z_1$, the intermediary's resources will not be equal to the sum of these claims, and he will have to default on at least some of them. If in such a case there is a premium to having been the first to present one's claims, then, as discussed by Diamond and Dybvig (1983), the prediction of a default by the intermediary may well be self-fulfilling. Such a prediction creates an incentive to run on the intermediary in order to be first in line; if everybody does so, the intermediary in fact has to default, and the prediction comes true. A bank run may well be an equilibrium of the 'claims game' between households at date 1.

The possibility of such bank run equilibria weakens the argument that competition forces intermediaries to offers contracts that will implement a second-best allocation. If they do offer such contracts, a second-best allocation will not be implemented after all if at date 1 consumers play the bank run equilibrium rather than the 'normal' equilibrium. Moreover the force of competition at date 0 may be blunted if consumers fear that a contract offer which under normal circumstances should be superior will actually be inferior because under this contract, at date 1 bank run equilibrium will be played.\footnote{For more detailed discussions of this effect in other contexts, see Yanelle (1989), Gale and Hellwig (1989).}

Fortunately, it is possible to design contracts that do not induce bank run equilibria. As discussed by Diamond and Dybvig (1983), the possibility of a run as an equilibrium phenomenon can be excluded if the initial contract contains a clause permitting the intermediary to suspend payments at date 1 once the aggregate of his payments at this date reaches the level $pc_1$. Such a clause protects the intermediary's long-term investments from excessive premature liquidations so his ability to honour his obligations at date 2 is not endangered by liquidations at date 1. Knowing this, the date 2 consumers have no incentive to present their claims at date 1, and the possibility of a bank run is excluded.
Given that contracts can be designed so that runs are excluded, the Bertrand argument actually goes through, so indeed competition among intermediaries will serve to implement a second-best allocation. The requisite contracts have a remarkably simple structure: at date 0, each household surrenders his initial capital $k_0$, with amounts $k_{01}$ to be put into short-term investments and $k_{02} = k_0 - k_{01}$ to be put into long-term investments. Subject to the intermediary's right to suspend payments at date 1, the household is free to claim a payment from the intermediary at date 1 or at date 2. Under the assumptions of Propositions 2 and 3, if the value of the parameter $\varepsilon$ is below the critical value $\bar{\varepsilon}$ in (12), then at date 1, the household may claim $\bar{c}_1$ as given by (9a); at date 2, he may claim $\bar{c}_2$ as given by (9b). If the value of parameter $\varepsilon$ exceeds the critical value $\bar{\varepsilon}$, then at date 1, the household may claim $\bar{c}_1$ as given by (10a); at date 2 he may claim $\bar{c}_2$ as given by (10b). In both cases, if $\varepsilon \leq \bar{\varepsilon}$ and if $\varepsilon > \bar{\varepsilon}$, the intermediary may suspend payments at date 1 if the fraction of the population that makes claims reaches $p$, the fraction of date 1 consumers in the population.

The equilibrium contract has a simple interpretation in terms of a securitization of returns from long-term investments. I show this first for the case $\varepsilon \leq \bar{\varepsilon}$. In this case if $\theta_2 \geq \bar{\theta}_2$, the consumer's claim at date 1 is just $\theta_1 k_{01} + \eta k_{02}/\bar{\theta}_2$, the sum of the return on the short-term investment and the present value at date 1 of the return on the long-term investment discount at the rate $\bar{\theta}_2$. Alternatively, his claim at date 2 is $\bar{\theta}_2 \theta_1 k_{01} + \eta k_{02}$, the sum of the return on reinvesting the first-period return on $\theta_1 k_{01}$ at the rate $\bar{\theta}_2$ and the second-period return on the long-term investment $k_{02}$. Similarly, if $\bar{\theta}_2 < \bar{\theta}_2$, the consumer's claim at date 1 is equal to

$$\frac{\theta_1 k_{01}}{p} = \frac{\theta_1 k_{01} + \eta k_{02}}{\bar{\theta}_2},$$

the sum of the return on the short-term investment and the present value at date 1 of the return on the long-term investment, discounted at the rate $\bar{\theta}_2 = p \eta k_{02}/(1 - p) \theta_1 k_{01}$. Alternatively, the consumer's claim at date 2 is

$$\frac{\eta k_{02}}{1 - p} = \bar{\theta}_2 \theta_1 k_{01} + \eta k_{02},$$

the sum of the return on investing $\theta_1 k_{01}$ at the rate $\bar{\theta}_2$ and the return on $k_{02}$. In general then, if one defines

$$\bar{R}_2^* = \max [\bar{\theta}_2, \bar{\theta}_2],$$

one finds that for $\varepsilon \leq \bar{\varepsilon}$, the consumer's claim on the intermediary is

$$\bar{c}_1 = \theta_1 k_{01} + \eta k_{02}/\bar{R}_2^*$$

(16a)
if the claim is submitted at date 1 and
\[ \tilde{c}_2 = R_2^\ast \theta_1 k_{01} + \eta k_{02} \]  \hfill (16b)
if the claim is submitted at date 2.

The variable $R_2^\ast$ can be interpreted as an intertemporal price ratio. Suppose that as of date 1 there was a Walrasian market for date 1 and date 2 consumptions good in which all – date 1 and date 2 – consumers participate with initial endowments consisting of $\theta_1 k_{01}$ units of the date 1 good and $\eta k_{02}$ units of the date 2 good. The variable $R_2^\ast$ that is defined by (15) is nothing but the equilibrium relative price of date 1 consumption versus date 2 consumption in such a Walrasian market. To see this, note that for a given value $R_2$ of the intertemporal price ratio in such a market, the date 1 consumers want to buy $\frac{\eta k_{02}}{R_2}$ units of the date 1 good in return for $\eta k_{02}$ units of the date 2 good. The date 2 consumers’ behaviour depends on the relation between $R_2$ and the realization of $\bar{d}_2$. If $\bar{d}_2 > R_2$, they prefer to reinvest their date 1 returns in their backyards; if $\bar{d}_2 < R_2$, they want to buy $R_2 \theta_1 k_{01}$ units of good 2 in return for $\theta_1 k_{01}$ units of good 1; if $\bar{d}_2 = R_2$, they are indifferent between the two alternatives. Given these excess demand behaviours, one easily verifies that the equilibrium intertemporal price ratio must be $R_2^\ast$ if $\bar{d}_2 < \bar{d}_2^\ast$, and $\bar{d}_2$ if $\bar{d}_2 > \bar{d}_2^\ast$.

The case $\varepsilon > \varepsilon^*$ is slightly more complicated because one must take account of premature liquidations of long-term investments. Let

\[ \theta_2 = c^*_2 \left( \frac{\eta}{\varepsilon} \frac{\eta}{\varepsilon} \theta_1 k_{01} + \eta k_{02} \right) / c^*_1 \left( \frac{\eta}{\varepsilon} \frac{\eta}{\varepsilon} \theta_1 k_{01} + \eta k_{02} \right). \]

From (10a)–(10c), premature liquidations under the second-best allocation are given as

\[ L = \begin{cases} \frac{1}{\varepsilon} \left[ p c^*_1 \left( \frac{\eta}{\varepsilon} \frac{\eta}{\varepsilon} \theta_1 k_{01} + \eta k_{02} \right) - \theta_1 k_{01} \right] & \text{if } \bar{d}_2 \leq \bar{d}_2^\ast, \\ \frac{p \eta k_{02} - (1-p) \bar{d}_2 \theta_1 k_{01}}{(1-p) \bar{d}_2^* + \rho \eta} & \text{if } \bar{d}_2 \in (\bar{d}_2, \bar{d}_2^\ast), \\ 0 & \text{if } \bar{d}_2 \geq \bar{d}_2^\ast. \end{cases} \]  \hfill (17)

Given (17), one can arrange (10a) and (10b) to obtain

\[ \tilde{c}_1 = \theta_1 k_{01} + \varepsilon L + \eta (k_{02} - L)/R_2^{**}, \]  \hfill (18a)

and

\[ \tilde{c}_2 = R_2^{**} [\theta_1 k_{01} + \varepsilon L] + \eta (k_{02} - L), \]  \hfill (18b)

where

\[ R_2^{**} = \max \left[ \bar{d}_2, \bar{d}_2^\ast \right]. \]  \hfill (19)
The interpretation of conditions (18a,b) and (19) is similar to that of conditions (15) and (16a,b) except that \( \theta_1 k_{01} \) and \( \eta k_{02} \) are replaced by \( \theta_1 k_{01} + \varepsilon L \) and \( \eta(k_{02} - L) \) to take account of liquidations. With (15)-(19), the present model of uncertainty about future technologies becomes a model of interest rate risk. The technological uncertainty affects the equilibrium consumption of date 1 or date 2 consumers if and only if it affects the value of the equilibrium intertemporal price ratio \( \bar{R}_2^* \), respectively \( \bar{R}_2^{**} \). In the range above \( \bar{\theta}_2 \), the equilibrium intertemporal price ratio in a Walrasian market - as well as the second-best allocation of consumption - depend on \( \bar{\theta}_2 \) because, with actual reinvestments of returns at date 1, \( \bar{\theta}_2 \) determines the relevant shadow price of the date 1 good in terms of the date 2 good from the perspective of date 2 consumers. For \( \varepsilon > \bar{\varepsilon} \), in the range between \( \theta_2 \) and \( \bar{\theta}_2 \), the equilibrium intertemporal price ratio \( \bar{R}_2^{**} \) still depends on \( \bar{\theta}_2 \) because through the incentive compatibility constraint (8d), \( \bar{\theta}_2 \) affects the liquidations of long-term investments and hence the endowment \( (\theta_1 k_{01} + \varepsilon L, \eta(k_{02} - L)) \) of consumers in the above account of an organized market at date 1. In contrast, fluctuations of \( \bar{\theta}_2 \) below \( \bar{\theta}_2 \) in the case \( \varepsilon \leq \bar{\varepsilon} \), or below \( \theta_2 \) in the case \( \varepsilon > \bar{\varepsilon} \), have no effect on the equilibrium intertemporal price ratio or on the second-best allocation of consumption because in this range, there is no reinvestment at date 1, and moreover the incentive compatibility constraint (8d) is not binding.

With this account of the relation between the technological parameter \( \bar{\theta}_2 \) and the intertemporal price ratio \( \bar{R}_2 \) in an organized Walrasian market at date 1, any solution to the second-best welfare problem is seen to involve a remarkably simple and clear-cut risk allocation: for a given choice \( (k_{01}, k_{02}, L) \) of real investments at date 0 and liquidations at date 1, the date 1 consumer bears all the valuation risk of the long-term investment, and the date 2 consumer bears all the reinvestment-opportunity risk of the short-term investment, the extent of both risks is determined by the equilibrium intertemporal price ratio \( \bar{R}_2^* \) respectively \( \bar{R}_2^{**} \). Notice that there is no transfer of valuation risk from date 1 consumers to date 2 consumers and no transfer of reinvestment-opportunity risk from date 2 consumers to date 1 consumers. Nor is either risk assumed by the intermediary.

At this point, we must refine our notions of liquidity provision by financial intermediaries. Under the contract that implements a second-best allocation, an intermediary provides households with liquidity in the sense that he enables even a date 1 consumer to benefit from the funds he had earmarked for long-term investments. However, he does not insulate his clients from the interest rate risks (valuation risks) to which their long-term investments are subjected. Nor does he insulate them from the interest rate risks that affect the reinvestment of returns on short-term investments. When we think of maturity transformation by intermediaries, we must distinguish between the market-making function by which the intermediary securitizes long-term investments
without assuming interest rate risks and a risk-shifting function by which the intermediary assumes some or all of the interest rate (valuation risks) risks of long-term investments. The analysis here suggests that liquidity provision by intermediaries should involve the market-making function without any assumption of interest rate risk by the intermediary.

Why are intermediaries actually needed? Couldn’t one implement a second-best allocation just as well through Walrasian markets? After all, the above discussion has shown that if the consumers were to meet in a Walrasian market at date 1, the market would clear at the intertemporal price ratio $\tilde{R}_2$, respectively $\tilde{R}_2^{**}$, with date 1 consumers obtaining the date 1 consumption corresponding to the second-best allocation and date 2 consumers obtaining the date 2 consumption corresponding to the second-best allocation.$^{11}$

There are three possible answers to this question. The first one is that the intermediary eliminates the need for consumers to meet and trade with each other at date 1. In principle, liquidity might just as well be provided through direct exchanges between date 1 consumers and date 2 consumers in organized markets at date 1. However, if the different consumers are spatially separated at this time, it may be more convenient to have exchanges coordinated by an intermediary. Whereas date 1 consumers and date 2 consumers do not meet, the intermediary as a market maker provides for the indirect exchange which enables date 1 consumers to trade their prospective returns on long-term investments against the date 2 consumers’ returns on short-term investments.$^{12}$

Secondly, in the case $\varepsilon > \bar{\varepsilon}$, one may observe that the equilibrium contract that is offered by an intermediary at date 0 provides a mechanism of commitment to the liquidation policy $\tilde{L}$ that corresponds to the second-best allocation. Without such a mechanism of commitment, in a Walrasian market at date 1, no consumer would have an incentive to liquidate long-term investments, i.e., to transform date 2 returns into date 1 returns at the rate $1/\tilde{R}_2^{**} = 1/\tilde{\theta}_2 > \varepsilon/\eta$.

It is not clear how strong the argument for intermediation as a mechanism of commitment actually is. If there are no organized markets at date 1, the argument is unproblematic, but then it is anyway clear that one needs

$^{11}$This objection is closely related to Jacklin’s (1987) observation that organized Walrasian markets at date 1 reduce the scope for financial intermediation because they eliminate the possibility, suggested by Diamond and Dybvig of using intermediation to shift consumption forward in time. Here the Diamond–Dybvig notion of having intermediaries to allow intertemporal consumption shifts has already been eliminated through the incentive compatibility constraint (8d), so the question is not whether organized markets at date 1 change the nature of the constraints under which intermediaries operate, but whether they might simply make intermediaries redundant.

$^{12}$For a thorough discussion of the role of spatial separation in this type of analysis, see Wallace (1988).
intermediaries to implement a second-best allocation. Suppose therefore that there do exist organized markets at date 1 as well as intermediaries offering contracts at date 0. In this case it seems that for \( \varepsilon > \delta \), any contracts between consumers and an intermediary that would implement the second-best allocation may not be renegotiation-proof: with an intertemporal price ratio \( \tilde{R}^{**}_2 = \max (\theta_2, \tilde{\theta}_2) < \eta/\varepsilon \) in the market, both a consumer and the intermediary would seem to have an incentive to abandon the initial contract in favor of a new one that involves no liquidations and instead permits the consumer to withdraw the rights to \( \theta_1 k_{01} - \delta \) units of the date 1 good and \( \eta k_{02} - \delta \) units of the date 2 good in order to trade on the open market; here \( \delta \) is a small positive number indicating the share of the renegotiation gain that goes to the intermediary. Whether in fact such a renegotiation can displace the initial contract depends on whether the consumer takes the price ratio \( \tilde{R}^{**}_2 \) in the market as given. If he expects that his coming to the market with the endowment \( (\theta_1 k_{01} - \delta, \eta k_{02} - \delta) \) will have an effect on the price, then he may prefer not to renegotiate the contract with the intermediary after all. The idea that one consumer – out of a continuum – should affect the price may seem absurd; however, one must realize that if all consumers accept contracts from intermediaries at date 0, then in fact there is no need for markets at date 1, i.e., at the equilibrium price there will be no transaction. In this case it is not clear that a null set of consumers coming to the market will in fact be too insignificant to move the price. In particular, it is not clear that a date 1 consumer who wants to sell \( \eta k_{02} - \delta \) units of the date 2 good will actually be able to raise his consumption above the \( \theta_1 k_{01} + \varepsilon L + \eta (k_{02} - L)/\tilde{R}^{**}_2 \) units that he has without renegotiation.\(^{13}\)

Thirdly, a need to rely on intermediaries rather than markets would arise in a more general model in which intermediaries also have monitoring or screening functions. Following Diamond (1984), we may think of an intermediary as an institution which invests in a well diversified portfolio of projects, monitoring each project and exploiting the fact that it can perform such monitoring just as well, but much more cheaply than one million households. In this interpretation, the parameter \( \theta_1 \) in the analysis here would correspond to the expected rate of return (net of monitoring costs) on a single short-term project; if the intermediary’s portfolio was sufficiently well diversified, \( \theta_1 \) would also correspond to the actual (certain) rate of return on this portfolio. The parameter \( \eta \) and the random variable \( \tilde{\theta}_2 \) can be similarly reinterpreted. The variables \( k_{01}, k_{02} \) and \( k_{12} = \theta_1 k_{01} + \varepsilon L - p\tilde{c}_1 \) must then be seen as aggregates of funds allocated to short-term investments at date 0, long-term investments at date 0, and short-term investments at date 1; these

\(^{13}\)The given argument can also be used to deal with the possibility that the intermediary itself might wish to cheat by using a market sale of the date 2 good for the date 1 good rather than liquidations to satisfy the date 1 consumers’ claims.
aggregates must be invested so as to exploit all possible gains from diversification. I assume that there is no moral hazard in this respect.

However, if intermediaries are assumed to perform a monitoring function, the incentive compatibility constraint (8d) in the second-best welfare problem needs to be reinterpreted. If a given realization $\theta_2$ of the random variable $\tilde{\theta}_2$ is interpreted as the rate of return which an intermediary obtains from a well diversified portfolio of projects at date 1, then one cannot assume that $\theta_2$ is also the rate of return that an individual date 2 consumer can obtain if he invests the date 1 good on his own. After all, the point of the monitoring model of intermediation is precisely that delegated monitoring through intermediaries enables agents to achieve higher returns on their investments.

The appropriate specification of incentive constraints now depends on the alternatives that are given to a date 2 consumer who dresses up as a date 1 consumer to make a premature claim on the intermediary. If he can merely watch his withdrawal go to rot between dates 1 and 2, then there is no incentive problem at all. Similarly, there is not much of an incentive problem if he can make, say a non-diversified investment in a single short-term project at date 1. However, in an unregulated competitive system, he may have access to a new intermediary offering a rate of return $\theta_2$ from a well-diversified portfolio of new short-term projects. In this case the incentive compatibility constraint (8d) is again the appropriate one; it reflects the need to immunize the second-best allocation from the threat of disintermediation through the competition of new intermediaries. Given this constraint, the results of the analysis here remain valid even if intermediaries serve a monitoring as well as a liquidity provision function.14

6. Concluding remarks: Interest rate risk, moral hazard, and the role of banking

In reality we observe that directly or indirectly financial intermediaries do bear a lot of interest rate risk. What then are we to make of the results of this paper? A simple response might be that (i) the positive predictions about the implementation of second-best allocations by Bertrand competition between financial intermediaries should not be taken too seriously and (ii) the discrepancy between the normative results and the observed practice

14By a similar consideration, one can use a strengthening of the incentive constraint (8d) to eliminate the need for the extra assumptions on $u(\cdot)$ and $\tilde{\theta}_2$ in Propositions 2 and 3. In section 4, (8d) was imposed as a purely technology-based constraint so that the analysis of the second-best welfare problem was separated from any consideration of institutions. If instead one assumes that there always is a system of Walrasian markets in the background and if agents are taken to believe that in these markets they arbitrarily trade back and forth at given prices, the technology induced constraint (8d) must be replaced by the market-induced constraint $\bar{R}_{2\hat{c}_1} = \bar{c}_2$ at which point one no longer needs any special assumptions on $u(\cdot)$ and $\tilde{\theta}$ to ensure that the incentive is always binding.
should be interpreted as evidence that the exposure of banks to interest rate risk is 'excessive', as presumed by the bank supervisors.

As a piece of positive analysis the paper has the weakness that it neglects all questions of moral hazard on the side of the intermediaries. Whereas I have assumed that the investment policy \((k_{o1}, k_{o2})\) as well as the liquidation plan \(L\) are part of the contract between an intermediary and its finances – and as such can be enforced – in practice depositors are not really able to control a bank's investment policy. Moreover, from the work of Stiglitz and Weiss (1981) and Rochet (1992), it is known that intermediaries which are financed by an issue of claims with outcome-independent payment obligations will have an incentive to take excessive risks as some of the risk of insolvency falls on their financiers rather than themselves. Unfortunately it is unclear whether this consideration entails the intermediary's taking excessive valuation risks from long-term investments or excessive reinvestment-opportunity risks from short-term investments. This requires further research.

If one accepts a moral-hazard explanation of the observed exposure of depository institutions to interest rate risk, one may wish to conclude that statutory regulation is needed to control this moral hazard. From this perspective the welfare analysis of this paper provides a point of reference as to what should be considered desirable, i.e., as to the direction towards which a regulation intended to restrain moral hazard ought to aim. As such, the paper calls for a clear reorientation of banking regulations: regulators ought to concern themselves less with the immunization of depositors from risk and more with the overall (nth best) efficiency of the risk allocation.

Such a reorientation of banking regulation seems the more important since in the past the very concern of regulators with the risklessness of deposits seems to have been a source of distortions that actually enhanced the interest rate risk exposure of banks and other institutions: in certain jurisdictions, regulatory restrictions on lending have lead to a concentration of activities on 'safe' home mortgages and other real-estate-related lending where interest-induced valuation risks are particularly high [Benston et al. (1991)]. Government provision of cheap deposit insurance has effectively subsidized 'risk free' demand and savings deposits relative to other forms of finance. Moreover for a long time deposit rate regulation at a low level has induced banks and savings institutions to regard demand and savings deposits a cheap source of funds devoid of refinancing risks [Lessard and Modigliani (1975), Baltensperger and Dermine (1987)]. Some of the difficulties that depository institutions have experienced from interest rate risk might perhaps have been

\[\text{Note that with interest rate risk a nondiversifiable risk, the proper price of deposit insurance should exceed an actuarily fair assessment of the cost to the taxpayer. The proper price of such insurance must include a pure risk premium. In contrast, the actual provision of deposit insurance has involved prices even below actuarily fair cost assessments.}\]
avoided if banking regulation had taken proper account of the non-diversifiable nature of this risk and reoriented itself accordingly.

The Basle Committee's proposals follow the tradition. By imposing capital adequacy requirements for 'interest rate risks' in the narrow sense while neglecting interest-induced credit and counterparty risks, the proposed measures will encourage the shifting of interest rate risk to debtors or to 'the market', with little control over the credit risks that are thereby induced. No consideration seems to be given to the possibility of shifting interest rate risk to depositors through securitization. This is the more remarkable since a transfer of interest rate risk to depositors would represent the only possibility of shifting interest rate risk without at the same time creating a potential credit risk that is correlated with the interest rate risk. One only needs to recognize that an efficient risk allocation requires the insurance of depositors' liquidity needs without at the same time insulating them from interest rate risk.

Appendix: Proofs

Proof of Proposition 1. If the constraint (2b) holds with a strict inequality, then for given \( k_{01} \) and \( k_{02} \), the interdependence of \( \xi_1, \xi_2, \) and \( L \) is entirely determined by (2c). In this case, one must have \( L = 0 \) since otherwise, with \( \tilde{\theta}_2 \xi < \eta \), a small decrease in \( L \) would make it possible to raise \( c_1 \) as well as \( \tilde{c}_2 \). The constraint (2c) can then be rewritten as

\[
\tilde{\theta}_2 p \tilde{c}_1 + (1 - p) \tilde{c}_2 = \tilde{\theta}_2 \theta_1 k_{01} + \eta k_{02},
\]

so, by a standard dynamic-programming argument, one must have \( \tilde{c}_1 = c_1^*(\tilde{\theta}_2, \tilde{\theta}_2 \theta_1 k_{01} + \eta k_{02}) \) and \( \tilde{c}_2 = c_2^*(\tilde{\theta}_2, \tilde{\theta}_2 \theta_1 k_{01} + \eta k_{02}) < \theta_1 k_{01}/p \), or, equivalently, if \( \theta_2 > \theta^*_2 \).

Alternatively, if \( \tilde{\theta}_2 \leq \theta^*_2 \), one has \( c_1^*(\tilde{\theta}_2, \tilde{\theta}_2 \theta_1 k_{01} + \eta k_{02}) \geq \theta_1 k_{01}/p \), so the constraint (2b) must hold with equality. I claim that in this case \( \tilde{c}_1 = \theta_1 k_{01}/p \), \( \tilde{c}_2 = \eta k_{02}/(1 - p) \) and \( L = 0 \). For suppose that \( \tilde{c}_1 > \theta_1 k_{01}/p \). From (2b) and (2c), for \( \tilde{c}_1 \geq \theta_1 k_{01}/p \), the tradeoff between \( \tilde{c}_1 \) and \( \tilde{c}_2 \) is given by the equation

\[
\frac{\eta}{\epsilon} p \tilde{c}_1 + (1 - p) \tilde{c}_2 = \frac{\eta}{\epsilon} \theta_1 k_{01} + \eta k_{02}.
\]

For \( \tilde{c}_1 > \theta_1 k_{01}/p \) to be chosen one must therefore have \( u'(\theta_1 k_{01}/p) > (\eta/\epsilon) u'(\eta k_{02}/(1 - p)) \), or \( \eta/\epsilon < \theta^*_2 \). Since \( \tilde{\theta}_2 < \eta/\epsilon \) with probability one, \( \eta/\epsilon < \theta^*_2 \) would imply that

\[
\tilde{c}_1 = c_1^*\left(\frac{\eta}{\epsilon}, \frac{\eta}{\epsilon} \theta_1 k_{01} + \eta k_{02}\right), \quad \tilde{c}_2 = c_2^*\left(\frac{\eta}{\epsilon}, \frac{\eta}{\epsilon} \theta_1 k_{01} + \eta k_{02}\right).
\]
and

$$L = \frac{1}{\varepsilon} \left[ pc_1^\ast \left( \eta, \eta, \frac{\theta_1 k_{02} + \eta k_{02}}{\varepsilon} \right) - \theta_1 k_{01} \right] > 0$$

with probability one. But then a small increase in $k_{01}$ together with a small decrease in $k_{02}$, matched by a reduction in $L$ would raise both $\tilde{c}_1$ and $\tilde{c}_2$, contrary to the assumption that the initial allocation is optimal. The assumption that $\tilde{c}_1 > \theta_1 k_{01}/p$ (with positive probability) thus leads to a contradiction and must be false. For $\tilde{\theta}_2 \leq \tilde{\theta}_2^\ast$, it follows that $\tilde{c}_1 = \theta_1 k_{01}/p, \tilde{c}_2 = \eta k_{02}/(1-p), \tilde{L} = 0$, as claimed. Q.E.D.

Before giving the proof of Proposition 2, I state a lemma, which provides the key to the argument. The proof of the lemma is standard and is left to the reader.

**Lemma.** Assume that the function $c \rightarrow u'(c)c$ is decreasing. Then for all $q > 0$ and $b > 0$, $c_1^\ast(q,b) \geq b/q$ and $c_2^\ast(q,b) \leq b$ as $q \rightarrow 1$.

**Proof of Proposition 2.** If the constraint (8b) holds with a strict inequality, then by the same argument as in the proof of Proposition 1, for given $k_{01}$ and $k_{02}$, one must have $\tilde{L} = 0$, and the tradeoff between $\tilde{c}_1$ and $\tilde{c}_2$ must be given by (A.1). Given that $\tilde{\theta}_2 \geq 1$ with probability one, the lemma implies

$$\tilde{\theta}_2 c_1^\ast(\tilde{\theta}_2, \tilde{\theta}_2 \theta_1 k_{01} + \eta k_{02}) \geq c_2^\ast(\tilde{\theta}_2, \tilde{\theta}_2 \theta_1 k_{01} + \eta k_{02}),$$

so for the second-best welfare problem the incentive-compatibility constraint (8d) must be binding if (8b) holds with a strict inequality. Thus one must have $\tilde{c}_1 = \theta_1 k_{01} + \eta k_{02}/\tilde{\theta}_2$ and $\tilde{c}_2 = \tilde{\theta}_2 \theta_1 k_{01} + \eta k_{02}$ if $\theta_1 k_{01} + \eta k_{02}/\tilde{\theta}_2 < \theta_1 k_{01}/p$, or equivalently, if $\tilde{\theta}_2 > \tilde{\theta}_2 = \eta k_{02}/(1-p)\theta_1 k_{01}$.

For $\tilde{\theta}_2 \leq \tilde{\theta}_2$, one has $\theta_1 k_{01} + \eta k_{02}/\tilde{\theta}_2 \leq \theta_1 k_{01}/p$, so the constraint (8b) must hold with equality. The tradeoff between $\tilde{c}_1$ and $\tilde{c}_2$ is then again given by (A.2), with the added requirement that $p\tilde{c}_1 \geq \theta_1 k_{01}$. If $u'(\theta_1 k_{01}/p) \leq (\eta/\varepsilon)u'(\eta k_{02}/(1-p))$, it is clearly undesirable to have $\tilde{c}_1 \geq \theta_1 k_{01}/p$, so for $\tilde{\theta}_2 \leq \tilde{\theta}_2$, one must have $\tilde{c}_1 = \theta_1 k_{01}/p, \tilde{c}_2 = \eta k_{02}/(1-p)$, and $\tilde{L} = 0$, which completes the proof of statement (a).

In contrast, if $u'(\theta_1 k_{01}/p) > (\eta/\varepsilon)u'(\eta k_{02}/(1-p))$, one has

$$c_1^\ast \left( \eta, \eta, \frac{\theta_1 k_{01} + \eta k_{02}}{\varepsilon} \right) > \frac{\theta_1 k_{01}}{p},$$

so, for $\tilde{\theta}_2 \leq \tilde{\theta}_2$, it is desirable to have

$$\tilde{c}_1 = \min \left[ c_1^\ast \left( \eta, \eta, \frac{\theta_1 k_{01} + \eta k_{02}}{\varepsilon} \right), \frac{(\eta/\varepsilon)\theta_1 k_{01} + \eta k_{02}}{(1-p)\tilde{\theta}_2 + p(\eta/\varepsilon)} \right],$$
\[ \tilde{c}_2 = \max \left[ c_2^*(\eta, \eta \theta_1 k_{01}, \eta k_{02}), \tilde{\theta}_2 \tilde{c}_1 \right], \]

going as close to \((c_2^*(\eta, \eta \theta_1 k_{01}, \eta k_{02}), (c_2^*(\eta, \eta \theta_1 k_{01}, \eta k_{02}))\) as the incentive-compatibility constraint \((8d)\) permits. Statement \((b)\) follows immediately. Q.E.D.

**Proof of Proposition 3.** The first statement of the proposition has been proved in the text, so it suffices to prove the last statement. Assume \((13)\) and suppose that, contrary to the proposition, one has \(\tilde{\varepsilon} \geq \theta_1\), i.e.

\[ \theta_1 u'(\theta_1 k_{01}/p) \leq \eta u'(\eta k_{02}/(1 - p)). \]

Then clearly \(\theta_1 k_{01}/p \geq c_2^*(\eta/\theta_1, \eta k_0), \eta k_{02}/(1 - p) \leq c_2^*(\eta/\theta_1, \eta k_0)\) and hence \(\tilde{\theta}_2 \leq \tilde{\theta}_2\), where \(\tilde{\theta}_2\), where \(\tilde{\theta}_2 := p\eta k_{02}/(1 - p)\theta_1 k_{01}\) as in \((11)\). Since \((13)\) implies \(\tilde{\theta}_2 \theta_1 \leq \eta\), it follows that \(\tilde{\theta}_2 \theta_1 \leq \eta\) for \(\theta_2 \in [\tilde{\theta}_2, \tilde{\theta}_2]\), and hence that

\[ \int_{\theta_2}^{\tilde{\theta}_2} (\theta_2 \theta_1 - \eta) \phi(\theta_2) \, dF(\theta_2) \leq 0, \]

where \(\phi(\theta_2) := pu'(\theta_1 k_{01} + \eta k_{02}/\theta_2) / \theta_2 + (1 - p)u'(\theta_2 \theta_1 k_{01} + \eta k_{02})\) for any \(\theta_2\). Given that \(u'(c)c\) is decreasing in \(c\), one easily verifies that \(\phi(\theta_2)\) is decreasing in \(\theta_2\), and hence that \((13)\) and \(F(\tilde{\theta}_2) < 1\) imply

\[ \int_{\theta_2}^{\tilde{\theta}_2} (\theta_2 \theta_1 - \eta) \phi(\theta_2) \, dF(\theta_2) < 0. \]

Upon combining \((A.3), (A.4), \) and \((A.5)\), one obtains

\[ [\theta_1 u'(\theta_1 k_{01}/p) - \eta u'(\eta k_{02}/(1 - p))] F(\tilde{\theta}_2) + \int_{\theta_2}^{\tilde{\theta}_2} (\theta_2 \theta_1 - \eta) \phi(\theta_2) \, dF(\theta_2) < 0. \]

But then the first-order condition for \(k_{01}', \; k_{02}'\) implies \(k_{01}' = 0, \; k_{02}' = k_0\), contrary to the above inequality \(\tilde{\theta}_2 \leq \tilde{\theta}_2\). The assumption that \(\tilde{\varepsilon} \geq \theta_1\) has thus led to a contradiction and must be false. If \((13)\) holds, one must have \(\tilde{\varepsilon} < \theta_1\) and the interval \((\tilde{\varepsilon}, \theta_1)\) must be nonempty. Q.E.D.

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